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# **MODELLING AND ANALYSIS OF SERIAL SUPPLY CHAINS IN UNCERTAIN ENVIRONMENTS**

. by  
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**December, 1997**

## ***SUMMARY***

The subject matter of this thesis is the modelling and analysis of serial supply chain (SC) behaviour in an uncertain environment. Main sources of uncertainty inherent in a serial SC and its environment have been identified, including customer demand, external supplier reliability, supply along the chain and lead times. A new approach to modelling and treating these uncertainties based on fuzzy sets theory has been proposed. It has been shown that the application of fuzzy sets is useful in cases where there is lack of available data about SC parameters, lack of certainty in data or when data does not exist.

A new original approach to SC analysis has been developed and implemented using C++ programming language. In this approach, two types of models have been combined: (1) SC fuzzy, analytical models and (2) SC simulation models. The new SC fuzzy analytical models have been developed which treat different SC uncertainties simultaneously. In these models, order-up-to levels for all inventories along an SC are determined in such a way as to minimise total possible inventory costs over a given time. Two SC control strategies which take into consideration different uncertainties and reflect different levels of SC integration have been proposed and built into the SC fuzzy models, including: (1) fully decentralised control, and (2) a new developed strategy of partially coordinated control. The aim of the new SC simulation models developed is to evaluate SC performance achieved by applying order-up-to levels and replenishment quantities recommended by the fuzzy models.

The SC fuzzy and simulation models, working in a coordinated manner, have been used to gain further insight into SC dynamic behaviour and its performance in an uncertain environment, and to enhance decision making on SC control parameters in the presence of uncertainty. The application of the developed SC tool in the various analyses has been demonstrated, including: (1) quantification and comparison of SC performance under different control strategies, such as decentralised and partially coordinated control, (2) quantification of the effects of changing uncertainty in SC data (e.g., customer demand) on SC behaviour and its performance, (3) analysis of the effects of uncertainty in external supplier reliability and investigation of the ways of making an SC less vulnerable to this uncertainty, and (4) application of two new procedures for one-site and multi-site compensation which have been developed to compensate for the negative effects of uncertainty in external supply.

## ***ACKNOWLEDGEMENTS***

I would like to express my sincere gratitude to Rajat Roy for his supervision and guidance throughout my research work and also for the provision of a grant which made this research possible. I wish to thank to the members of the Simulation Group within Warwick Manufacturing Group, University of Warwick, for creating a very pleasant working atmosphere. I gratefully acknowledge the support of the Mihajlo Pupin Institute in Belgrade, particularly the Automation Laboratory, I have been working with.



## ***DECLARATION***

I declare that the work described in this Ph.D. thesis, unless otherwise acknowledged in the text, is my own work and has not been previously submitted for any academic degree.

Signed:

*Dobrica Petrovic*  
Dobrica Petrovic

17 December, 1997

*“The closer one looks at a real-world problem,  
the fuzzier becomes its solution.”*

(Lotfi Zadeh, 1973)

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## NOTATION

- Time characteristics:

- $T_h$  - time horizon,  
 $T$  - number of the unit time intervals within  $T_h$ ,  
 $t$  - index of a particular time unit within  $T_h$ ,  $t = 1, \dots, T$ .

- Inventory characteristics:

- $N$  - number of inventories,  
 $n$  - superscript used to identify a particular inventory,  $n = 1, \dots, N$ ;  
 starting from the superscript for the end-product inventory 1, to  
 the superscript for the raw material inventory  $N$ ,  
 $R^n$  - number of the time units within the review period of inventory  $n$ ,  
 $n = 1, \dots, N$ ,  
 $k^n$  - index of a particular review/replenishment period of inventory  $n$   
 within  $T_h$ ,  $n = 1, \dots, N$ ,  $k^n = 1, \dots, K^n$ ,  
 $c_h^n$  - holding cost per one item per unit time interval at inventory  $n$ ,  
 $n = 1, \dots, N$ ,  
 $c_s^n$  - shortage cost per one item at inventory  $n$ ,  $n = 1, \dots, N$ ,  
 $FR_{h,k^n}^n$  - holding cost incurred at inventory  $n$  during replenishment period  $k^n$ ,  
 $n = 1, \dots, N$ ,  $k^n = 1, \dots, K^n$ , a discrete fuzzy set,  
 $FR'_{h,k^n}^n$  - possible value of  $FR_{h,k^n}^n$ ,  $n = 1, \dots, N$ ,  $k^n = 1, \dots, K^n$ ,  
 $\mu_{FR_{h,k^n}^n}$  - possibility distribution of  $FR_{h,k^n}^n$ ,  $n = 1, \dots, N$ ,  $k^n = 1, \dots, K^n$ ,

- $FR_{s,k^n}^n$  - shortage cost incurred at inventory  $n$  during replenishment period  $k^n$ ,  
 $n = 1, \dots, N, k^n = 1, \dots, K^n$ , a discrete fuzzy set,  
 $FR'_{s,k^n}^n$  - possible value of  $FR_{s,k^n}^n, n = 1, \dots, N, k^n = 1, \dots, K^n$ ,  
 $\mu_{FR_{s,k^n}^n}$  - possibility distribution of  $FR_{s,k^n}^n, n = 1, \dots, N, k^n = 1, \dots, K^n$ ,  
 $FR_{k^n}^n$  - possible total cost incurred at inventory  $n$  during replenishment period  
 $k^n, n = 1, \dots, N, k^n = 1, \dots, K^n$ , when uncertainty in demand is  
taken into consideration,  
 $SQ^n$  - supply from facility  $n+1$  to facility  $n, n = 1, \dots, N-1$ , a discrete fuzzy set,  
 $\mu_{SQ^n}(s_{u^n}^n)$  - possibility distribution of  $SQ^n$ , where  $s_{u^n}^n$  represents the undelivered  
quantity of items,  $n = 1, \dots, N-1, u^n = 1, \dots, U^n$ ,  
 $FRS_{k^n}^n$  - possible total cost incurred at inventory  $n$  during replenishment period  
 $k^n, n = 1, \dots, N-1, k^n = 1, \dots, K^n$ , when uncertainties in demand from  
facility  $n-1$  and supply from facility  $n+1$  are taken into consideration,  
 $S_{d,k^n}^n$  - order-up-to level for inventory  $n$  for review period  $k^n, n = 1, \dots, N$ ,  
 $k^n = 1, \dots, K^n$ , when uncertainty in demand is taken into consideration,  
 $S_{p,k^n}^n$  - order-up-to level for inventory  $n$  for review period  $k^n, n = 1, \dots, N$ ,  
 $k^n = 1, \dots, K^n$ , when uncertainties in demand and supply are taken into  
consideration,  
 $S_{l,k^n}^n$  - order-up-to level for inventory  $n$  for review period  $k^n, n = 1, \dots, N$ ,  
 $k^n = 1, \dots, K^n$ , when uncertainties in demand and the lead time to  
inventory  $n$  are taken into consideration,  
 $S_{k^n}^n$  - order-up-to level for inventory  $n$  for review period  $k^n, n = 1, \dots, N$ ,  
 $k^n = 1, \dots, K^n$ , determined in either of the ways listed above.

• Lead times:

- $L^n$  - lead time to inventory  $n, n = 1, \dots, N$ , in time units; a real number or a  
discrete fuzzy set,



$\mu_{L^n}(l_{m^n}^n)$  - possibility distribution of  $L^n$ , where  $l_{m^n}^n$  is the number of time units within lead time to inventory  $n$ ,  $n = 1, \dots, N$ ,  $m^n = 1, \dots, M^n$ .

• Demand characteristics:

$D_t$  - customer demand in time unit  $t$ ,  $t = 1, \dots, T+L^1$ , a discrete fuzzy set

$\mathcal{D}_t$  - domain of  $D_t$ ,  $t = 1, \dots, T+L^1$ ,

$\mu_{D_t}(d_t)$  - possibility distribution of  $D_t$ ,  $d_t \in \mathcal{D}_t$ ,  $t = 1, \dots, T+L^1$ ,

$d_t'$  - actual customer demand recorded during time unit  $t$ ,  $d_t' \in \mathcal{D}_t$ ,  
 $t = 1, \dots, T$ ,

$D_t^n$  - demand from facility  $n-1$  on facility  $n$  in time unit  $t$ ,  $n = 1, \dots, N$ ,  
 $t = 1, \dots, T+L^n$ , and  $D_t^1 = D_t$ , a discrete fuzzy set,

$\mathcal{D}_t^n$  - domain of  $D_t^n$ ,  $n = 1, \dots, N$ ,  $t = 1, \dots, T+L^n$ , and  $\mathcal{D}_t^1 = \mathcal{D}_t$ ,

$\mu_{D_t^n}(d_t^n)$  - possibility distribution of  $D_t^n$ ,  $d_t^n \in \mathcal{D}_t^n$ ,  $n = 1, \dots, N$ ,  $t = 1, \dots, T+L^n$ ,  
and  $\mu_{D_t^1}(d_t^1) = \mu_{D_t}(d_t)$ ,

$DR_{k^n}^n$  - demand from facility  $n-1$  on facility  $n$  during replenishment period  $k^n$ ,  
 $n = 1, \dots, N$ ,  $k^n = 1, \dots, K^n$ , a discrete fuzzy set,

$\mathcal{DR}_{k^n}^n$  - domain of  $DR_{k^n}^n$ ,  $n = 1, \dots, N$ ,  $k^n = 1, \dots, K^n$ ,

$\mu_{DR_{k^n}^n}(dr_{k^n}^n)$  - possibility distribution of  $DR_{k^n}^n$ ,  $dr_{k^n}^n \in \mathcal{DR}_{k^n}^n$ ,  $n = 1, \dots, N$ ,  
 $k^n = 1, \dots, K^n$ ,

$DL^n$  - demand from facility  $n-1$  on facility  $n$  during the lead time,  $n = 1, \dots, N$ ,  
a discrete fuzzy set,

$\mathcal{DL}^n$  - domain of  $DL^n$ ,  $n = 1, \dots, N$ ,

$\mu_{DL^n}(dl^n)$  - possibility distribution of  $DL^n$ ,  $dl^n \in \mathcal{DL}^n$ ,  $n = 1, \dots, N$ .

$P_{k^n}^n$  - length of replenishment period  $k^n$  of inventory  $n$ ,  $n = 1, \dots, N$ ,

$k^n = 1, \dots, K^n$ , a discrete fuzzy set,

$\mu_{P_{k^n}^n}(p_{i^n, k^n}^n)$  - possibility distribution of  $P_{k^n}^n$ , where  $p_{i^n, k^n}^n$  is the possible length of  
replenishment period  $k^n$  of inventory  $n$ ,  $n = 1, \dots, N$ ,  $k^n = 1, \dots, K^n$ ,

- $i^n = 1, \dots, I^n$ ,  
 $DR_{i^n, k^n}^n$  - demand on inventory  $n$  during replenishment period  $k^n$ , when the length of replenishment period is  $p_{i^n, k^n}^n$ ,  $n = 1, \dots, N$ ,  $k^n = 1, \dots, K^n$ ,  
 $i^n = 1, \dots, I^n$ , a discrete fuzzy set,  
 $\mu_{DR_{i^n, k^n}^n}(dr_{k^n}^n)$  - possibility distribution of  $DR_{i^n, k^n}^n$ ,  $dr_{k^n}^n \in \mathcal{DR}_{k^n}^n$ ,  $n = 1, \dots, N$ ,  
 $k^n = 1, \dots, K^n$ ,  $i^n = 1, \dots, I^n$ ,  
 $DL_{m^n}^n$  - demand on inventory  $n$  during lead time of  $l_{m^n}^n$  time units,  $n = 1, \dots, N$ ,  
 $m^n = 1, \dots, M^n$ , a discrete fuzzy set,  
 $\mu_{DL_{m^n}^n}(dl^n)$  - possibility distribution of  $DL_{m^n}^n$ ,  $dl^n \in \mathcal{DL}^n$ ,  $n = 1, \dots, N$ ,  $m^n = 1, \dots, M^n$ .

- External supply characteristics:

- $SP$  - percentage of raw material order, delivered by the external supplier, a discrete fuzzy set,  
 $\mu_{SP}(sp)$  - possibility distribution of  $SP$ ,  $sp \in [0, 100]$ ,  $sp$  is a possible value of the percentage of raw material order, delivered by the external supplier,  
 $sp_t'$  - actual percentage of raw material order, delivered at the beginning of time interval  $t$ ,  $t = 1, \dots, T$ .

- State and performance of inventory  $n$  during  $t$ ,  $n = 1, \dots, N$ ,  $t = 1, \dots, T$ :

- $I_t^n$  - stock level,  
 $Q_t^n$  - quantity of items ready for delivery from facility  $n+1$  to facility  $n$ ,  
 $BS_t^n$  - part of backlogged quantity ready for delivery from facility  $n$  to facility  $n-1$ ,

- $BR_t^n$  - backlogged quantity not available for delivery from facility  $n$  to facility  $n-1$ ,
- $O_{d,t}^n$  - quantity ordered from facility  $n$  to facility  $n+1$ , when uncertainty in demand is taken into consideration,
- $O_{p,t}^n$  - quantity ordered from facility  $n$  to facility  $n+1$ , when uncertainties in demand and supply from facility  $n+1$  to facility  $n$  are taken into consideration,
- $O_{l,t}^n$  - quantity ordered from facility  $n$  to facility  $n+1$ , when uncertainties in demand and the lead time to inventory  $n$  are taken into consideration,
- $O_t^n$  - quantity ordered from facility  $n$  to facility  $n+1$ , calculated in either of the ways listed above,
- $repl_t^n = \begin{cases} 1, & \text{if inventory } n \text{ expects delivery from inventory } n+1 \\ 0, & \text{otherwise} \end{cases},$
- $rew_t^n$  - counter of the current time interval  $t$  within the review period of inventory  $n$ ,  $rew_t^n \in [0, \dots, R^n-1]$ ,
- $F_{h,t}^n$  - cumulative holding cost charged on inventory  $n$  during time units  $1, \dots, t$ ,
- $F_{s,t}^n$  - cumulative shortage cost charged on inventory  $n$  for failure to meet demand during time units  $1, \dots, t$ ,
- $F_t^n$  - cumulative total cost of inventory  $n$  incurred during time units  $1, \dots, t$ ,
- $NS_t^n$  - cumulative number of shortages at inventory  $n$  during time units  $1, \dots, t$ .

- Performance of inventory  $n$ ,  $n = 1, \dots, N$  during  $T_h$ :

- $FR_{T_h}^n$  - fill rate,
- $\Delta FR_{T_h}^n$  - difference between the fill rates achieved with and without an unit stock increase,

$F_{T_h}^n$	- total cost,
$FD_{T_h}^n$	- total cost per item demanded from inventory $n$ ,
$FH_{T_h}^n$	- total holding cost,
$FHD_{T_h}^n$	- total holding cost per item demanded from inventory $n$ .

- SC performance during  $T_h$ :

$FR_{T_h}$	- SC fill rate,
$F_{T_h}$	- SC total cost,
$FD_{T_h}$	- SC total cost per end-product demanded,
$FH_{T_h}$	- SC total holding cost,
$FHD_{T_h}$	- SC total holding cost per end-product demanded.

# **CHAPTER 1**

## **INTRODUCTION**

Recent years have seen an acceleration of interest in the analysis, management and control of supply chains (SCs). More and more production/manufacturing, distribution and retail organisations are addressing the question of how to develop an effective SC as an important factor in improving their productivity and delivery performance.

There are a few trends that characterise a modern environment in which an SC operates, such as: (1) a growing demand for ever-higher levels of service and quality, (2) globalisation of industry; this means that materials are sourced world-wide and manufacturing and product sales take place at many locations in different parts of the globe, (3) importance of time related issues and quick responsiveness due to shortened product life-cycles or requirements for just-in-time deliveries. Rapid changes have been introduced in manufacturing in the last few decades, including flexible manufacturing systems (FMS), new approaches to inventory control based on material requirements planning (MRP) and just-in-time methods, emphasis on total quality management (TQM), and so on. Further, continuous development of information technology and telecommunication brings new applications. Among them, Electronic Data Interchange (EDI) for exchange of information in standardised forms between data centres of different organisations is increasingly used to facilitate the flow of information along an SC. All these factors have a strong impact on the philosophy, perspectives and concepts of SC management and control.

SC management and control activities span from the procurement of raw materials through production/manufacturing to distribution of end-products to customers. Efficient SC management and control require an interdisciplinary approach



drawing on various aspects of manufacturing, economics, marketing, logistics, organisational behaviour. SC management and control differ significantly from conventional material handling and production/manufacturing control. A new approach to SC integration, not simply interface between various SC segments, is recognised as a key issue in SC management and control (Jones, 1989, Maloni and Benton, 1997). The paradigm of the integrated SC denotes an SC that is managed as a single system, where both material flows and the flows of related planning and control information are considered as an integrated process, rather than, as it was often the case in the past, managing the SC segments as a series of independent activities (Schmidt, 1993). Although "natural" conflicts between different segments in the SC may be inevitable, effective SC management and control call for a focus on the overall objectives rather than simply a set of local objectives of constituent parts of the SC.

In order to put the research concerning this thesis in the context, the general concept of the supply chain, SC management and control problems and complexities associated with them are introduced first in this chapter. The importance of treating uncertainty inherent in an SC and its environment is then emphasised, before stating the objectives of the research.

## **1.1. The concept of the supply chain**

An SC is generally viewed as a network of facilities that performs the procurement of raw materials, their transformation into intermediate and end-products, and distribution and selling of the end-products to customers. The facilities in the SC network include raw material inventories, production facilities, in-process and end-product inventories and distribution facilities that encompass warehouses and retailers. The links in the SC network represent two flows in opposite directions: the flow of orders from customers towards suppliers, and the flow of materials, including raw materials, components, manufacturing parts and packaging materials, which is directed from the suppliers



towards customers. The main processes along an SC, the flow of orders and the flow of materials that link these processes, are depicted in Figure 1.1.

SC management and control problems are concerned with the movement of materials and goods over time and space. Decisions made at any node in an SC network have an impact throughout the entire SC and influence SC behavior and performance. Thus, the main objective of SC management and control is to link the distribution, manufacturing and procurement activities in such a way as to provide customers with products of competitive quality, price, and variety and to service customers at a higher level and yet at a lower cost.

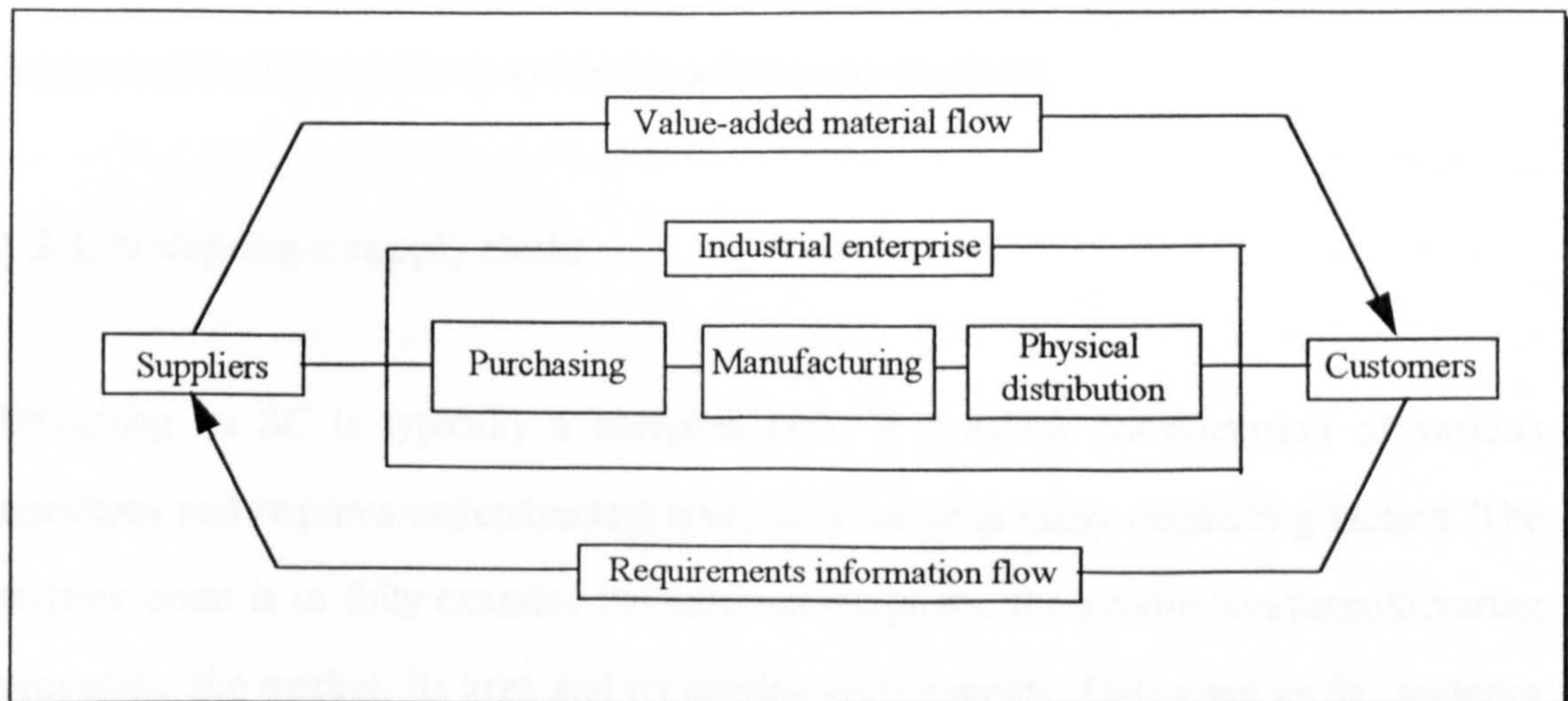


Figure 1.1. Processes along an SC, (Christopher, 1992)

The concept of the SC is actually not new, but there is now a greater emphasis on its complexity and importance (Hicks, 1997a, 1997b, 1997c). Gattorna and Walters (1996) support the opinion that the theory on which SC management relies is the theory of integrated logistics and it has become just a preferred name for the "actualisation of integrated logistics theory". It combines such traditionally distinct activities as forecasting, purchasing, manufacturing, distribution, sales and marketing into one continuous process of interrelated activities. Christopher (1992) considers the



concept of SC management as an extension of logistics, where the logic of integration is extended outside the boundaries of an organisation to include "the rest of the world": suppliers and customers.

## **1.2. Complexity of supply chain management and control**

SC management and control are faced with various complexities. Complexity appears in designing an SC, its integration, variety of SC management and control problems that have to be addressed, defining SC performance measures, SC dynamics, uncertainty inherent in an SC and its environment, and so on. The characteristics of these complexities are discussed in the subsequent sections.

### **1.2.1. Designing a supply chain**

Designing an SC is typically a complex task. It involves consideration of various questions and requires understanding trade-offs between many conflicting factors. The starting point is to fully examine the external suppliers, the production/manufacturing processes, the market, its area and its service requirements. Designing an SC includes designing a base of suppliers, setting a global strategy for manufacturing, designing a global distribution network and sometimes designing a global network for spare parts and repair (Arntzen, et al., 1995). Designing a base of suppliers encompasses tasks such as selection of suppliers and determination of which suppliers should supply each production facility. Setting a global strategy for production/manufacturing is focused on deciding on how many plants need to be involved, where to locate them, what technologies and capacities each should have, which plants should produce/manufacture certain types of products and at what volume, etc. Designing a global distribution network addresses the questions such as how many distribution centres there should be, where they should be located, what policies of resupply and

capacity each should have, which distribution centres should serve certain customers, what are the transit times and costs of various transportation modes and others. If a network for spare parts and maintenance is included then the questions arise on general issues concerning preventive/corrective maintenance policy, how many repair centres there should be, which products should each repair, and what are the ways of shipping spare parts between plants, vendors and customers. Designing a world-wide SC becomes even more complex, bringing into attention issues such as export regulations, duty rates, locations of tax havens, locations and availability of inexpensive skilled labour and many other economic and legal aspects.

A designed SC can have various structures, from a serial to a complex network structure. A wide variety of links may exist between SC actors indicating their mutual arrangements and inter-relationships. As a natural consequence of the trend toward specialisation in industry and focusing on a smaller range of activities, the number of organisations involved in an SC increases. Therefore, an SC may link from several to thousands of organisations, forming a large scale system.

### **1.2.2. Supply chain integration**

Different segments within an SC usually have different objectives, which are based on those aspects of their activities that lead to their best performance. However, some of these objectives may conflict with overall SC objectives and with objectives of other SC segments (Gattorna and Walters, 1996). For example, a large product range and frequent product changes may be objectives for marketing, that can widen its offer to customers and increase the sales. On the other hand, a large product range and frequent product changes have disadvantages for procurement and materials management and production; these objectives cause larger stocks of raw materials and increase in production costs. A production/manufacturing segment of an SC tends to minimise its cost by long production runs, reducing set-ups and product standardisation costs; these objectives lead to a number of negative consequences,

such as a build up of end-product inventory, increased need for warehousing space and tying up of capital.

Good reasons exist, both practical and theoretical, to study and to manage the functional areas of purchasing, manufacturing, distribution and sales as a single entity. SC integration is essential because: (1) the cost of quality and service levels which are perceived by customers are the result of the cost, quality and service levels of all the SC actors and their operations condition the success of the entire SC and (2) there is a common belief that managing the entire SC as a single entity can significantly improve cost and service performance (Forza, 1996).

Although, the benefits of taking a system point of view in SC management and control are apparent, in practice it is difficult to achieve SC integration. Standard organisational barriers, functional divisions and hierarchy are the main obstacles to SC integration.

Stevens (1989) recognises four stages in SC integration, which are illustrated in Figure 1.2. In stage 1, different activities along an SC are planned and controlled independently; responsibility for them is distributed among separate departments within a company. In stage 2, there is a limited degree of integration between adjacent activities, for example, distribution and inventory management, purchasing and material control; still, distribution is effectively decoupled from manufacturing. The natural next step is stage 3 that requires full internal integration; management and control of inward goods are integrated with management and control of outward flow of goods to customers, and typically all the activities of an SC under the control of one company are internally integrated. Full SC integration is achieved in stage 4, where the scope of management and control is extended outside a company to embrace customer requirements, on one side, and mutual support and cooperation with the suppliers, on the other side.

It is important to emphasize that these stages in SC integration are followed by the corresponding stages of information integration; from separate information systems for each functional area in an SC in stage 1, to a fully integrated information system, in



stage 4, which contains all relevant data about the whole SC. Information technology is the force behind the improvement of the information flow and allows sharing of information between all SC segments.

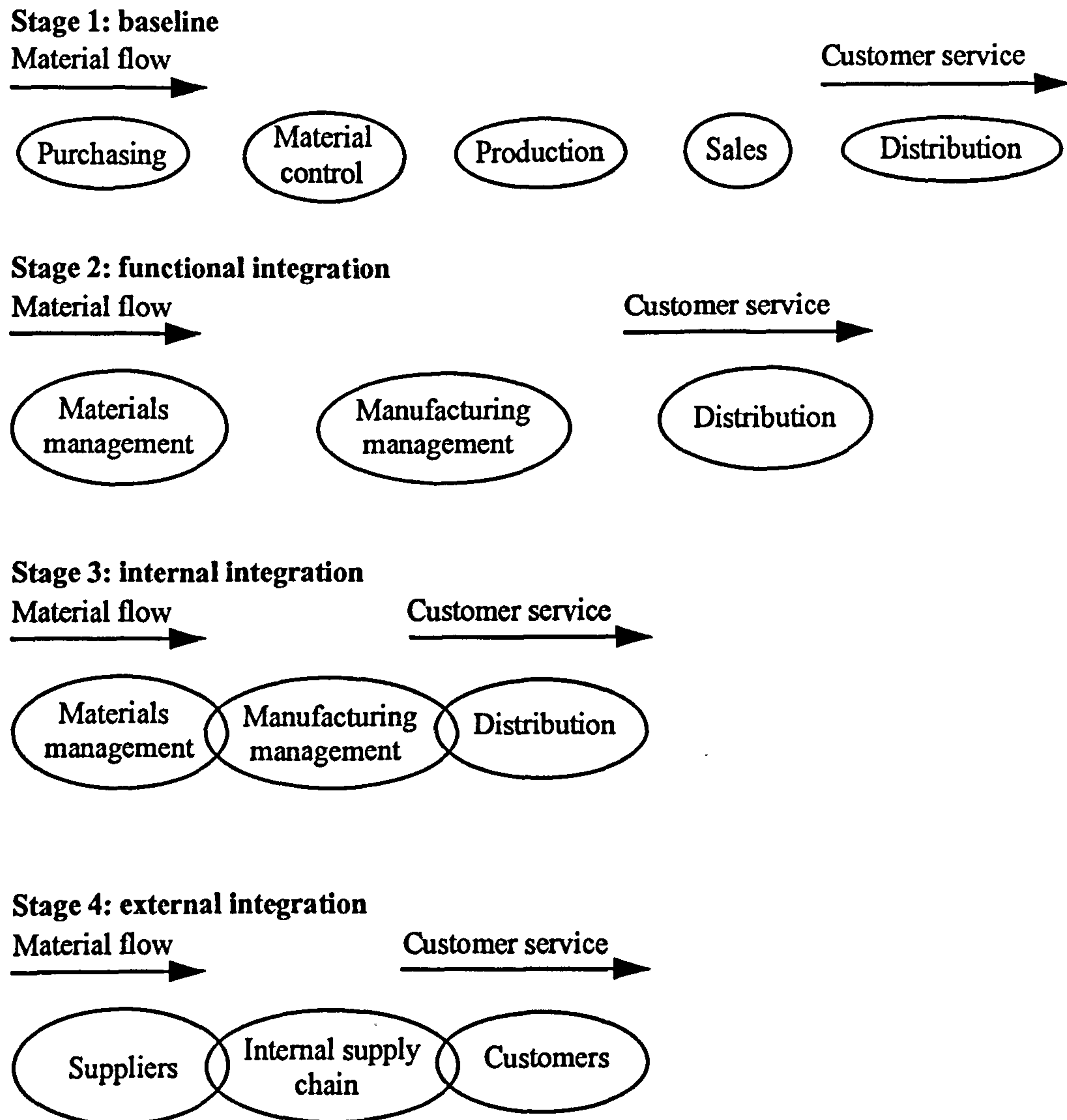


Figure 1.2. Stages in achieving an integrated SC, (Stevens, 1989)

### 1.2.3. Supply chain management and control problems

There is a wide spectrum of SC management and control problems. They can be grouped into three levels with respect to the issues they deal with: strategic, tactical and operational level (Gattorna and Walters, 1996). At the strategic level the issues dealt with are focused on development of objectives and policies for an SC. The questions to be answered are, for example, what market segments should be served, to what extent, what are the prices to be charged, what is the total capital investment required, how many facilities of each type are required and where, what are their main characteristics (location, products handled, capacities, etc.), what is the necessary level of their vertical, horizontal and lateral interrelations, which facilities in an SC should be added, expanded or closed and when, and others. Issues considered at the tactical level are directed to finding the means by which the strategic objectives can be effectively realised. Questions to be solved at the tactical level are, for example, what products should be produced, where and in what quantities, what supply sources should be used and how, what products should be held in an inventory, where and in what quantities, what transportation arrangements should be made for moving materials along an SC, etc. Issues raised at the operational level are devoted to efficient short-term operations of an SC. Operational questions are, for example, where each customer order should be shipped from, what transportation rates and routes should be used to move particular products, what production plans are needed to provide the necessary inventories in the right place and at the right time, etc. These three levels of SC management and control cover different time horizons (Vidal and Goetschalckx, 1997). The strategic level considers usually time horizons of several years, while the operational level uses time units of an hour, day or week. The time horizons treated at the tactical level fall between these two time horizons.

Each of the questions mentioned above is a complex problem on its own. In addition, there are interactions and inter-dependencies among these problems, that bring even more complexity in SC management and control.

#### **1.2.4. Supply chain performance**

What must be kept in mind in choosing adequate and good performance measures is that the basic function of an SC is to provide service to customers. There is an extensive range of issues that characterise customer service (Christopher, 1992), such as: (1) order cycle time, i.e., the elapsed time from customer order to delivery, (2) stock availability expressed as the percentage of customer demand that can be met from available inventory, (3) frequency of delivery, (4) delivery reliability, expressed as the proportion of total customer orders delivered on time, (5) order completeness, expressed as the proportion of orders that are delivered complete, i.e., with no back-orders or part shipments, (6) documentation quality, related to the error rate on invoices, delivery notes and other means of communication with customers, (7) technical support, concerning support provided to customers after the sale, and so on. Customer service needs vary substantially from segment to segment in the market and even from customer to customer. Also, different industries and even companies in the same industry give different preference and priority to each of these customer service issues.

Clearly, to satisfy a target service level and customer needs, an SC will utilise certain amount of resources, comprising financial, human and physical resources. The objective of SC management and control is to minimise the total amount of resources required to provide the target level of customer service. Therefore, SC performance measures are related to amount of resources required to provide the target service level to customers.

#### **1.2.5. Supply chain dynamics**

An SC is a dynamic system, and therefore its behaviour and performance have to be considered over time. For example, periodic economic swings are inevitable and

generate disturbances in an environment in which an SC operates. They are reflected in changes in customer demand and supplier reliability. It is shown in both theory (Forrester, 1961) and practice (e.g., Houlihan, 1987) that they result in inefficiency of SC operations. Of particular interest is to analyse SC dynamic behaviour in response to changes in the SC environment. Small disturbances in one part of the SC can very quickly become magnified as the effect spreads along the chain, due to natural inertia of an SC itself and its decision making mechanism, time delays in transferring information from one to another part of the SC and restricted visibility that purchasing and production/manufacturing have on customer demand.

Houlihan (1987) describes these effects apparent in practice as follows. Let us consider an increase in customer demand that produces shortages or, an inventory stock below a target level at some point in an SC. The reaction of management to any threat of shortage is local protection, that leads to over-ordering. Additionally, most methods for order forecasting use data about orders placed in the past. Hence, this surge in ordering has an impact on the new forecast. Similarly, inventory control logic dictates that unreliable supply should be compensated for by additional inventory investment. This causes further demand and order amplification along the SC. In these ways, aggregation and accumulation of changes along the SC dynamically create the illusion of massive swings, even in the case of small customer demand or supply variations. It contributes to a poor SC performance, inadequate customer service and excessive inventory costs.

Making an SC less vulnerable to external changes and smoothing SC reaction to demand and supply variations are the goals of modern SC management and control. However, it must be emphasised that they are complex and difficult problems to solve.

#### **1.2.6. Uncertainty in a supply chain**

An important and real cause for complexity of SC management and control is the presence of uncertainties within environments in which SCs operate. Three distinct



sources of uncertainty can be identified (Davis, 1993, Braithwaite, 1987): suppliers, production/manufacturing processes and customers. Uncertainty in supply can be caused by uncertainty in suppliers' own production/manufacturing or in transportation, while uncertainty in a production/manufacturing process can be caused by random failures of production machines or repair times, etc. It may be uncertain when customer demand will occur and, once it happens, what the quantity of end-products ordered will be. In addition, orders may be cancelled. Uncertainty propagates through an SC from both ends. It propagates from external suppliers side toward the downstream SC part, i.e., selling points, and typically manifests in delays in delivery or in delivering less quantities than ordered. Uncertainty also propagates in the opposite direction, from customer demand side toward the upstream SC part, (i.e., raw material inventories), and it reflects in uncertainty in orders placed from one to another SC part.

Uncertainty plagues SC performance. It is essential for effective SC management and control that each source of uncertainty is understood and that each uncertainty is addressed, properly described and measured. It is important to understand the system wide impact of different sources of uncertainty and how they combine to affect delivery to customers.

In practice, the impact of uncertainty can be reduced by some strategic initiatives, such as choosing good suppliers or encouraging reliable delivery, investigation of more reliable transportation modes, changing product design to stabilise manufacturing processes, or redesign the whole SC structure (Davis, 1993). However, not all sources of uncertainty can be eliminated in these ways. They have to be taken into account at all levels of decision making along the SC. In particular, it is important to consider the sources of uncertainty in determination of adequate inventory stocking policies to apply along the SC.

### 1.3. Inventory management in a supply chain

The principal roles of inventory, in general, and in various segments of industry, such as marketing, production, purchasing or finance, have been explained in the literature by many authors (e.g., Ballou, 1978, Smith, 1989). Inventory management has an important role in the overall SC management. Efficient and effective management of SC inventories can significantly improve service provided to customers (Lee and Billington, 1992). Developments in information technology, such as Electronic Data Interchange (EDI) and Electronic Point of Sale (EPOS), and acceptance of just-in-time concepts and computer based manufacturing methods also have significant impact on inventory control, leading to the reduction of stock levels held along an SC.

Linking inventories in an SC impose different management and control problems in comparison with management and control of an isolated inventory. Figure 1.3. illustrates the way in which goods flow through an SC from suppliers to customers. Inventories stored at different points in the SC have a different impact on cost and service performance of the chain (Lee and Billington, 1993). For example, the unit cost of holding stock depends on the value of goods stored and consequently, it depends on the place of the inventory in the chain. The unit cost is higher for finished products and lower for raw materials. The degree of flexibility that can be associated with an inventory also depends on its place in the chain. More flexibility may be achieved by storing raw material, because it may be turned into different end-products. Inventories at different places along the SC have different levels of responsiveness. End-products can be shipped to customers without delay and, therefore, end-product inventories have the highest responsiveness, whereas a transformation of materials into end-products takes a lead time prior to shipment to customers. All these issues have to be considered in deciding where to position inventories along the SC and how much to stock at each location.

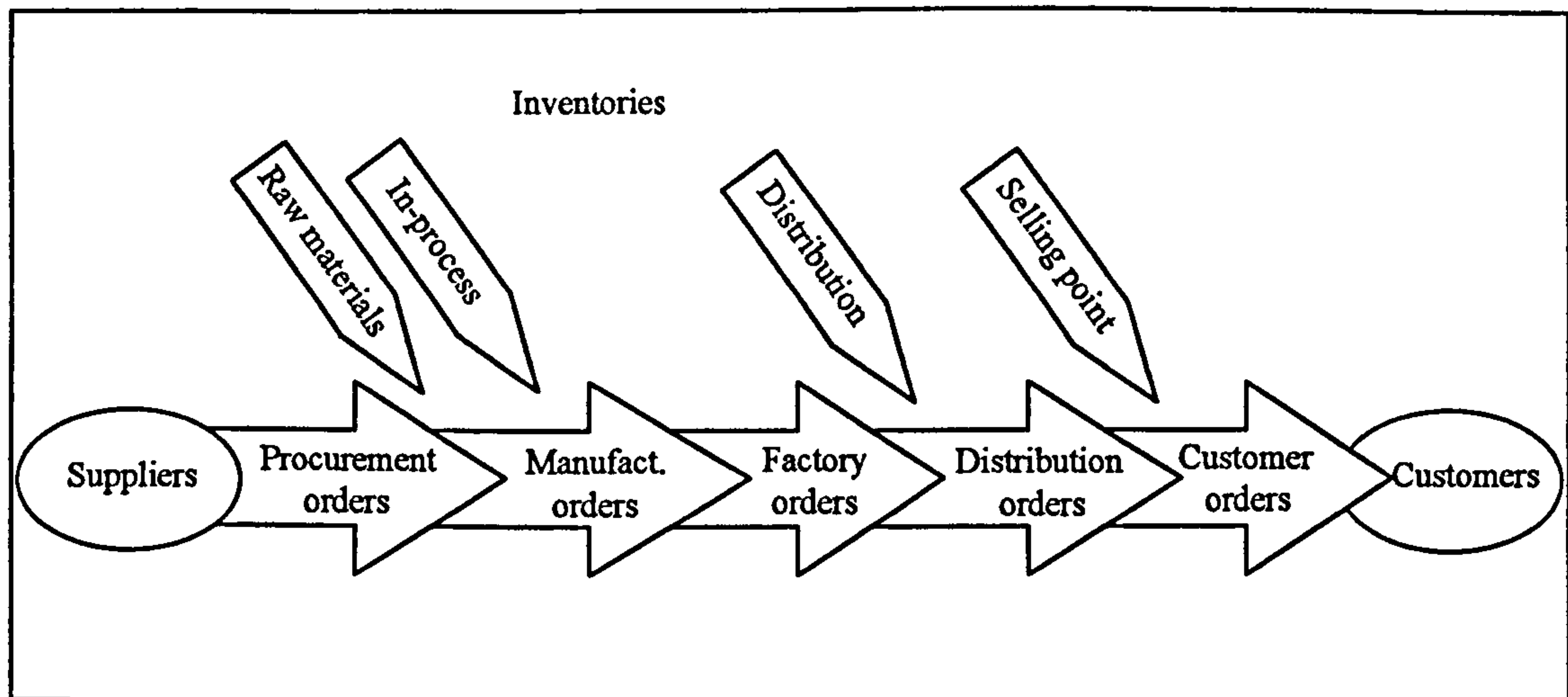


Figure 1.3. Inventories along an SC, (Jones and Riley, 1985)

There are complex relationships between behaviour of inventories in an SC and their control. If, for example, one part of the SC decides to decrease its inventory to reduce cost, the reduction may deteriorate the overall SC service performance. Compensation can be achieved by increasing inventory at some other point in the chain. This may lead to a situation where only a very large stock held at one part in the SC enables another part to operate with minimal stock; overall, there may be an increase in stocks held in the SC as a whole and an increase in the total inventory holding cost incurred. The sum of all inventory holding cost may represent a large proportion of the total SC cost. Thus, sophisticated methods are required for the overall management and control of inventories along an SC.

#### 1.4. Supply chain modelling

SC modelling is a complex and challenging problem, supported by a desire to provide appropriate solutions to real-world problems. Main roles of SC models can be summarised as follows (Gattorna and Walters, 1996): (1) to better understand an SC, its structure, activities, inter-dependency between SC constituent parts, inter-relationships with its operating environment, (2) to generate potential SC strategies,



i.e., to use an SC model as a strategy suggestive tool, and (3) to evaluate different strategic alternatives; an SC model is then used as a strategy assessment tool.

SC models should have various capabilities in order to be beneficial to managers (Lee and Billington, 1993). An SC model should provide inventory and service benchmarking by relating inventory investments throughout the SC with customer service performance. There is a need for SC models to support operational planning and control; they should be used to determine how much safety stock to hold at each inventory, when to initiate orders for material from the upstream part, etc. SC models should facilitate what-if analysis that can examine effects of various factors on SC operations and performance, such as changes in market demand, the design of the SC network, the transportation mode used or production capacity.

There are various complex issues in modelling of SCs as large scale dynamic systems (Schwarz, 1981a, Federgruen, 1989). As an illustration, some of these complexities are highlighted here. With acceptance of Integrated Supply Chain Paradigm a system view of an SC has to be taken. Simultaneous optimisation of SC resources is required, instead of finding optimal policies for each part of the SC separately. However, simultaneous SC optimisation becomes a very complex, computationally intensive and analytically intractable task, even in the case of a relatively simple SC structure. Due to the complexity of this problem, it is necessary to make a number of restrictive assumptions and simplifications in the specification of both the inter-relationships of the SC subsystems and the demand/supply sides of the SC. That is why SC models often suffer from a number of limitations caused by artificial assumptions. Another source of complexity is the demand process along an SC. Customer demand placed on one end of the SC, in combination with production/inventory policy, determine demand at upstream part of the chain. Even when customer demand is modelled by a standard, well-known probability distribution and the SC has a relatively simple structure, using straightforward operating policies still yield demand processes along the chain which are quite difficult to model analytically. In addition, treatment of shortages becomes more complex in an SC than

in the case of a single isolated inventory consideration. The consequences of a shortage recorded, for example, at one retail inventory depends on mutual arrangements with other retailers and their inventory status. Various sources of uncertainty inherent in an SC and its environment bring further complexity to SC modelling. In order to make SC models as close as possible to real-world SC problems, these uncertainties have to be represented in an appropriate way and included in the models. Data requirements for SC modelling are very extensive. Therefore, it is very important to define the scope of the model in order to focus the data acquisition process, i.e., to determine where the SC management function begins and ends, which SC activities are relevant, what type of costs should be included in the model.

A wide range of techniques is available and has been used for SC modelling, including optimisation tools for solving analytically tractable SC problems, heuristic procedures for obtaining near optimal solutions, and simulation techniques for treating analytically intractable SC problems or for evaluating SC analytical models.

A gap that often exists between research and practice is also apparent in the case of SCs. It is not surprising, due to all the complexities that characterise SC management and control problems in both theory and practice, that research results obtained often lack realism or they are too complex to be easily interpreted. There is a need to develop SC models which can be used to gain more understanding of SC behaviour and inter-relationships of various disparate parts of an SC and which enable the systematic design and analysis of SCs and optimisation of their resources (Schmidt, 1993). One of the recognised pitfalls of SC modelling has been the failure to address the impact of uncertainties (Lee and Billington, 1992).

SC modelling that is carried out in this research is motivated by two basic ideas: (1) including different sources of uncertainty inherent in real SCs and their environment into SC models can contribute to effective SC management and control and to better understanding of SCs and their processes, and (2) representing and treating uncertainty in a more natural way and in a form acceptable by practitioners can bring SC models nearer to real-world problems.

## 1.5. Treating uncertainty

Standard concepts of random variables and probability have been usually employed by researchers. By modelling an SC as a system where random processes appear, it is appreciated that the future is not precisely determined by the present. Therefore, probability distributions are used to describe the future state of the SC.

In this research, the author has taken an attitude that in the study of such a complex system as an SC is, there is a need to turn to even wider aspects of uncertainty such as: lack of understanding of processes under consideration, unreliable sources of data and information, conflicting or complementary sources of facts, the abundance of irrelevant data, even the imprecision of natural language. In the literature, there are various misgivings concerning the relevance of probability theory as an appropriate mathematical structure to treat different types of uncertainty. Although stochastic models, in general, provide valuable insights into system behaviour under certain assumptions, they are rarely implemented in practice. The reasons for this are various. Basic assumptions of a stochastic model might be too restrictive and difficult to understand, the model on its own might be too complex, or information required to determine the values of model parameters cannot be obtained or they are not available (Turksen, 1991). In such cases, the models may be inadequate to help practitioners to cope with real-world problems.

In the last few decades, some new approaches and methods for treating uncertainty have been proposed and analysed. Among them, fuzzy sets theory (Zadeh, 1965) attracted the most attention from researchers and its applicability in practice has been already fully recognised. Based on the motto "precision is not truth", fuzzy sets theory offers a framework in which uncertain data, relationships between them and system behaviour can be expressed by vague, imprecise or ambiguous linguistic terms. Use of fuzzy sets may become fruitful in cases when information is not readily



captured in terms of probabilistic estimates due to the lack of appropriate statistical data. Fuzzy sets theory also offers an efficient way of accounting for vagueness and imprecisions in human judgment.

Having in mind the complex nature of SC management and control problems and various sources of uncertainty that may exist in an SC and its environment on the one hand, and flexibility and applicability of fuzzy sets theory in treating various types of uncertainty on the other, the use of fuzzy sets theory in treating SC problems offers potential advantages in SC modelling. Clearly, it is an area that requires to be researched.

## **1.6. Objectives of the research**

This research is focused on SC management and control in the presence of uncertainty. The main objectives of the present research are as follows:

1. to identify various sources of uncertainty in data that characterise SC operations and its environment,
2. to investigate the potential of using fuzzy sets for describing and representing the uncertainties identified,
3. to develop a new software tool for analyses of SC dynamics in uncertain environment. The tool should include: (1) new SC models to be developed for SC control in the presence of various sources of uncertainty described by fuzzy sets, and (2) SC simulation models to be developed to provide a dynamic view of SC operations and to evaluate order-up-to level decisions made using the SC fuzzy models. The application of the SC fuzzy models and SC simulation models should be combined in such a way as to enhance decision making on SC control parameters in order to improve SC performance in the presence of uncertainty,
4. to demonstrate the application of the developed tool in various SC analyses which



lead to better understanding of SC behaviour and its performance in the presence of uncertainty. To support this objective, various analyses should be carried out, in order to:

- examine SC behaviour and performance under different control strategies which consider different sources of uncertainty,
- analyse the impact of actual or potential changes in uncertainty in the SC and its environment,
- investigate ways of compensating for the negative effects of uncertainty on SC performance and to develop corresponding procedures.

## 1.7. Thesis outline

This thesis is organised in the following way.

SC models developed and reported in the literature are reviewed in Chapter 2. Special emphasis is placed on the sources and types of uncertainty and the ways they are treated in SC models.

Chapter 3 is devoted to the selected parts of fuzzy sets theory and its applications in production and inventory management and control. It contains basic definitions, concepts and techniques relevant to this research, and reviews fuzzy inventory models developed and reported in the literature so far.

The main sources of uncertainty identified in an SC and its environment, and the methods to represent them formally using fuzzy sets are discussed in Chapter 4. The conceptual simplicity of using fuzzy sets to represent uncertainties in customer demand, external supplier reliability, supply along the chain and lead times is underlined.

In Chapter 5, new original SC fuzzy analytical models are presented. Optimal order-up-to levels and replenishment quantities are determined for all inventories in an

SC which operates in a fuzzy environment. Two SC control strategies which take into consideration different sources of uncertainty and reflect different levels of SC integration are defined: (1) decentralised control, and (2) a new concept of partially coordinated control.

A new original tool for SC analyses developed by the author is described in Chapter 6. In this tool, two types of new SC models are combined: (1) the SC fuzzy analytical models and (2) SC simulation models developed to evaluate SC performance achieved by applying the SC fuzzy models.

The application of the developed SC tool in analyses of SC behaviour and performance in uncertain environments, is demonstrated in the next two chapters. SC behaviour and performance when there is uncertainty in customer demand are analysed using illustrative SC examples in Chapter 7.. The negative effects of uncertainty in customer demand, its propagation along an SC and the effects of changing uncertainty in customer demand on SC behaviour and performance are examined. Effectiveness of the two SC control strategies, decentralised and partial coordination, is compared.

Illustrative examples of analyses of SC behaviour and performance when uncertainty exists in both customer demand and external supply are given in Chapter 8. Different methods for compensating for uncertainty in external supply are examined and corresponding procedures are developed. Furthermore, the effects of changing uncertainty in external supply are analysed.

The main conclusions of this research and directions for future research are presented in Chapter 9.

## CHAPTER 2

### SUPPLY CHAIN MODELLING - A LITERATURE REVIEW

SC analysis, management and control have recently received a great deal of attention by both academics and practitioners. A modern environment in which an SC operates, characterised by increased competitive pressures and market globalisation, complex structures and relationships between its constituent parts, make SC analysis, management and control very complex and challenging tasks.

SCs have been studied from various aspects. A number of books, monographs, scientific or practical papers devoted to various SC matters have been published. SC related topics regularly occupy considerable number of papers in internationally leading periodicals such as *Management Science*, *International Journal of Production Economics*, *European Journal of Operational Research*, *Interfaces*, etc. Recently, a new journal *Supply Chain Management: An International Journal*, completely dedicated to SC matters, has appeared.

In this chapter, a sample of the broad spectrum of research and applications developed in the area of SC management and control is presented. Although extensive, this literature review is not intended to be exhaustive, but rather it is directed toward identifying some of the key aspects of SC management and control problems that have been treated in the SC models.

In the first three sections, some important issues regarding SC modelling are addressed, including different SC structures, activities within SCs and measures of their performance. There are many ways in which SC models can be classified. They can be grouped following the general model classifications, such as analytical or

simulation models, generative or evaluative models, static or dynamic models, deterministic or stochastic models, etc. Based on the management aspects, the SC models can be classified as strategic or operational. A wide range of methods and techniques have been applied in the SC models so far, encompassing optimisation, heuristics and simulation (Brierly, 1993). In this review an attempt has been made to classify the SC models into specific groups, with respect to types of SC management and control problems that these models consider. There are:

- models which describe coordinated SC management and control, starting from coordination of SC constituent parts only, to the global coordination that involves the whole SC, (Section 2.4.),
- models where different SC control strategies are considered, (Section 2.5.),
- SC dynamics models, (Section 2.6.),
- SC stochastic models, (Section 2.7.), and
- real-world SC models (Section 2.8.).

Summary of the literature review and main conclusions made are given in Section 2.9.

## 2.1. Supply chain structures

Generally, an SC is viewed and modelled as a network of facilities in which nodes represent the facilities and links represent the flow of goods between these facilities. There is a number of ways in which the facilities are combined to form the SC. In simplified terms, the structure of an SC is defined by a structural diagram which indicates the mutual arrangements and interrelationships between the facilities in the chain. Three basic structures that constitute the SC network can be identified (Schwarz, 1981b):

- **serial structure**, where each node in an SC network, except the first and the last one, has exactly one predecessor and one successor node,



- **assembly structure**, where each intermediate node has exactly one successor, but possibly several predecessors; most often this structure includes an assembly facility within the production part of an SC, connected with a number of intermediate product suppliers,
- **arborescent structure**, where each intermediate node has exactly one predecessor, but possibly several successors; the distribution part of an SC is usually structured in this way.

The three structures and a general SC network that encompasses these three structures are illustrated in Figure 2.1. A substantial part of the published results is devoted to the three basic structures ((a), (b) and (c) in Figure 2.1) and particularly to serial configurations (Hanssmann, 1962). In a serial system, a raw material inventory supplies the succeeding production facility, which in turn replenishes the succeeding inventory stock and so on, towards the end-product inventory, which receives customer demand.

## 2.2. Supply chain activities

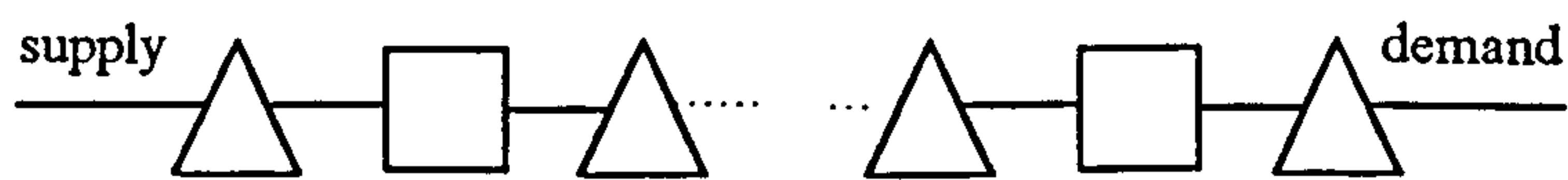
Activities along an SC network unroll in the following way (Cohen and Lee, 1988). Raw materials and/or intermediate products, which can be purchased from different vendors, are inputs to production nodes. The production is accomplished by transformation and assembly of raw materials and the intermediate products. This can be viewed as a network of processing and stocking point nodes. The production outputs are finished goods (i.e., end-products) that can be stocked or shipped directly to appropriate locations within a distribution network and to end customers. The distribution part of the SC network contains stocking point nodes only, which represent warehouses that stock goods for distribution to other warehouses and to retail or peripheral stores where customer demand for these goods originates. As a

result, three parts are recognisable in an SC: (1) procurement of raw material, (2) production and (3) distribution. Activities along the SC have been classified accordingly.

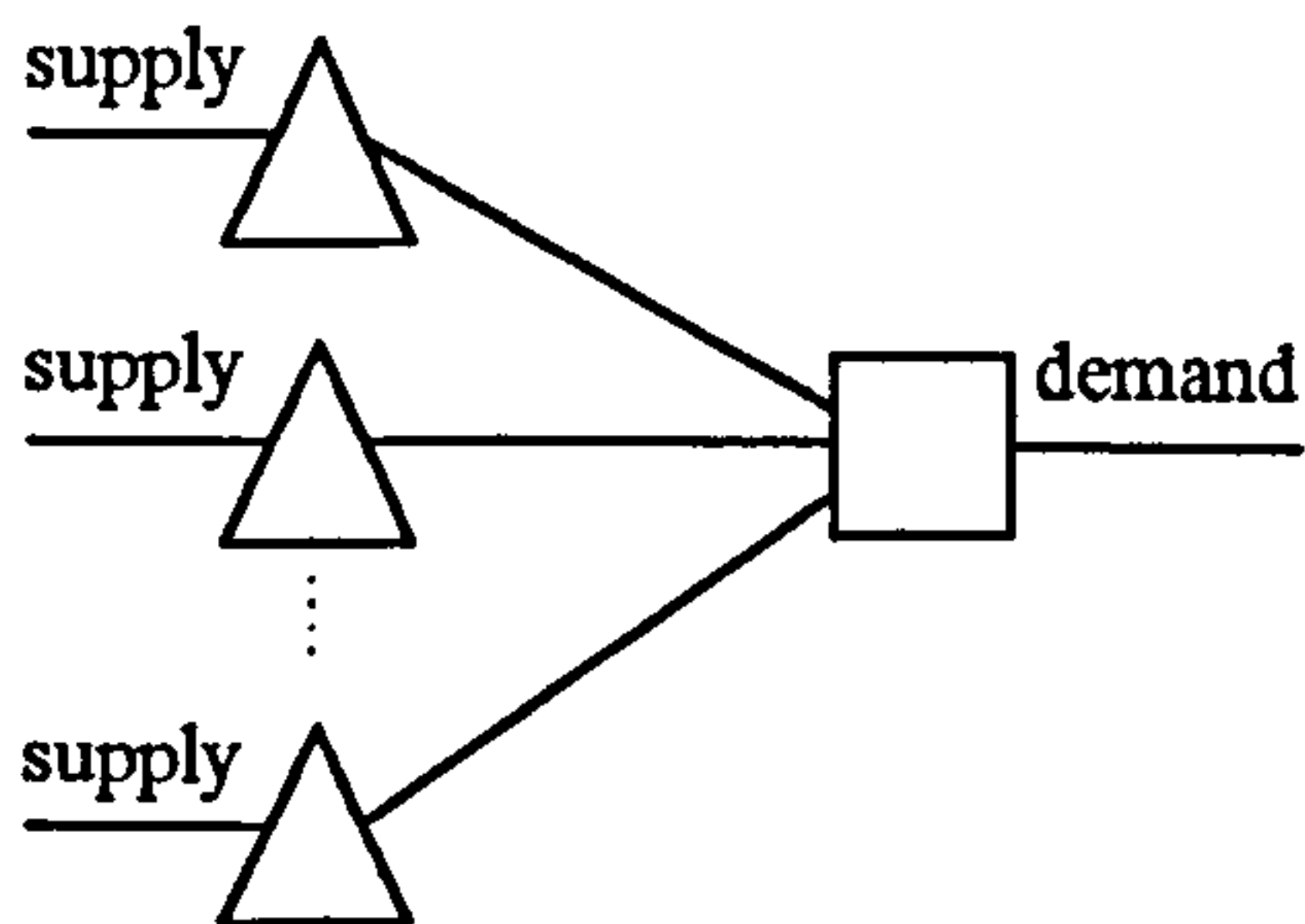
Slats et. al. (1995) focus attention on the informatisation of SCs. They divide activities within an SC into three categories: (1) a feed-forward flow of goods, including transportation, material handling and transformation activities (manufacturing, assembly, packaging, etc.), (2) a feed-back flow of information, including information exchange activities related to ordering, delivering, transporting, etc., and (3) management and control, encompassing purchasing, marketing, forecasting, inventory management, planning, sales and after sales service activities. The use of EDI (Electronic Data Interchange) within an SC facilitates very high levels of data integrity which is, according to modern views, fundamental to the credibility of the SC (Motogami, 1991).

Gattorna and Walters (1996) identify and list activity and information processes along an SC and classify them into suppliers, production/distribution and customers related activities and information. This is illustrated in Figure 2.2.

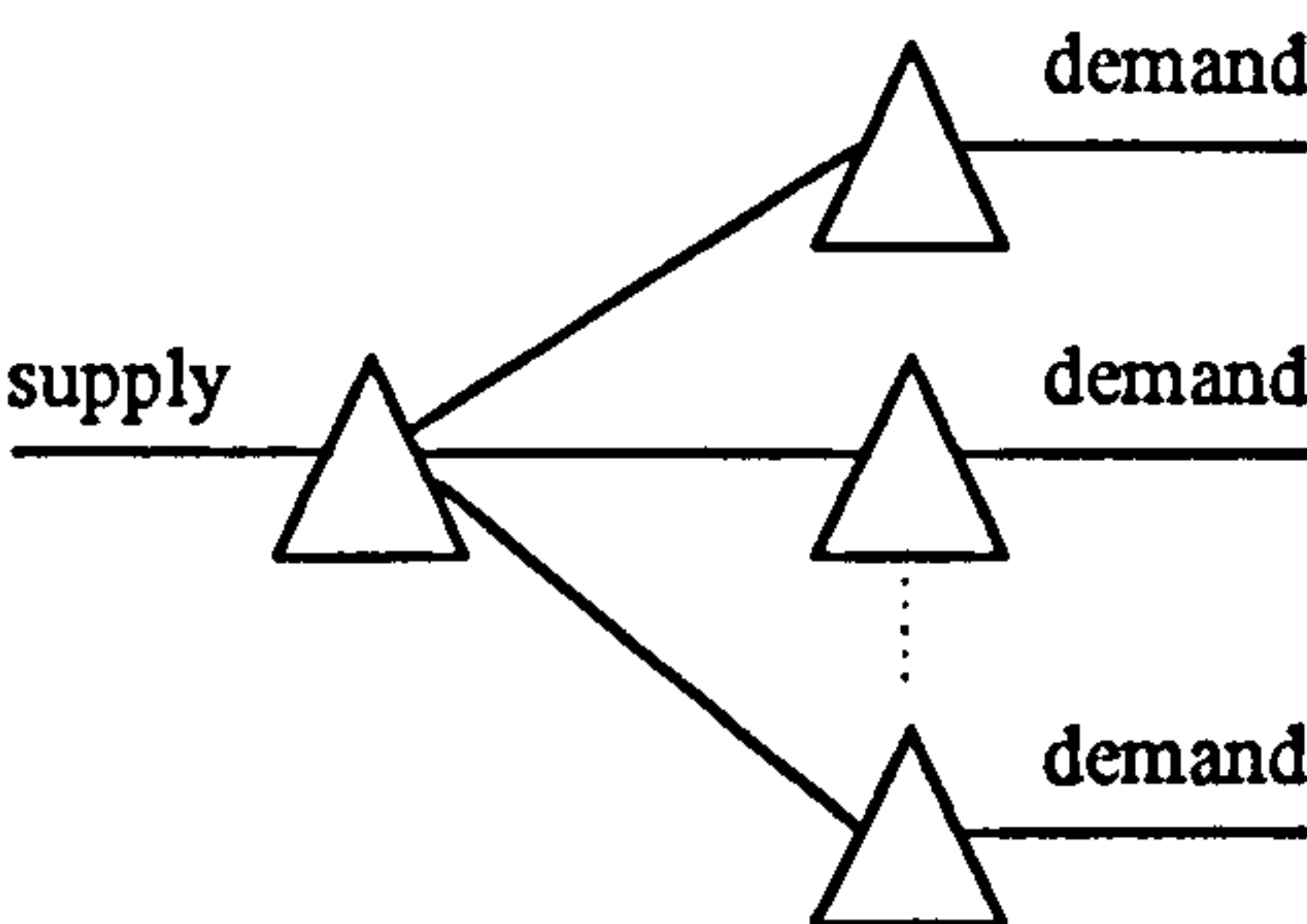
All the activities along an SC are interrelated; activities at one facility affect control and performance of other facilities and the performance of the SC as a whole. Hence, models that attempt to link the activities across all parts of an SC receive, nowadays, a great deal of attention (Titly, 1989, Thomas and Griffin, 1996).



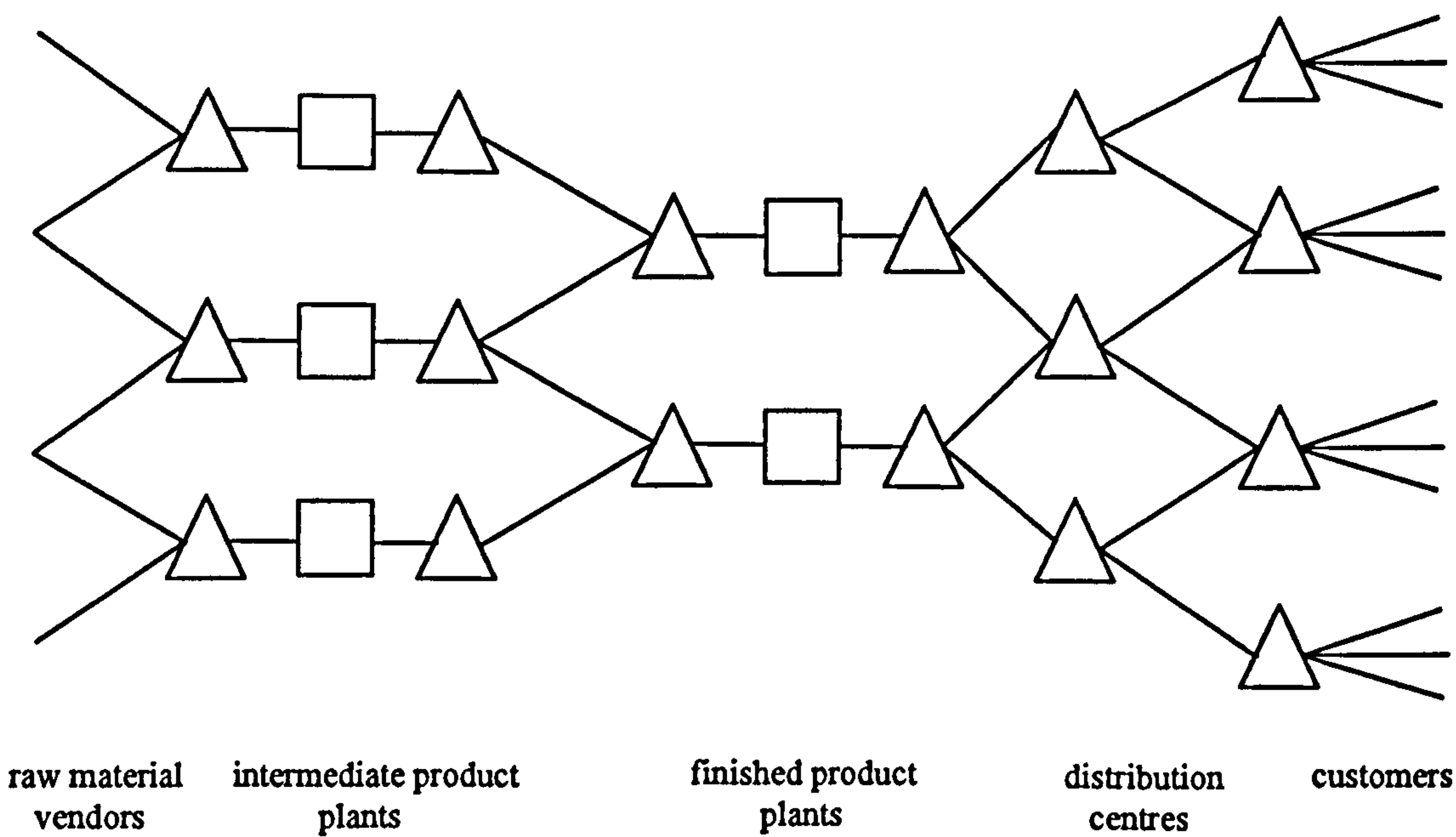
(a) Serial structure



(b) Assembly structure



(c) Arborescent structure



(d) A complex SC network (Cohen and Lee, 1988)

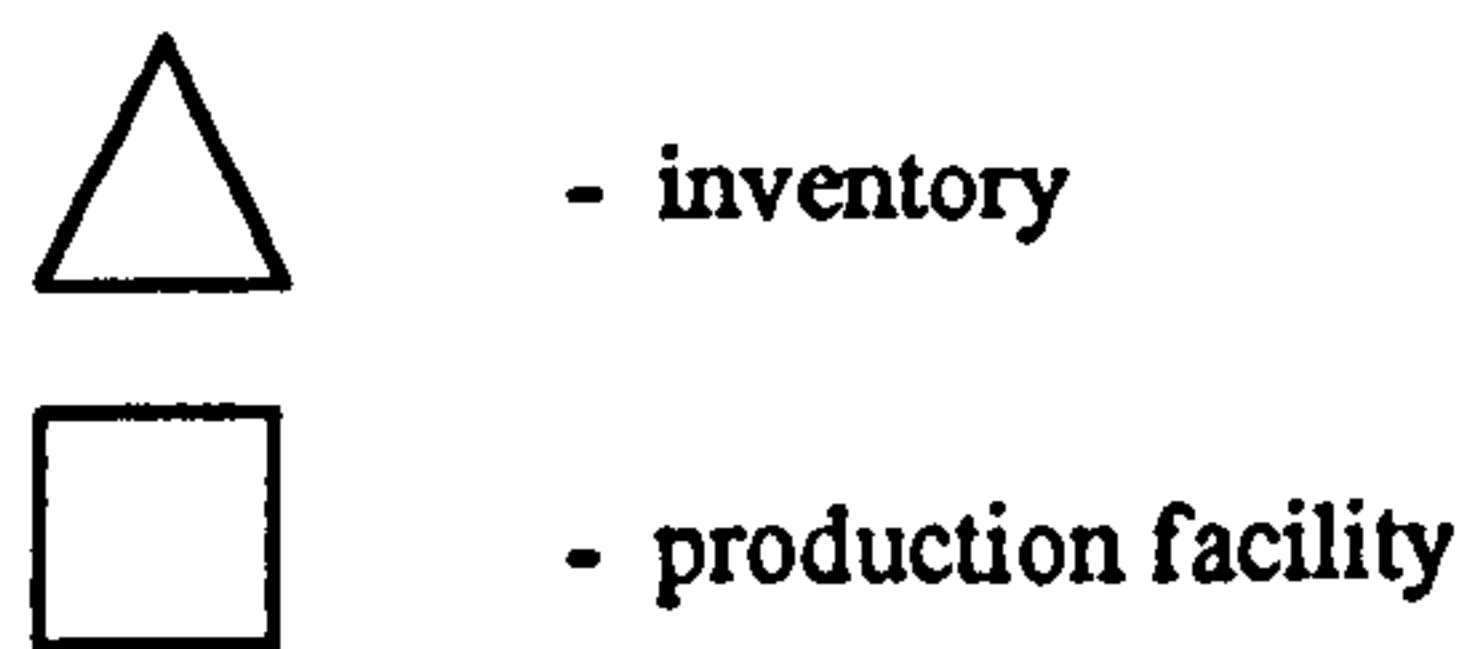


Figure 2.1. Basic SC structures

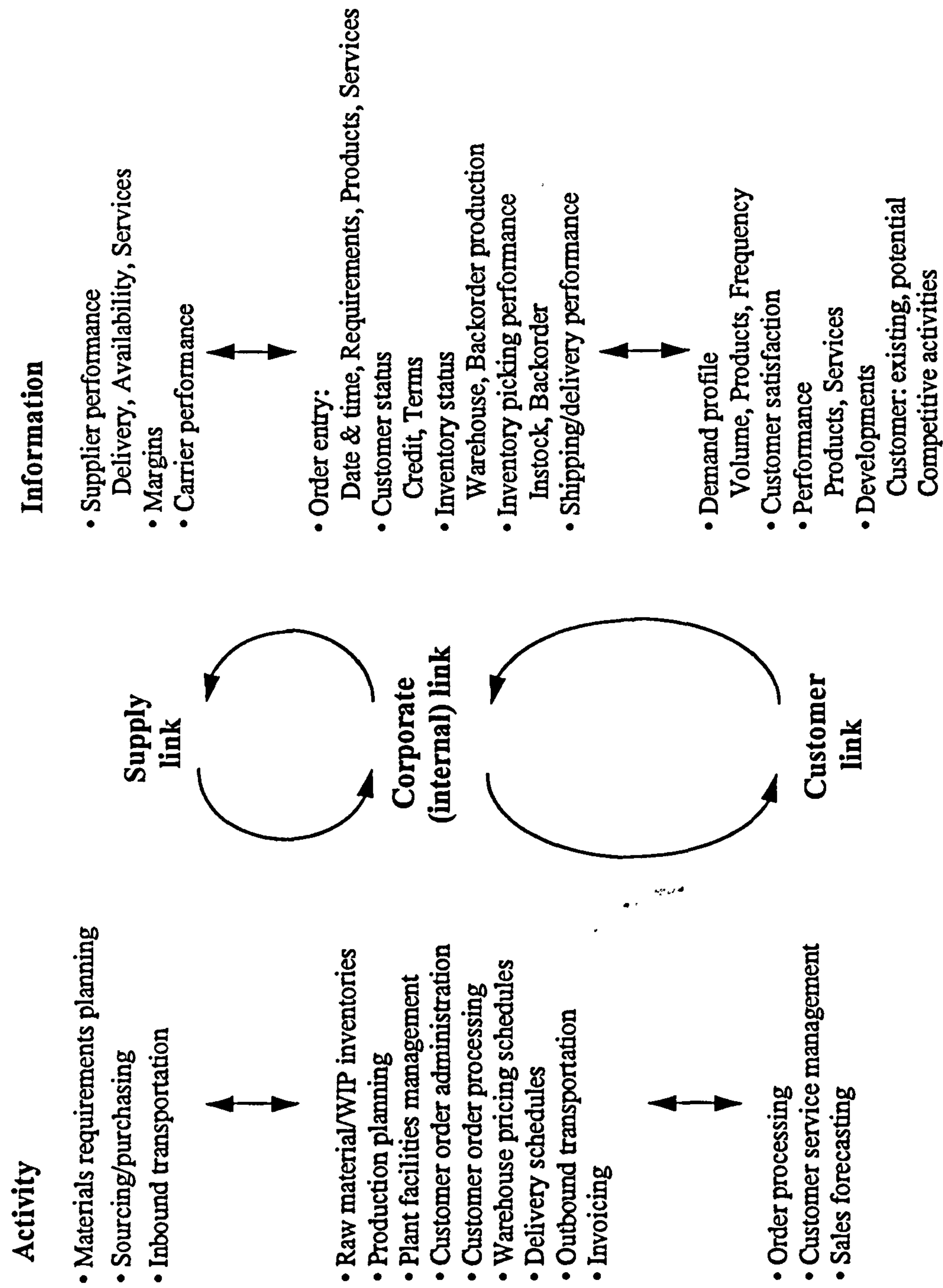


Figure 2.2. Activity and information processes along an SC, (Gattorna and Walters, 1996)



### 2.3. Supply chain performance measures

Selection of appropriate measures of SC performance is a very important issue for effective SC management and control. A problem of defining operationally significant performance measures of an SC as a whole exists. Two main questions emerge: what should be measured and how should standards be set for the measures (Fitzgerald and Moon, 1996). Regular monitoring of SC performance is of high importance and the requirement for it stems from management expectations of direct financial benefits.

Two different types of performance measures can be distinguished: external and internal performance measures (Diks, de Kok and Lagodimos, 1996). External performance measures are related to the service provided to external customers at the most downstream end-product stock points of an SC. Services from one to the succeeding facility along the SC are assessed by internal performance measures. Having in mind that the basic function of an SC is to provide service to external customers, in some sense, internal SC performance on its own is irrelevant. On the other hand, performance of each facility influences service at the downstream part of the SC, and consequently, affects external customer service.

Many of the analytical SC models have been oriented towards associating a single performance measure, so that an SC management and control problem can be expressed in terms of a classical single criterion optimisation task. This has generated a need for careful analysis and comparison of different potential measures of SC performance. However, it is not easy to interpret what is a service goal and its actual true value for a customer. In general, service provided by an SC is associated with availability of products to external customers, but availability of products itself is a vaguely defined concept. Customer service is impacted by many factors, such as, stock availability, frequency of delivery, reliability of delivery, order cycle time and many others (Christopher, 1992). A number of SC performance measures, most often used in the SC models, are singled out in the remaining part of this section.

Stock availability is usually expressed by a **fill rate**. The fill rate is defined as a fraction of customer demand that is immediately satisfied from an end-product inventory. Although, this measure of performance is associated with stock-out situations at the end-product inventory only, the fill rates of upstream inventories in the SC are taken into account through their influence on the fill rate of the end-product inventory. The fill rate is a traditional measure of supply behaviour, but it does not represent how well different customers are supported, and this sometimes might be important.

It may be argued that when a shortage occurs, the backorder delivery may take various times. Therefore, a reasonable modification can be made in the very definition of the fill rate. A time of acceptable delay of full customer demand satisfaction can be introduced. Then, the fill rate is defined as the fraction of customer demand that is fulfilled within a certain given period of time. Such a modified definition of the fill rate is justified from the practical point of view, but it makes SC models more complex.

**Number of backorders** can be used as an SC performance measure in the case when excess customer demand is backordered. Although the number of backorders is recorded at end-product inventories only, it is indirectly affected by backorders at the upstream part of the SC.

SC management and control decisions are based on making a trade-off between a customer service level provided by an SC and a total SC cost required to achieve this service level. Various types of costs are incurred at the different constituent parts of an SC, for example, raw material, intermediate and end-product holding costs, ordering costs, raw material purchasing costs, production set-up costs, transportation costs, etc. In addition, a shortage cost, that is correlated with a customer fill rate, is defined to take into consideration negative effects of stock-out situations. By introducing the concept of a shortage cost, as a consequence of an unfilled demand, the total SC cost is used as an overall measure of SC performance. It is calculated as the sum of all the costs incurred and the shortage cost. Although there are claims that the notion of a shortage cost is not very usable in practice, it helps to state an SC optimisation



problem (Silver, 1981). In this case, it is important to recognise the influence of the activities at each facility on the total SC cost.

Changes in an environment in which an SC operates bring into attention new measures of SC performance. For example, in a modern environment where product life-cycles are becoming shorter, i.e., where the product becomes obsolete very quickly, a suitable measure of performance can be **order cycle time**, i.e., the time that elapses from placing a customer order to its delivery. Application of just-in-time philosophy requires more frequent deliveries, making **frequency of delivery** an important measure of SC performance. Also, trends towards globalisation, bring international scenarios into SC performance considerations. The calculation of the SC total cost has to be changed, by taking into account additional factors, such as exchange rates or different taxes and duties.

## 2.4. Towards global supply chain modelling

There are many interesting papers with both scientific and practical values that have treated activities along an SC, including material procurement, production and scheduling, transportation and distribution, separately. Although these models represent the basis for global SC modelling, linking the activities across different parts of an SC brings new problems into the focus. Coordinated SC management and control have become more and more important. Although the term **supply chain** is relatively new, the idea of coordinated multi-level production-inventory systems is not. The study of multi-level production-inventory systems began in the late fifties (Hanssmann, 1959), and since then a large amount of literature has been published in this area.

Generally, a multi-level production-inventory system involves two or more facilities, where each facility is defined to be an entity that produces, services or holds an inventory (Schwarz, 1981b). For example, typical facilities in a distribution

inventory system include retailers, regional warehouses, district warehouses, etc., while a production system includes facilities such as production sites, intermediate product inventories, vehicles and containers used for transportation between production sites and so on. Different terminologies have been introduced to denote these systems. Some of the authors (e.g., Silver and Peterson, 1979) refer to production-inventory systems which contain mixed production and inventory facilities as **multistage systems** and to inventory-distribution systems with inventory facilities only as **multiechelon systems**, while others (e.g., Schwarz, 1981a) use these terms as synonymous.

Scanning the literature, one can notice that most of the past efforts were devoted to coordination of just parts of, what is now called, an SC. Recent attention has been oriented towards global SC management and control, from raw material suppliers through production to delivery of end-products to customers.

SC models are generally concerned with two levels of coordination: **operational** and **strategic**. Operational SC models are targeted at issues such as selection of production batch sizes, transportation modes, safety stocks for inventories along an SC, while strategic SC models deal with problems of plant or distribution centre objectives, locational issues, configuring a new SC, evaluation of changes in the flow of a particular product through the chain, SC informatisation.

Thomas and Griffin (1996), classify operational models that coordinate just parts of an SC into three categories: buyer-vendor, production-distribution and inventory-distribution coordination models. A lot of research has been done regarding multi-echelon inventory-distribution systems (Diks, de Kok, Lagodimos, 1996), while models that coordinate the buyer-vendor and production-distribution SC systems have appeared in the last decade only (Thomas and Griffin, 1996).

Buyer-vendor coordination models are related to procurement of raw material or intermediate products (i.e., subassemblies), which represents the first activity in supplying goods along an SC. Most of these models aim to find order quantities for purchasing that are jointly optimal for both buyers and vendors (Goyal and Gupta,



1989). Different buyer-vendor structures have been modelled, starting from a single vendor - single buyer structure (e.g., Monahan, 1984, Lee and Rosenblatt, 1986, Banerjee, 1986), to single buyer - multiple vendors (e.g., Lau and Lau, 1994) and single vendor - group of buyers structures (e.g., Kohli and Park, 1994).

Although the literature addressing production planning or distribution planning is rich (e.g., Lambrecht, Eecken and Vanderveken, 1981, Bhatnagar, Chandra and Goyal, 1993), there is only a small number of models that deal with these problems simultaneously, taking into consideration interactions between them. Thomas and Griffin (1996) note a few possible reasons for the shortage of models in this area. First, many problems concerned with these activities separately are already difficult to solve; second, in practice, production and distribution are often separated by inventory buffers and, third, different departments are usually responsible for these activities.

Production-distribution problems are mainly stated to determine production and distribution batch sizes or to select a transportation mode to minimise production-distribution costs incurred (Williams, 1981, Haq et al., 1991, Benjamin, 1990, Chandra and Fisher, 1994). The costs are of different nature and include production cost, setup cost, holding cost and/or transportation cost. Byrne and Bakir (1996) demonstrate how an analytical model, working in cooperation with a simulation model, can provide a better solution of a multi-period multi-product production planning problem, than either of the models alone. It is emphasised that probabilistic characteristics of a system can be easily added into consideration using this hybrid approach. Zijm and de Kok (1988) consider a multi-stage, multi-item production/inventory system. They use analytical and simulation methods to analyse the influence of various factors on work-in-process inventory levels, including a production lead time, the length of a review period of the work-in-process inventory, uncertainty in demand for work-in-process and changes in a product structure towards more standardisation.

Much effort has been made to model coordinated inventory-distribution systems, starting from Clark and Scarf's model (Clark and Scarf, 1960) for a serial

inventory system with stochastic customer demand. The coordination between inventories is achieved by introducing a penalty cost. The penalty cost charged on an inventory in the chain is assumed to be equal to the expected increment in the total cost of the succeeding inventory, incurred due to the failure of the inventory under consideration to deliver the ordered quantity.

Most of the inventory-distribution models are oriented towards determination of optimal inventory policies for stocking points in a system. A lot of attention has been placed on demand modelling, particularly on: (1) modelling of customer demand that is independently placed on inventories at the lowest echelon of an inventory-distribution system, (e.g., Svoronos and Zipkin, 1991, Ernst and Pyke, 1993), (2) modelling of correlated demand when there is a correlation between demand in successive periods or between demand for the same product at different locations, (e.g., Erkip, 1990), and (3) predicting and modelling of secondary demand, placed on the next echelon in the system starting from the lowest (Sand, 1981, Kelle and Milne, 1996).

One of the representative analytical model which considers the whole SC network is that developed by Cohen and Lee (1987). Their SC model consists of four sub-models for different SC subsystems, namely material control, serial production, finished goods inventory control, and distribution. First, some simplifying assumptions are made in order to find an optimal operating policy for each of the separate SC subsystems that provides the specified fill rate of the subsystem at the minimum local cost. However, due to the nonlinear nature of each of the submodels, the overall SC optimisation is not feasible and a heuristic optimisation procedure is proposed. The relationships between the local control policies and overall SC performance is investigated.

Vidal and Goetshalckx (1997) give a critical review of the strategic SC models proposed in the literature. They conclude that very few strategic models include stochastic aspects and treat uncertainty in customer demand and lead times. Additional

sources of uncertainty, such as government stability or general infrastructure of a particular country, are identified to be the critical issues for the strategic design of an SC, especially in an international environment. However, the authors point out that it is very difficult to include these factors in formal models using traditional methods and techniques. These problems are outlined for future research. A few interesting strategic models developed and used for real-life SCs are presented in some detail in Section 2.8.

Slats et al. (1995) highlight the drawbacks of SC analytical models that make unrealistic assumptions and ignore uncertainty in order to reduce complexity and to achieve computational tractability. To overcome these problems, the authors introduce a new concept of "a logistic laboratory" that has to include a whole set of various SC models, based on different operational research and management science methods and tools, including optimisation, heuristics and simulation. The objectives of the logistic laboratory as Slats et al. report are: (1) to facilitate quick building of a new SC model or easy modification of an existing one, (2) to support the analyses of an entire SC from various perspectives and for each level of SC management and control (strategic, tactical and operational), and (3) to conduct various experiments related to existing or potential SC configurations. The logistic laboratory aims at providing different functions, including: (a) market oriented functions which are used for analysing customer demand, for selecting the type of problem to be considered and logistic goals to be achieved, etc., (b) functions for configuring facilities and defining the structure of an SC, (c) functions for specifying management and control rules to be applied along the SC under consideration, (d) functions for defining activities of constituent SC parts and for selecting appropriate parameters, that result in a specific model of the SC under consideration, (e) functions for running experiments to validate the SC model, for evaluating results obtained and for trade-off and sensitivity analyses.



## 2.5. Modelling of supply chain control strategies

Different SC control strategies have been proposed and analysed in the literature, from totally decentralised control to fully centralised control (Lee and Billington, 1993). Under decentralised control, each individual facility in an SC makes decisions based on local information and local objectives only. In a centrally controlled SC, decisions for all the facilities are made based on information about the entire system, such as data about material status of each facility, customer demand, lead times along the SC, etc., and objectives of the SC as a whole. Often in practice, there are organisational barriers between facilities along an SC, restrictions regarding information flows or, perhaps, partially conflicting objectives of parts of the SC. In these cases, fully centralised SC control is neither feasible nor even necessarily desirable. On the other hand, integrating individual activities along a chain can result in better overall SC performance, for example, higher customer service level, lower total SC cost, reduction in levels of inventory stocks, or better resource utilisation.

Modelling and optimisation of a fully centralised SC control strategy is a very difficult and complex task, analytically intractable, even for relatively simple SC structures. Different approaches have been taken to overcome this problem. By reviewing the existing SC models described in the literature, a few approaches most often used can be identified: (1) in order to model fully centralised control and to make the optimisation task feasible, an approximation is used and only a part of an SC is taken into consideration, (2) heuristics that lead to satisfactory solutions instead of optimal solutions are used, (3) to simplify the overall optimisation task, restrictive assumptions are made, or (4) the problem is decomposed and fully centralised control strategy is replaced by some forms of coordination between SC facilities.

A review of few illustrative SC models, based on these approaches, now follows.



The idea of decentralised and centralised control was initially applied to multiechelon inventory-distribution systems, leading to two general inventory stock policies: **installation** and **echelon stock policies**. In a decentrally controlled multiechelon inventory system, an ordering decision made at one inventory facility (also called an installation) is based on the current installation stock level only (also called the installation inventory position). Formally, the installation inventory position is calculated as the sum of the stock on hand and on order, reduced by a backlog quantity. The obvious advantage of such inventory policy is that it does not require any information about the inventory situations at other installations. The cost effectiveness of an installation stock policy is, however, limited by the lack of information about the entire system. Hence, Clark and Scarf (1960) introduce the concept of the echelon stock that is still attractive. The echelon inventory position is obtained by adding the inventory position at the installation under consideration and the inventory positions at all downstream installations, i.e., the installations towards customers.

It is interesting to notice that despite the fact that an echelon based stock policy requires information about all downstream inventories, in a way decentralised control can be retained (Axsater and Rosling, 1993). Due to the fact that an echelon stock position is completely determined by the initial echelon stock position, the replenishments and customer demand, it is not necessary to keep track of each downstream installation inventory position explicitly. Still, each echelon needs to know customer demand.

Axsater and Rosling (1993) investigated inventory control rules under which the echelon stock based policy was shown to be superior to the corresponding installation stock based policy, for serial and assembly configurations. In addition, it is shown by Axsater and Juntti (1996) that in the case of a distribution system, only under specific circumstances, such as a long warehouse lead time, an echelon stock based policy outperforms an installation stock based policy.

Lawrence (1977) considers an inventory-distribution system that includes a central warehouse and a number of branch warehouses. The control problem is defined as a single criterion optimisation task to find the total minimum safety stock to be held in the system to achieve a required branch service level to customers. The importance of jointly calculating safety stocks for both the central and the branch warehouses is emphasised. The author does not describe the stochastic mathematical model used, but gives its main characteristics. Uncertain customer demand imposed on the branch warehouses during each time period is forecast using the exponential smoothing technique. It is interesting to note that the calculation of the order quantities in this particular inventory-distribution system is based on a simple approximation of the Economic Order Quantity formula. This is justified by the lack of accuracy of the re-order cost estimate, that makes the development of more refined computation procedure unnecessary. Approximate analytical solutions that determine the necessary levels of safety stocks at the central and the branch warehouses for various branch service levels to customers are checked by simulation. Based on the simulation results, the relation between the central warehouse service level and the total safety stock level in the system is established, as the rules: (1) if the central warehouse service level to the branches is increased by raising its stock level, the branch inventory stock can be reduced while still maintaining the same service to customers and (2) in the case when a higher service level to customers is required (e.g., 0.95, that means that 95% of customer demand is satisfied from the branch stock on hand), the central warehouse service level becomes very important; on the other hand, when a lower service level to customers is required (e.g., 0.90), the balance of the stock held at the central and the branch warehouses is not very critical; in other words, in order to provide the required service level to customers, the central warehouse service level can have different values with almost no influence on the total safety stock level to be held in the system.

Different coordination control strategies for a manufacturer-supplier system are proposed and analysed by Tzafestas and Kapsiotis (1994). This paper demonstrates

applicability of control theory and classic discrete optimisation in SC management and control problems. The chain under consideration consists of a manufacturer and a number of its suppliers, all linked in series. The models developed to support the control strategies are deterministic in nature. Three scenarios, reflecting different coordination control strategies, are defined:

1. In the first scenario, the manufacturer is regarded as "an SC leader" that determines the policy for the rest of the chain. No cooperation between the chain levels is assumed. The manufacturer optimises its own cost and generates corresponding order patterns that are imposed on the next supplier level. The immediate supplier takes the manufacturer's order as an uncontrollable input and minimises its own cost, generating orders for the next supplier level and so on. Each level in the chain optimises its own cost, accepting the orders imposed by the preceding level without any bargain.
2. In the second scenario, the control of the chain is fully centralised. There exists a common objective. The optimal ordering policies for the manufacturer and all the suppliers are determined in such a way as to minimise the total cost of all the levels involved.
3. In the third scenario, a partially coordinated control is suggested to overcome the drawbacks of decentralised control. There is an objective for each level. The cost of each level is minimised taking into account possible conflicts with the next level in the chain. The conflict is measured as a difference between the order pattern generated by the level under consideration and the demand pattern "preferred" by the next level.

The three control strategies are compared with respect to their cost effectiveness and computational aspects. The total chain cost obtained under the first and the third control strategies are compared with the minimum total cost incurred under the fully centralised control strategy. Using an illustrative example, it is shown that the partially coordinated control leads to a total cost that is quite near to the minimum total cost. The first strategy is worse with respect to the total cost incurred.



The conclusion is made that the first and the third control strategies have similar computational requirements, much lower than the fully centralised strategy.

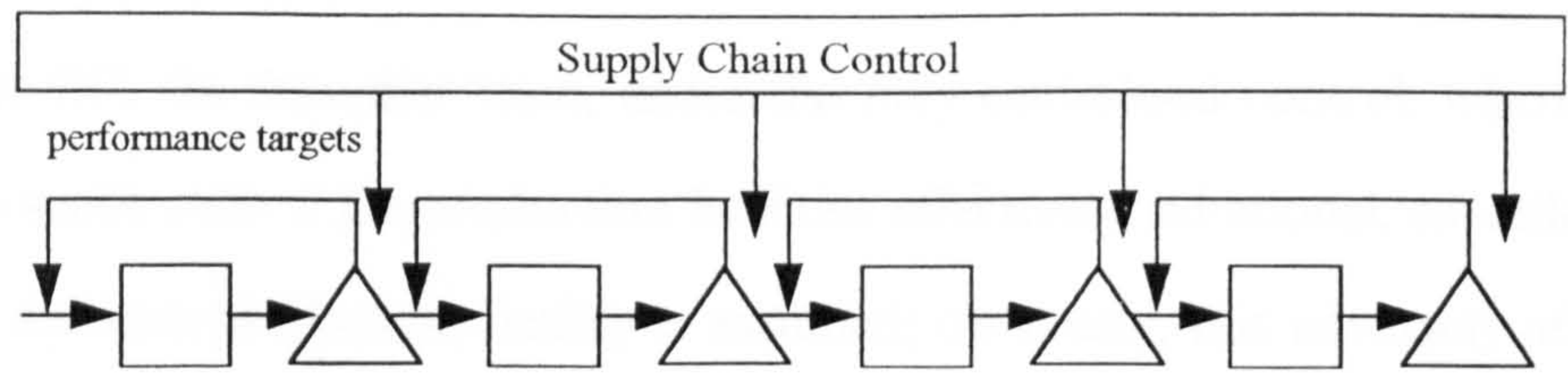
An important contribution to the field of SC performance measurement is made by De Kok and Bertrand (1995). They measure SC performance, namely stock levels along a chain and customer service level, under different SC control strategies. The strategies defined are based on hypothetical allocation of control responsibility among an SC manager and production managers, as follows:

1. In a decentrally controlled SC, each production manager is responsible for operational control of a production facility in the chain and replenishment of the inventory stock that follows the production facility. The objective of production control is to achieve the performance targets at the stock points, which are set by the SC manager. The authors of the paper observe that most of the SCs nowadays are controlled using this strategy.
2. Centralised operational control strategy gives a high responsibility to the SC manager, regarding operational control of the production facilities and control of the inventory stocks. The production managers are responsible for the technical aspects of the production organisation only.
3. Under decomposed operational control, the SC manager is responsible for the inventory stocks between the production facilities and for setting a target customer service level. A production manager controls a production facility in order to replenish the succeeding inventory stock within an agreed lead time.

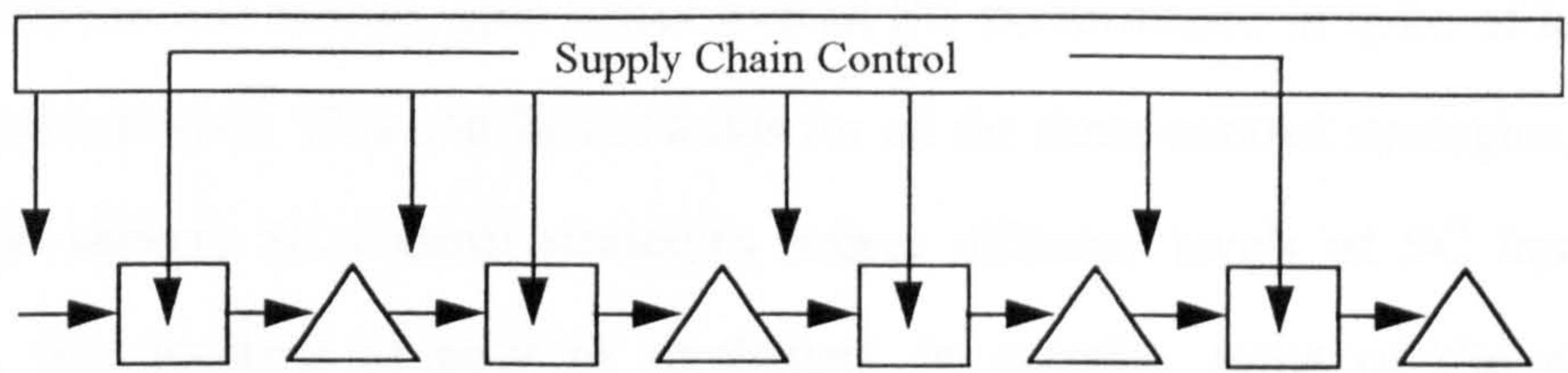
The three control strategies are graphically illustrated in Figure 2.3.

A corresponding analytical model based on heuristics is developed for each control strategy. Simulation is used to verify the heuristics applied and to model some aspects of interactions between facilities in a chain, ignored by the analytical models.

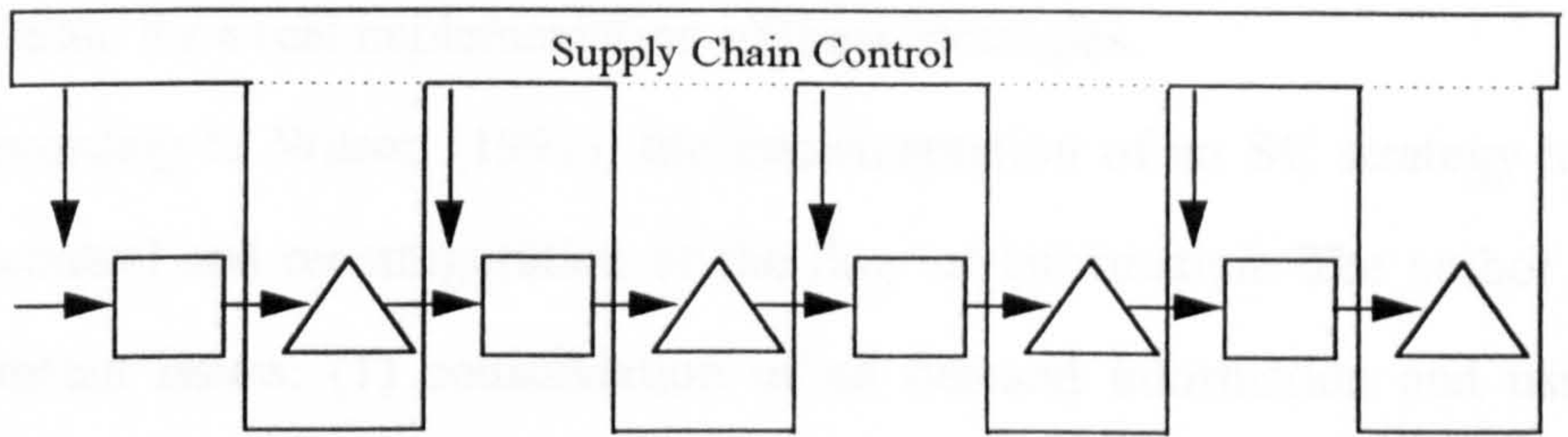




(a) Decentralised operational SC control strategy



(b) Centralised operational SC control strategy



(c) Decomposed operational SC control strategy

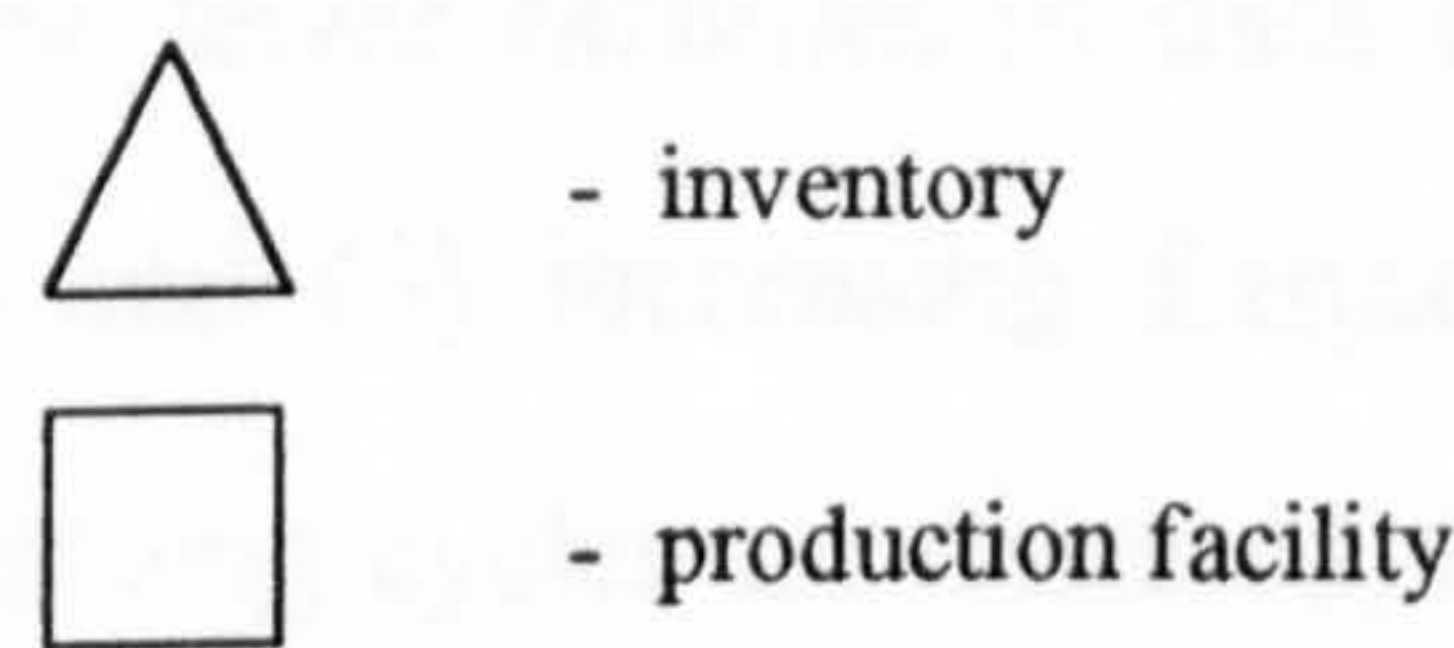


Figure 2.3. SC control strategies, (De Kok and Bertrand, 1995)

The control strategies are compared using as an example a serial SC that contains an intermediate product stock, an assembly facility, a central warehouse and a regional warehouse. The performance achieved under the decomposed control are better than the performance under the two other control strategies due to the following reasons. In order to provide high customer service level under the decentralised control, the tendency is to set high service levels at the upstream stock points so that the downstream stock points are not affected by lack of material from the upstream



part of the SC. On the other hand, under the fully centralised control, where the SC manager cannot control the production facilities effectively, additional, so-called safety lead time for each production facility is assumed; obviously, this adversely affects the overall SC performance. The authors offer an interesting, but non-expected conclusion: lower upstream stock levels (in the chain considered, this is the lower intermediate product stock), give better overall SC performance in spite of low costs of upstream materials. This conclusion holds for all the three control strategies.

The various SC control strategies reflect different levels of SC integration. However, the question of how to implement in practice some of the strategies proposed, particularly those that require a higher level of SC integration, remains open. The feasibility of management across different organisations along an SC is an important issue for a real implementation of these strategies.

According to Wilson (1991), the implementation of an SC strategy has at its heart the control and reconfiguration of the flow of information. The author outlines three important issues: (1) consolidation of all demand information and using it in decision-making along the chain, (2) shortening the length of communication channels, by involving fewer facilities in data processing, all focused on reducing information lead times and (3) increasing frequency of information management; for example, demand planning cycles are changed, from monthly to weekly.

## 2.6. Supply chain dynamics

One of the most often quoted example which describes the essence of SC dynamics is given by Houlihan (1987). It shows amplification of demand variation as transmitted along five companies (i.e., factories) linked in series, as illustrated in Figure 2.4. Demand variation is amplified between each two companies, reflecting on the capacity required. If one starts with an acceptable ripple of a few percent in company *A*, high fluctuations and even 20 times amplified values in company *E* are observed. As a

consequence, company *A* operates in an almost level-scheduling situation, characterised by a constant or slowly varying production rates, while the operation of company *E* is very disturbed due to the oscillations, increased in magnitude over time.

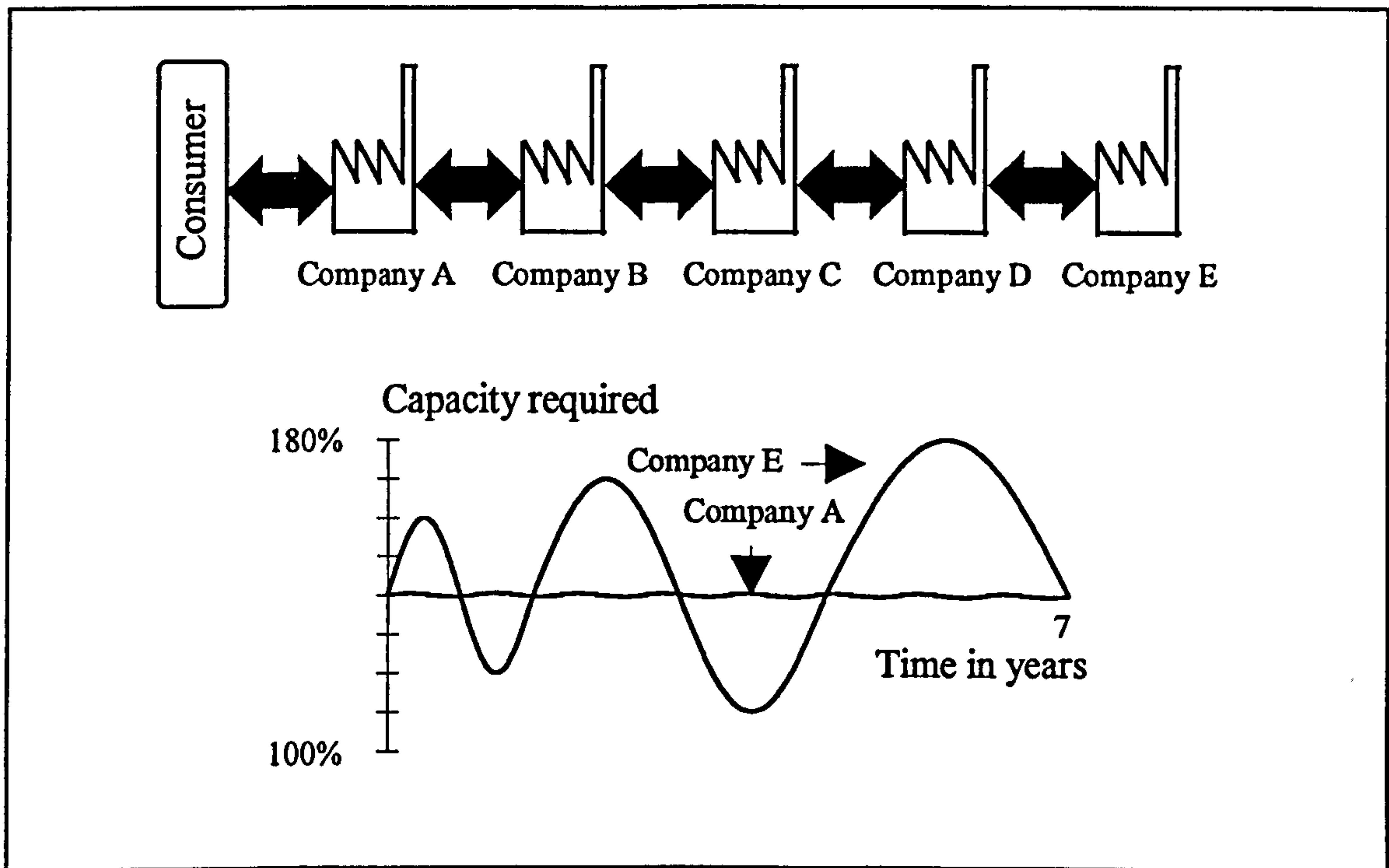


Figure 2.4. Amplification of demand as transmitted along an SC, (Houlihan, 1987)

The Law of Industrial dynamics, coined by Burbidge (1984), refers to this problem. It is postulated that "if demand for products is transmitted along a series of inventories using stock control ordering, then the demand variation will increase with each transfer". This phenomenon appears everywhere, in capacity planning, production rates and stock levels along an SC.

Research on these phenomena began in the early sixties, when Forrester (1961) stated the main principles of **industrial dynamics**. Forrester used basic concepts and techniques of control theory to study dynamic behaviour of industrial systems. The philosophy and methodology of industrial dynamics were introduced by an example of a production-distribution system, encompassing four stages: a factory, a factory

warehouse, a distributor inventory and a retailer inventory. There are flow of orders and flow of goods between each two neighbouring stages along the chain. Time delays exist in both flows; in the flow of orders they embrace clerical and postal delays and in the flow of goods there are manufacturing lead time and transport delays. The first industrial dynamics model developed is still used as a reference model and, therefore, it will be explained in some detail.

Generally, an industrial dynamics model is made up of vector difference equations of the first order, that represent the state of a system at discretised time intervals. Computation of the current system state is based on information available in the present and from the past, i.e., the system state in the previous time interval. Actually, this is one of the basic ideas of industrial dynamics approach; to link decisions and actions in an industrial system by information-feed back loops. The Forrester model of the production-distribution system includes 73 difference equations that determine relationships between the stock levels, rates of orders flow, rates of goods flow, unfilled orders, and other variables, for each stage in the system. Once the mathematical description of the system is identified, it is used to examine its dynamic behaviour. The flow of orders and the flow of goods along the chain are simulated and traced, step by step over time.

Forrester examined system behaviour in four typical situations. The three experiments performed are related to variations in retailer sales, characterised by (1) a small step increase in sales, (2) periodic variations in sales, or (3) random retailer sales. The fourth experiment involved a limited factory capacity. In all these experiments great fluctuations in the order rates, factory output, stock levels and unfilled orders along the chain are observed. For example, in the first experiment an increase in retailer sales of only 10% caused great swings in the retailer, distributor and manufacturing orders and the factory production output. Time delays in transferring orders and goods between the retailer and the distributor cause fluctuation in the order rate; retailer orders placed on the distributor reach the desired increase of 10% only after some time, but then continue to grow, exceeding the desired level. The



phenomenon of overshoot appears. These negative effects are transmitted and amplified along the stages in the system towards the factory, making the system dynamically unstable. A long time period is required before all the ordering rates and the production rate at the factory are stabilised.

Forrester concludes that the causes of system instability are organisational relationships and management policies at the factory, distributor and retailer. In order to improve system behaviour and to make it less vulnerable to variability of customer demand, Forrester explores three approaches:

1. faster order handling; however, reduced time delays do not improve system stability significantly and orders and production peaks remain very high,
2. eliminating the distributor stage; placing retailer orders directly to the factory warehouse reduces fluctuations along the system to a large extent,
3. changing inventory policies; various experiments were conducted to examine the effects of changing timing of order placement and quantities ordered; the more often and the more gradual the inventory corrections, the more stable the system was found to be.

Forrester identifies six interacting flows in an industrial system: materials (goods), orders, money, personnel, capital equipment and information. Although a possibility of including the flow of money is mentioned, the dynamic models developed so far do not include cost factors.

Forrester initiated the development of the industrial dynamics methodology. It is further expanded into the system dynamics methodology for modelling not just manufacturing, but also business systems, embracing both continuous and discrete control techniques. Towill (1993a, 1993b) discusses how both servo-mechanism theory, applied traditionally to technical systems, and cybernetics dealing with organisational and human systems, have influenced the system dynamics methodology. Software packages to support system dynamics modelling and simulation have been developed. Among them the most often used is DYNAMO (Pidd, 1984).

Three standard strategic approaches to meeting variation in demand in a production-inventory system are discussed by Towill (1991), and Towill, Naim and Wikner (1992). They are illustrated in Figure 2.5. One strategy is to fix the process rate and to make the plant busy evenly, all the time; due to the fact that all of the demand fluctuations are met from inventory, fluctuations in the stock level are generated. The alternative strategy to meet variation in demand is to keep the inventory level fixed and to vary the process rate; although this approach provides a constant inventory level which can be very low some disturbances in the production process can cause negative effects on the customer demand side. Besides, from the production point of view, large and frequent variations in the process rate are not desirable at all. The most reasonable approach is a compromise by which the demand variation is met combined from stock and by varying the process rate. Quantification of this approach is not at all an easy task.


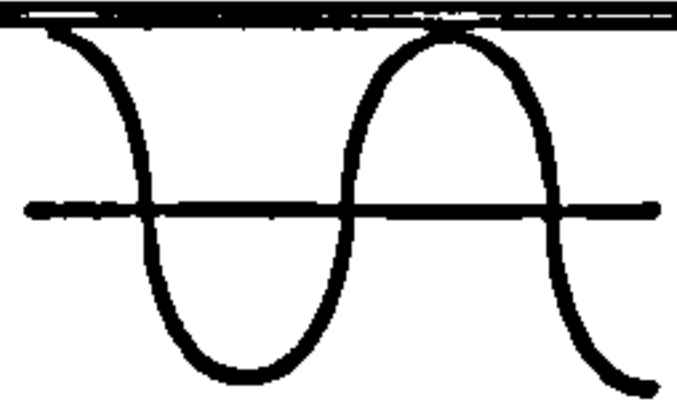
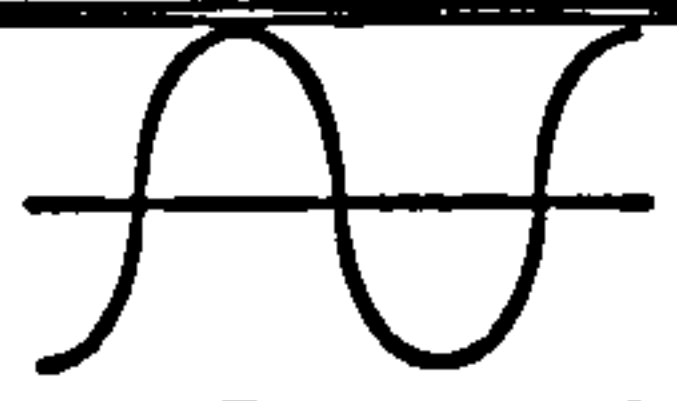


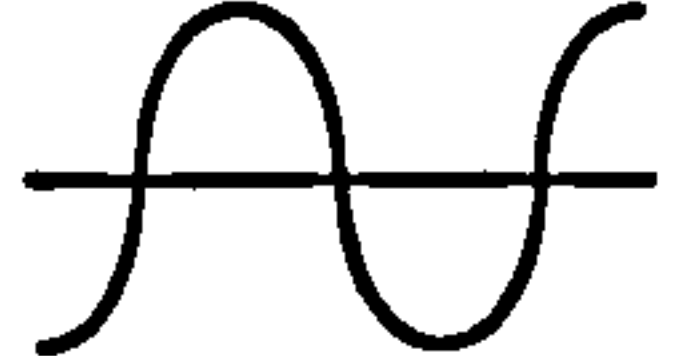


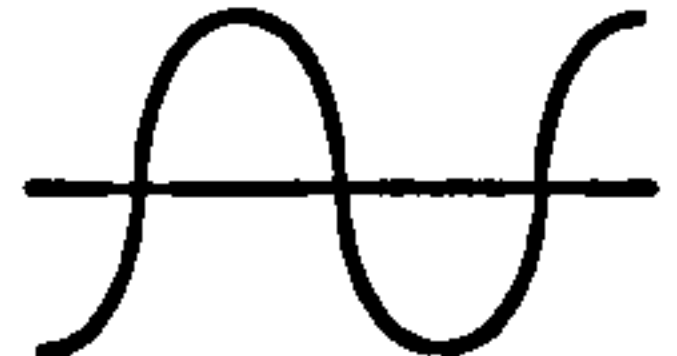
Strategy	Process	Inventory	Demand
Fixed process rate			
Fixed inventory level			
Compromise			

Figure 2.5. Trade-off between process rate and inventory level, (Towill, 1991)

Towill (1991) considers system dynamics models as tools for designing SCs in such a way as to reduce demand amplification along the chains. Strategic SC design takes into consideration the number of stages in the chain, level of information flow integration, parameters of ordering, lead times along the chain, etc. Desirable properties of an SC dynamic model given by Towill are: (1) it should reproduce

historical data to an acceptable degree of accuracy, i.e., having as inputs data recorded in the past, the model should generate outputs similar to those recorded, (2) it should predict behaviour of the SC in the future, when inputs (e.g., customer demand) are similar to data used in the model development, (3) it should enable tracing causes of demand amplification along the chain, and (4) it should predict circumstances under which demand amplification would occur. Towill states various causes of demand amplification. The author emphasises that interactions between individual "players" in an SC are significant contributing factors to demand amplification, and that the stability of an individual player in the SC depends not only on its own planning and control policy, but also on the design of the whole SC. It is concluded that integrating various decision-making mechanisms and the information flow along the SC can lead to the substantial reduction in order amplification and stock level swings. Another cause of demand amplification recognised is a time delay incurred in both the flow of orders and the flow of goods, by both "value-added" and "idle" activities (such as stocking, queueing, etc.). The negative effects of time delays can be reduced by shortening the times for transferring orders and goods and the production lead time, or by removing one of the stages in the chain.

The Forrester model has shed much light on SC dynamic behaviour. Since then, the system dynamics methodology has often been used to analyse distortion of information and data in flows through an SC and its impact on SC dynamic behaviour. Application of the system dynamics methodology for analysing SC dynamic behaviour and for improving total SC dynamic performance are noted here through a few representative models.

Pidd (1984) reports on a system dynamics model developed to give an insight into demand amplification problem in a real-life distribution system and gives practical guidelines on system dynamics model building. The system considered contains one central warehouse which supplies a number of regional warehouses, which in turn supply the next echelon of retailers. The echelons in the system apply different



inventory review policies, periodic at the retailer outlets and continuous at the regional and central warehouse. Also, different demand forecast methods are used; an exponential smoothing method at the retailer outlets, a weighted moving average at the regional warehouses and a double exponential smoothing method at the central warehouse. The author states that the smoothing forecast methods dampened out sudden or rare variations in demand level, but on the other hand cause a delay in introducing actual changes of demand in the corresponding forecast.

Four system dynamics models are developed; one for each of the echelons separately and one aggregate model for the whole distribution system. The models are developed based on the Forrester approach and follow two steps. First, the level-rate diagram is created for each model, to represent graphically the echelons and their activities, including the flow of orders, flow of goods, decision functions by which the flows are controlled, the beginning and final destination of each flow, their effects on the inventory levels and time delays. In the second step, the level-rate diagrams are mapped onto the corresponding relationships: (1) the level equations, which describe the changes of stock levels over time and (2) the rate equations, which represent control policies. The delays in receiving the orders and delivering goods are modelled by difference equations also. The dynamic behaviour of the distribution system is then simulated. The author of the paper emphasises the importance of the proper steps of time discretisation. Large steps of time discretisation would give too coarse modelling of system behaviour, while too short time increments would not just waste simulation time, but would require a large amount of data to be collected and analysed.

A response of the distribution system to small step wise increase in customer demand was investigated. The behaviour of each echelon considered separately under such change of demand was analysed first. The steady state of demand forecast and the fluctuations in the stock level and orders were observed at all three echelons. The negative effects of the customer demand change became even more evident when echelons are linked into the system. The typical familiar patterns of order fluctuations along the system, that characterise the demand amplification phenomenon are



illustrated in Figure 2.6. The changes in customer demand are amplified through the system substantially. The point is that larger amplification is observed at echelons that are further from the place where external customer demand is generated.

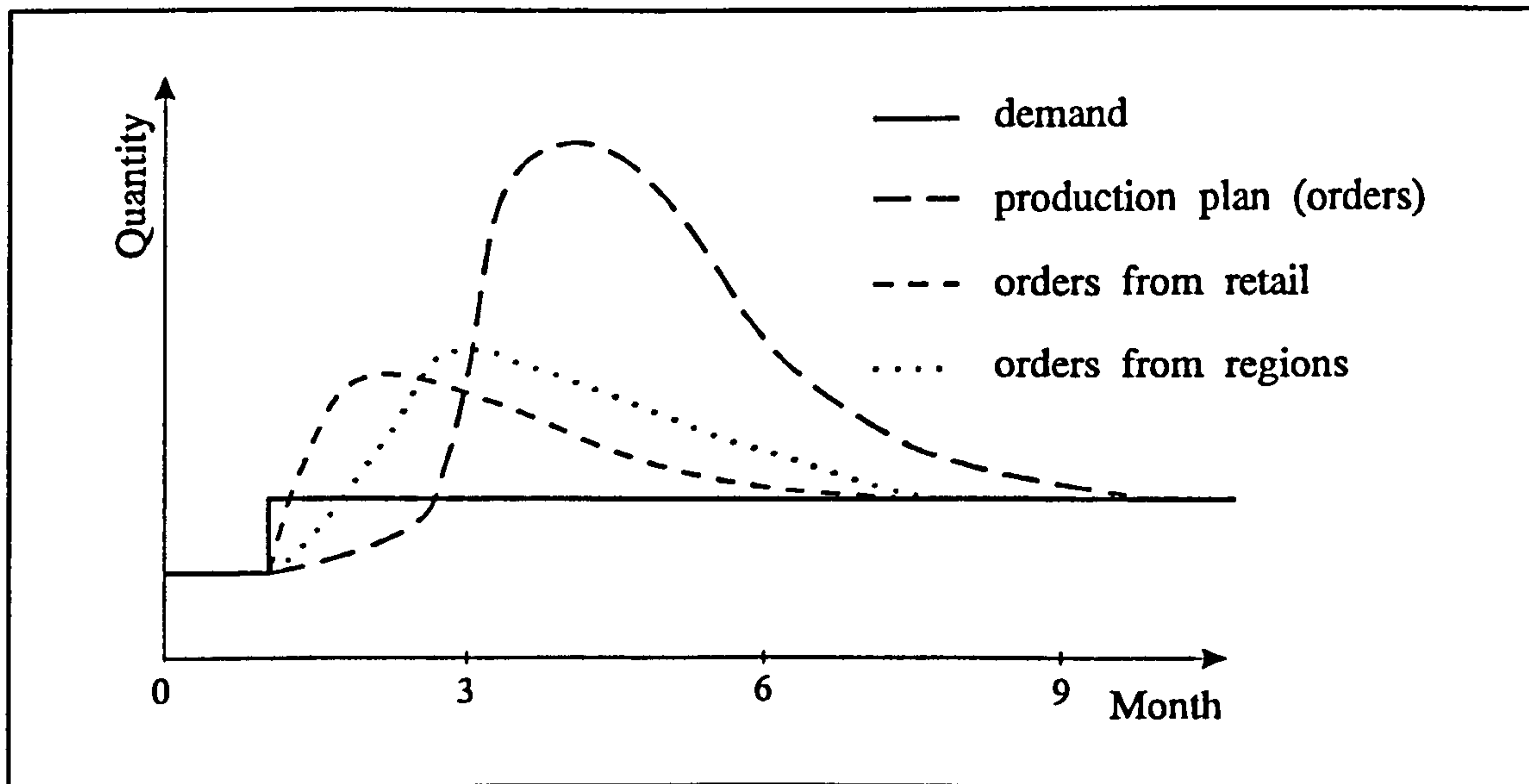


Figure 2.6. Effects of a step wise increase in customer demand, (Pidd, 1984)

Towill (1984) considers a simple production system which contains a factory and one inventory location only. It is recognised that one of the drawbacks of the Forrester type model that causes high demand amplification is that the factory does not have information about customer demand. In the Towill model the production order rate depends on two variables: the inventory deficit (i.e., the difference between the desired inventory level and the level achieved) and the average sales rate (i.e., average customer demand). This means that two control loops for determining the production rate exist: feed-back control based on the inventory level in the previous time interval and feed-forward control based on the demand forecast for the next time interval. Two parameters by which the production rate is controlled are specified: *TAI* - the time over which the present inventory deficit has to be recovered, and *TAC* - the time over which customer demands are recorded and used for demand forecast. Towill shows how

these system design parameters can be tuned to improve system dynamics behaviour. Extensive simulation experiments were conducted to establish the relationship between the two system design parameters and the key output variables: the inventory level and the production rate.

Edghill, Olsmats and Towill (1988) study dynamic behaviour of a simple production-distribution system with two stages only: a factory and a distribution stage which places orders directly on the factory. The calculation of an order from the distributor is based not only on the current distributor stock level, but also on average customer demand, which makes the system more responsive to demand changes, desired stock level, expressed as a multiple of averaged demand, and factory balance, i.e., difference between orders scheduled on the factory and actual production output. The shop floor schedules at the factory are set based on the orders received from the distributor and a fraction of outstanding arrears, remaining from previous production orders. The main contribution of this paper is that the model developed considers uncertain production expressed by an average percentage of a target production rate achieved and its standard deviation.

System dynamics characteristics, when a step wise increase in demand is applied, were examined. Two situations are considered: (1) production is uncertain and (2) the factory produces exactly to its scheduled target. Interesting results are observed. Demand amplification appears in both situations. However, it is noticed that to a certain extent uncertainty in production negates the effect of order amplification, reducing a peak overshoot in the distribution orders to the factory. On the other hand, uncertainty in production generates higher factory arrears and there is a tendency to eliminate it by increasing the production schedule; as a result, a high peak overshoot in the production schedule is observed.

Wikner, Towill and Naim (1991) made further progress in studying SC dynamics. They propose and analyse five strategies for improving SC dynamics, i.e., for smoothing demand and inventory level fluctuations along a chain. The Forrester

type model is used as a benchmark for evaluating the strategies proposed. The necessary level of SC integration and organisational changes required to implement each of the strategies in practice are discussed. The strategies examined are:

1. tuning the existing echelon decision rules, such as the time to smooth orders and the times to adjust inventory and pipeline deviations from the target levels,
2. reducing the system delays,
3. removing the distribution echelon from the SC,
4. improving the decision rules made at each echelon; for example it is shown that taking into account an average value of pipeline delays instead of a variable pipeline delay in the setting of the target inventory level at one echelon lead to better system dynamics,
5. integrating the information flow in various ways and making some of the information available at every echelon.

It is shown that the most effective are the strategies 3 and 5, and combination of all the strategies.

The application of control theory concepts in modelling industrial systems and, in particular, SCs may be arguable and it is still not a fully accepted approach (Towill, 1993a). However, Southall, Mirbagheri and Wyatt (1988) show that demand amplification phenomenon can be studied using the technique of discrete event simulation. The authors develop a discrete event simulation model of a three stage production-distribution system which involves a wholesaler inventory, a central warehouse and a factory. Uncertainty inherent in customer demand and a factory lead time is treated and the importance of examining the sensitivity of SC behaviour to uncertainty is highlighted. Uncertainty is introduced in the model by applying probability concepts. Customer demand is represented by a random variable that follows a normal distribution. The factory lead time can be either precisely known or a random variable, uniformly distributed over a time interval. In addition to the standard



Forrester type experiment that shows the effects of step wise change in customer demand on fluctuations in the stock levels and orders, experiments were performed to assess the impact of changes in: (1) customer demand uncertainty, expressed by its dispersion, (2) the stock target level at the central warehouse, (3) factory response time, (4) the frequency of the central warehouse orders placed on the factory and (5) uncertainty in the factory lead time. Conclusions concerning uncertainties in the system are the following: uncertainty in customer demand causes system instability, that propagates upstream towards the factory; the uncertainty in factory lead time has similar effects, but in the reverse direction. When both sources of uncertainty exist, the adverse effects are accumulating, causing high instability at the central warehouse. The authors of the paper suggest that the results obtained in the experiments performed require further explanation and additional study.

Lee, Padmanabhan and Whang (1997) have focused on the demand information flow in an SC. They analyse the same phenomenon of systematic distortion in demand information as it is passed along the SC, as Forrester did. In contrast with Forrester, the authors consider this phenomenon within the framework of classical inventory theory and have used optimisation models to represent rational behaviour of SC members. They have proved that orders to a supplier tend to have a larger variance than sales to a buyer and the distortion propagates upstream in an amplified form. This phenomenon has been termed “the bullwhip effect”. The authors examined four sources of the bullwhip effect: (1) demand signal processing where demand is non-stationary and past demand information is used to update demand forecasts, (2) rationing game that refers to the strategic ordering behaviour of buyers when supply shortage is anticipated, (3) order batching applied to gain economies in pricing and transportation and (4) price variations that refer to nonconstant purchase prices of products. Metters (1997) has investigated the impact of the bullwhip effect on SC costs. The author has shown that reducing demand seasonality and/or demand variance



can cause substantial reduction of the SC costs, particularly in the cases of tight capacity conditions and large penalty cost for lost sales.

## 2.7. Stochastic aspects of supply chains

Browsing the literature devoted to SCs, models can be found that treat uncertainties inherent in real-life SC management and control problems. These are stochastic models, where uncertain data are described by random variables, assuming that the types of their probability distributions, expected values, standard deviations or other distribution parameters are known. The effects of uncertainty on a single inventory or production facility operations have been studied in the literature. For example, Lau and Zaki (1982) give a deep insight into the effects of the shape of probability distribution of demand during lead time on some basic inventory decisions.

The problem becomes much more complicated when one considers a whole SC network. Uncertainty propagates through the network in both directions. From one side, there may be uncertainty in customer demand that is transmitted through the network, creating uncertainty in demand for materials, and from another side, there is uncertainty of external supply of materials that causes uncertainty of supply from one to the succeeding facility in the chain. Uncertainty may exist in all SC activities, including procurement of raw material, production and distribution.

A few stochastic SC models are discussed here to identify the main sources of uncertainty that have been considered and the ways in which they have been represented.

There are models devoted to buyer-vendor problems in SCs which include uncertainty of buyer demand for raw materials or subassemblies, vendor delivery of raw materials or subassemblies, or vendor delivery lead time. For example, Anupindi and Akella, (1993) describe uncertain delivery in three different ways: (1) by a probability that all goods ordered from a vendor arrive in the current period and a

complementary probability that the goods arrive in the next period, (2) by a random fraction of an order delivered in the current period, with the assumption that the portion of the order quantity not delivered is cancelled or (3) by a random fraction of an order delivered in the current period and the full delivery of the remainder in the next period.

Although, there are many sources of uncertainty inherent in the production and distribution activities, attempts have been made to model only a few of them. For example, Pyke and Cohen (1994) treat randomness in stationary demand for products, and a number of periods between replenishment batches ordered by a distribution centre.

Most of the inventory-distribution models use probability concepts to model uncertain customer demand (Van Houtum, Inderfurth, and Zijm, 1996). In addition to uncertain demand, some of the inventory-distribution models assume stochastic transit times between stock locations (for example, Svoronos and Zipkin, 1991).

Cohen and Lee's model (1987), which covers a whole SC network, includes two key uncertain parameters: (1) random demand for material requested by a plant over a delivery lead time and (2) random demand for a product placed on a plant by a distribution centre during a production lead time. The two uncertain parameters reflect interactions between different SC activities, material procurement and production, and production and distribution, respectively.

Lee and Billington (1993) developed a stochastic model that includes three main sources of uncertainty in a real-life SC: customer demand, supply and production processes. Demand is assumed to be normally distributed, with a known mean and standard deviation. Uncertainty of supply is characterised by uncertain time in delivering ordered quantities; it involves various times, expressed by their average (expected) values and standard deviations, including a material delay time caused by shortage of material needed to manufacture ordered components, a manufacturer time and a transit time. Uncertainty of the production time is affected by random occurrence of production downtimes and by uncertain duration of downtime on the production

line. In order to simplify the model, it is assumed that the number of downtime occurrence is Poisson distributed.

With respect to uncertainty, Southall, Mirbaghen and Wyatt's study (1988) is very interesting, because they do not just model uncertainty in an SC, but also investigate the effects of changing uncertainty associated with customer demand and factory lead time on SC behaviour. Changes of uncertainty in customer demand are represented by changing dispersion of the corresponding normal distribution, while changes of uncertainty in the factory lead time are expressed by varying the boundaries of the corresponding uniform distribution. The authors just initiate a study of the complex problems related to uncertainty in SC management and control.

A conclusion can be made that further research is required to better understand the sources of uncertainty and the effects that uncertainty has on SC operations, behaviour and performance.

## **2.8. Supply chain models in practice**

The development of some of the SC models reviewed in the previous sections have been motivated by real-world SCs (for example, Lawrence, 1977, Southall, Mirbaghen and Wyatt, 1988, Pidd, 1984, etc.). In addition, two inspirational and representative strategic real-world SC models, which further reduce a gap between the SC theoretical consideration and practice, are presented in this section. They are selected among other models because they treat the whole, complex SC networks, from external suppliers to end-customers, give some practical guidelines for building real-world SC models, and show how SC models can be used for solving various actual SC management and control problems.

Lee and Billington (1993) consider an SC for one of Hewlett-Packard product - printers. The SC analysed is very complex, involving about 300 external suppliers and



more than 200 components. In the authors' opinion a comprehensive SC model, although accurate, would be difficult for analysis and would require a large amount of data. Therefore, only those components that constitute the bulk of the cost incurred or the components for which supply is highly uncertain are brought into focus. As a result two main manufacturing stages, that can be treated as SCs themselves, and three distribution centres, that are world-wide located, are identified. The SC under consideration is decentrally controlled.

First, a basic model for a single production-inventory facility is developed. All the facilities are then treated together, linked by two processes, named by the authors as: (1) the demand transmission process, where demand placed on one facility translates, via a bill of material, to demand for materials or components that is transmitted to the supplying facilities and (2) the supply transfer process, where the availability of input materials or components at a production facility is determined by the service performance for these materials or components at its supplying facilities. The problem of specifying target service levels or target inventory levels throughout the SC is pointed out. In the current implementation of the SC model, a simple heuristic is used to find a combination of service levels or inventory levels throughout the chain that support the desired end-customer service level. This problem is identified as an area for further research.

The authors report on valuable experiences of applying this SC model in a few on-going projects:

1. Assessment of the existing SC performance, including evaluation of current inventory stocking level effectiveness, analyses of impact of supplier improvement programs, assessment of alternative ways of shipping finished products, etc.,
2. Analysis of alternative manufacturing/distribution strategies for launching a new product, and
3. Cost/benefit analysis of changes in the printer design and their effects on SC operations and delivery service to customers.



Usage of the SC model in a new product launching is discussed in (Lee and Billington, 1993). The structure of the new SC network is slightly changed, containing one more assembly facility. Due to the fact that no historical data about demand for the new product exist, it is assumed that it will be the same as demand for the already existing product. It is shown that an additional link to the new assembly facility would require a substantial increase in finished product stock level. Strategies for improving SC performance are analysed, such as (1) using a new inventory stock to hold components ready for shipment to the new assembly, (2) shortening the transit time to the new assembly facility, (3) placing the manufacturing facility that produces the components for assembly and the new assembly facility at the same location and (4) shipping the products from the new assembly facility to the distribution centres more frequently. All these strategies are compared using a standard inventory/service trade-off curve, and by analysing the inventory requirements, expressed by number of weeks of finished products supply to be held at the distribution centres, to meet a target service level.

The SC model developed is also used to address the consequences that different printer design alternatives have on SC costs and delivery service to customers (Lee, Billington and Carter, 1993). Traditionally, product design evaluation takes into account the impact of the design on manufacturability, cost and quality. However, different markets may have different requirements for products. In the case of printers, for example, appropriate power supply modules and manuals should be provided for different countries. The authors define a term "design for localisation" to give an importance to the relationship between a design for a product and requirements of different markets, and conclude that the design for localisation affects the SC operational cost and its delivery service to customers. The SC model is used to determine a location in the SC at which it is the most cost effective to perform printer localisation, i.e., to prepare a printer for a specific market. Two alternatives are investigated: (1) factory-localisation, where the printer localisation is done at the factory and already localised printers are shipped to the distribution centres; to

implement this strategy the factory has to hold the inventory of the localisation materials, and (2) distribution-centre-localisation, where the distribution centres, equipped with the localisation materials, perform printers localisation; the implementation of this strategy requires some changes of the already existing printer design. It is shown that in the case of distribution-centre-localisation a target service level can be achieved with lower inventory levels.

Arntzen et al. (1995) report on applying an SC model for redesigning and rationalising a world-wide supply and delivery network within Digital Equipment Corporation. The model developed is deterministic, based on cost and/or production and distribution time optimisation. It is a very complex model that includes many variables and constraints, and complex optimisation methods have to be applied to find an optimal solution. The authors mention various strategic analyses that have been successfully performed using this SC model, such as:

- to find an SC configuration that lead to the minimum SC cost,
- to find the shortest cycle time (i.e., the cumulative manufacturing and distribution time) for a product,
- to generate a cost/time trade-off curve,
- to compare SC performance, measured by cycle time and SC cost, achieved with the existing network and a network configuration recommended by the model,
- to determine the influence that different suppliers have on the cycle time and the SC cost incurred,
- to quantify the contributions of various costs (e.g., inventory charges, production charges, distribution expenses via multiple modes, duties, labour costs, taxes, etc.) to the total SC cost,
- to examine the influence that local content requirements have on the cycle time and the SC cost, where a local requirement, for example, specifies a percentage of product parts that has to be manufactured in a particular country.

## 2.9. Summary

The key aspects of SC management and control problems treated in the literature have been identified. The literature reviewed shows that the interest in SC analysis has moved from coordination of just parts of SCs, to the global SC coordination, including raw material supply, production and delivery of end-products to customers. A wide range of methods and techniques have been applied in SC management and control, encompassing optimisation, heuristics and simulation.

A great deal of research has been done in the area of SC dynamics. A widely used approach to studying SC dynamics has been based on the system dynamics methodology, which includes concepts and techniques of control theory. First, the interest has been focused on examining the effects of changes in an external environment on an SC, usually described as step increases or oscillations in customer demand. These SC dynamics models are deterministic models. Another approach used to study SC dynamics has been based on modelling an SC as a discrete event system. In this approach some uncertainties in SC data have been modelled by stochastic variables.

The importance of addressing uncertainty inherent in SCs and their environments has been recognised in the literature. Uncertainties in all the SC models developed so far have been treated as random processes and modelled by stochastic variables and probability distributions. The stochastic SC models have been usually limited to treating uncertainty in customer demand. Only a small number of the SC models have included and treated some other sources of uncertainty, and most often, it has been lead time.

Although, the problem of treating uncertainty in an SC is very real and important, reviewing the SC literature has revealed that it is an open area for research. In order to gain better understanding of uncertainty that realistic processes in SCs and their environments introduce, research is undertaken by the author to develop a new



fuzzy approach to modelling various sources and different types of uncertainties in SCs, which take place simultaneously.

## **CHAPTER 3**

### **TREATING UNCERTAINTY USING FUZZY SETS THEORY A SURVEY**

The problem of representation of and reasoning with uncertain data and knowledge is of importance in a broad range of disciplines, e.g., engineering, manufacturing, artificial intelligence and expert systems, decision theory and many others. Dealing with uncertainty has been debated widely in the literature. However, it is evident that the management of uncertainty presents a complex and still not fully understood problem. The term uncertainty has been given a wide interpretation, encompassing different sources of uncertainty that can be distinguished, such as noisy data, random processes, lexical imprecisions (i.e., vague, ill defined, imprecise and ambiguous concepts and definitions), incomplete data, contradictory data, uncertain knowledge about causal links in a process under consideration.

For a long time, scientists, philosophers, mathematicians and statisticians have used the concepts of a random event and probability to model uncertainty. Nevertheless, during the last few decades limitations of probability theory as the only means for dealing with uncertainty have been recognised. As a result, several approaches that have different perspectives of uncertainty have been developed and analysed (Henkind and Harrison, 1988), such as Fuzzy Sets Theory (Zadeh, 1965, Zadeh, 1983), Certainty Factor Methods (Shortliffe and Buchanan, 1984) and Dempster-Shafer Theory (Shafer, 1976). Among them, fuzzy sets theory has developed the most rapidly, including both a theoretical framework and practical application of the theory.

Fuzzy sets theory was introduced by Lotfi Zadeh (Zadeh, 1965). It is initially developed in the context of complex systems analysis. Zadeh's Principle of incompatibility relates the degree of complexity of a system to the precision inherent in a model of the system, and asserts that complexity of a system and ambiguity and imprecision are correlated: "As the complexity of a system increases, our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics", (Zadeh, 1973).

Last decade witnessed tremendous interest in fuzzy sets theory and its applications. Thousands of publications are now available in the area. Let us mention a few of the titles, the most often referred to: excellent overviews of fuzzy sets theory and its mathematical foundations are given in (Dubois and Prade, 1980, Klir and Folger, 1988), and Zimmermann (1991) and Ross (1995) cover and analyse a wide range of fuzzy set applications.

The incorporation of fuzzy sets theory has proved to be very fruitful in areas where intuition and judgement play a significant role. In fuzzy sets theory, the notion of a standard set defined by a binary membership, i.e., a certain membership or a certain non-membership in the set is extended to accommodate a gradual transition from a membership to a non-membership through various degrees. It offers a variety of concepts and techniques for treating different types of uncertainty inherent in real-world problems. It can be used in situations characterised as follows:

- Data values and relations between them are uncertain and imprecise. Their estimation is based on the subjective beliefs of individuals.
- It is difficult to measure data, either because there is no unit of measurement or there is no quantitative criterion for representing their values.
- Some of the parameters that characterise a problem under consideration are vaguely and unclearly defined.
- The knowledge available about a problem is complex, limited or incomplete.



- Human reasoning, perception or decision making are inextricably involved in problem solving.

This chapter gives a brief survey of fuzzy sets theory connected with the subject of the thesis. In Section 3.1. basic definitions, notions and properties of fuzzy sets, that have been used in the research presented in the thesis, are given. The comparison of the two approaches in treating uncertainty, fuzzy sets theory and probability theory, occupied a lot of researchers' attention. Some aspects of this comparison are discussed in Section 3.2. Different ways of measuring uncertainty represented by fuzzy sets are defined in Section 3.3. Fuzzy sets theory has been applied in variety of disciplines. Application of fuzzy sets and techniques in production management and control problems, and particularly in inventory problems, are reviewed in Section 3.4. Conclusions made are given in Section 3.5.

### 3.1. Basic definitions in fuzzy sets theory

#### 3.1.1. Fuzzy sets concepts, properties and operations

This section introduces some of the basic concepts, properties of and operations on fuzzy sets. Many of the definitions of the classical (crisp) sets theory are generalised in fuzzy sets theory, but some of them are unique to the fuzzy sets framework.

Definition of a fuzzy set. Let  $X$  denote a universal set with elements of  $X$  denoted as  $x$ . A fuzzy set  $A$  of  $X$  is characterised by a membership function  $\mu_A(x)$  where  $\mu_A(x): X \rightarrow [0,1]$  associates each element  $x$  with a degree of membership of  $x$  in  $A$ .

Naturally, a complete membership of an element  $x$  in fuzzy set  $A$  is represented by the membership degree 1 and a complete non-membership by the degree 0. As in the classical sets theory, the membership degree of any element  $x$  in null set  $\emptyset$  is 0 and the membership degree of any element  $x$  in universal set  $X$  is 1.

When the universal set is discrete and finite,  $X = \{x_1, x_2, \dots, x_I\}$ , a conventional way of representing fuzzy set  $A$  is:

$$A = \{\mu_A(x_1)/x_1, \mu_A(x_2)/x_2, \dots, \mu_A(x_I)/x_I\} \quad (3.1)$$

Also, in this case, fuzzy set  $A$  is often expressed in the following form:

$$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \dots + \mu_A(x_I)/x_I = \sum_{i=1}^I \mu_A(x_i)/x_i \quad (3.2)$$

where  $+$  denotes the collection of the elements.

Similarly to this, in the case when universal set  $X$  is continuous and infinite, a fuzzy set  $A$  is expressed by

$$A = \int_{x \in X} \mu_A(x) \quad (3.3)$$

A key difference between a classical (crisp) set and a fuzzy set is in their membership function. A crisp set has a unique membership function. On the other hand, an uncertain concept can be modelled by fuzzy sets with different membership functions. This means that uniqueness is sacrificed, but flexibility is gained because a membership function can be adjusted for a particular application.

The basic concept of a fuzzy set is extended in different ways, introducing a type 2 fuzzy set (Zadeh, 1975a, 1975b and 1975c), a level 2 fuzzy set (Zadeh, 1971) and an interval-valued fuzzy set (Turksen, 1986).

Type 2 fuzzy set. The accuracy of any membership function is necessarily limited. Although, a representation of fuzziness using membership degrees that are themselves precise real numbers does not pose a serious problem for many applications, it is nevertheless possible to extend the concept of a fuzzy set to allow imprecise degrees of its membership function. Elements of a type 2 fuzzy set have membership degrees that are represented by standard fuzzy sets. Similarly, higher types of a fuzzy set can be defined, but they are hardly of any interest to practical applications.

Level 2 fuzzy set. This extension of the fuzzy set concept involves creating a fuzzy set whose elements are standard fuzzy sets. This fuzzy set is named a level 2 fuzzy set. Generally, a level  $k$  fuzzy set can be defined, where  $k$  indicates the depth of nesting. Very often, in practical applications it is useful to transform a level 2 fuzzy set into a standard fuzzy set and to perform standard fuzzy set operations on it.

s-fuzzification (Zadeh, 1971) is a method of transforming a level 2 fuzzy set  $A$  into an ordinary fuzzy set,  $s\text{-fuzzif}(A)$ , defined in an universal set  $X$ . Let  $A$  take  $J$  fuzzy values  $a_j(x)$ ,  $x \in X$ ,  $j = 1, \dots, J$  with the possibilities  $\mu_A(j)$ . The membership function  $\mu_{s\text{-fuzzif}(A)}$  of the corresponding standard fuzzy set  $s\text{-fuzzif}(A)$  is calculated as:

$$\mu_{s\text{-fuzzif}(A)}(x) = \sup_{j=1, \dots, J} \mu_A(j) \cdot \mu_{a_j}(x), \quad x \in X \quad (3.4)$$

Interval-valued fuzzy set. The requirement for a precise membership function of a fuzzy set can be relaxed by allowing membership degrees to be represented as intervals of real numbers within the  $[0,1]$  interval. This type of a fuzzy set is a special case of a type 2 fuzzy set and it is called an interval-valued fuzzy set.

Standard relations of set equality and set inclusion are defined in fuzzy sets theory as follows.

Equal fuzzy sets. Fuzzy sets  $A$  and  $B$  are equal,  $A = B$ , iff  $\mu_A(x) = \mu_B(x)$ , for every  $x \in X$ .

Fuzzy sets inclusion. A fuzzy set  $A$  is a subset of a fuzzy set  $B$ ,  $A \subset B$ , iff  $\mu_A(x) \leq \mu_B(x)$ , for every  $x \in X$ .

Fuzzy sets theory was initially based on the following definitions of the three basic fuzzy set operators, namely set complement, union and intersection. Let  $A$  and  $B$  be fuzzy sets of an universal set  $X$  with membership functions  $\mu_A(x)$  and  $\mu_B(x)$ ,  $x \in X$ , respectively.



The complement of a fuzzy set  $A$  is a fuzzy set  $A^c$  with the membership function

$$\mu_{A^c}(x) = 1 - \mu_A(x), x \in X \quad (3.5)$$

The union of fuzzy sets  $A$  and  $B$  is a fuzzy set  $A \cup B$  with the membership function

$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x)), x \in X \quad (3.6)$$

The intersection of fuzzy sets  $A$  and  $B$  is a fuzzy set  $A \cap B$  with the membership function

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x)), x \in X \quad (3.7)$$

It is important to note that in the case when membership degrees are restricted to only two values, 0 and 1, the definitions of the fuzzy set operators given above, reduce to the corresponding crisp set operators. In addition, different membership functions that have certain axiomatic properties have been proposed and analysed for each of the basic fuzzy set operators (Klir and Folger, 1988). This makes fuzzy sets theory very flexible, but on the other hand generates a problem of selecting fuzzy set operators suitable for a particular application. Some very general guidelines that may be helpful in the selection are provided in the literature (for example, Zimmermann, 1991). However, in most of the applications the standard initial definitions of the fuzzy set operators are used, due to their simplicity and the desirable properties they have.

Fuzzy sets and crisp set operators have many properties in common. However, the laws of excluded middle and contradiction do not hold for fuzzy sets in their traditional forms and this is one of the most important difference between the two theories.

The laws of excluded middle and contradiction for a fuzzy set  $A$  and its complement  $A^c$  are expressed by the following relations:

$$\begin{aligned} A \cup A^c &\neq X \\ A \cap A^c &\neq \emptyset \end{aligned} \quad (3.8)$$

In other words, the two fundamental laws of the classical sets theory are broken. A fuzzy set and its complement may overlap and consequently, an element may belong to both, the fuzzy set and its complement.

Linguistic hedges are additional operators, defined in fuzzy sets theory. A linguistic hedge is an operation that modifies the meaning of a linguistic term. The modification is performed by changing the corresponding membership function of the fuzzy set which represents the linguistic term. Two most often used hedges are (Mendel, 1995):

Concentration of a fuzzy set  $A$  generates a fuzzy set  $Con(A)$  with the membership function

$$\mu_{Con(A)}(x) = [\mu_A(x)]^2, x \in X \quad (3.9)$$

It is used to strengthen the meaning of a linguistic term represented by a fuzzy set  $A$  and can model an adverbial or an adjectival qualifier such as *very*, *rather* and so on.

Dilation of a fuzzy set  $A$  generates a fuzzy set  $Dil(A)$  with the membership function

$$\mu_{Dil(A)}(x) = [\mu_A(x)]^{1/2}, x \in X \quad (3.10)$$

It is used to weaken the meaning of a linguistic term represented by a fuzzy set  $A$  and can model a linguistic qualifier such as *fairly*.

The exponents used in the hedge definitions are quite arbitrary and reflect a particular interpretation of the hedges.

### 3.1.2. Membership functions

The concept of a fuzzy set is based on a membership function. The membership function embodies uncertainty, imprecision or fuzziness associated with data. Special features that characterise a membership function are defined as follows.

The support of a fuzzy set  $A$  in an universal set  $X$  is a crisp set,  $support(A)$ , that contains all the elements of  $X$  that have a nonzero membership degree in  $A$ , i.e.,

$$support(A) = \{x \in X \mid \mu_A(x) > 0\} \quad (3.11)$$

The core of a fuzzy set  $A$  is a crisp set,  $core(A)$ , that contains all the elements of  $X$  that have a full membership degree in  $A$ , i.e.,

$$core(A) = \{x \in X \mid \mu_A(x) = 1\} \quad (3.12)$$

The height of a fuzzy set  $A$ ,  $height(A)$ , is the largest membership degree attained in  $A$  by any element in  $X$ , i.e.,

$$height(A) = \max_{x \in X} \mu_A(x) \quad (3.13)$$

A fuzzy set  $A$  is normalised when at least one of its elements attains the maximum membership degree 1, i.e.,  $height(A) = 1$ .

A cross over point of a fuzzy set  $A$  is an element in an universal set  $X$  whose degree of membership in  $A$  is 0.5.

Creating membership functions. A fuzzy set is represented by its membership function and, hence, it is important to consider how an appropriate membership function is created. The membership function of a fuzzy set can be obtained based on one's experience, subjective belief, intuition and contextual knowledge about the concept modelled (Zimmermann, 1978). This approach to providing membership functions prevailed in the literature and practical applications for a long time. However, subjectivity in determining membership functions has been considered as the weakest point in fuzzy sets theory. Some basic statistical methods based on polling a group of people have been used for creating membership functions (Dubois and Prade, 1986). One method for generating a membership function from a probability distribution is described in Section 3.2. In recent years, a lot of attention has been devoted to this issue. New approaches to creating membership functions are surveyed in (Ross, 1995).



One of the methods widely applied relies on neural network techniques. A neural network uses a learning mechanism and a set of available data to derive a membership function. There is a growing number of research papers that combine neural networks and fuzzy sets theory to form a powerful framework for solving various problems (Kosko, 1992).

### 3.1.3. Fuzzy to crisp conversions

Modelling, computing and reasoning that involve fuzzy sets generate results in the form of fuzzy sets. Very often, especially in engineering and manufacturing applications, it is necessary to convert a fuzzy result to a crisp solution, action or decision. A fuzzy set can be reduced: (1) to a crisp set, called an  $\alpha$ -cut set or (2) to a representative scalar; this conversion is known as arithmetic defuzzification. These conversions are defined as follows.

$\alpha$ -cut of a fuzzy set  $A$  is a crisp set  $A_\alpha$  that contains all the elements of an universal set  $X$  that have membership degrees in  $A$  greater or equal to the specified value  $\alpha$ , i.e., formally written:

$$A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\} \quad (3.14)$$

$\alpha$ -cut is an important concept in fuzzy sets theory. Due to the fact that  $\alpha$ -cut of a fuzzy set is a crisp set, using this conversion of a fuzzy set into a crisp set, concepts from classical sets theory can be translated into fuzzy sets theory.

Arithmetic defuzzification methods. Arithmetic defuzzification is a conversion of a fuzzy set into a scalar that "the most suitably" and "correctly" represents the fuzzy set. Several arithmetic defuzzification methods have been proposed. Defuzzification methods that are most often applied are summarised here.

Maximum method selects an element  $x^*$  from the support of a fuzzy set  $A$  which attains the maximum membership degree, i.e.,

$$\mu_A(x^*) \geq \mu_A(x), \quad x \in X \quad (3.15)$$

Different modifications of this method are defined in the case when the element with the maximum degree in a fuzzy set is not unique, including Mean-maximum method which calculates a mean of all the values that attain the maximum membership degree and First (or Last) of maxima which selects the smallest (or the largest) element from the support with the maximum membership degree.

Centroid method (also called Moment rule) finds an element  $x^*$  in the support of a fuzzy set  $A$  at which a line perpendicular to the axis passes through the centre of the area formed by the corresponding membership function. Value  $x^*$  is calculated by the following expression:

$$x^* = \frac{\int_{x \in X} x \cdot \mu_A(x)}{\int_{x \in X} \mu_A(x)} \quad (3.16)$$

These two methods are the most widely used for defuzzification. There is no theoretically grounded approach to deciding which defuzzification method is to be preferred. However, some important characteristics of the methods help in the selection (Graham and Jones, 1988). The use of the Maximum method leads to discontinuities in the decision space. This means that not every change in input data described by fuzzy sets causes a change in a decision made. Changes in input data induce a change in an output fuzzy set, but the elements that reach the maximum membership degree might remain the same. As opposed to such a robust method, the Centroid method provides a continuous response to variations in input data on an output fuzzy set, and it is more useful for continuous problems.

However, in some situations more than one output fuzzy sets exist and a scalar is to be found that represents all the output fuzzy sets. In such cases, the two defuzzification methods previously described are applied to one fuzzy set obtained by aggregation of the output fuzzy sets. Most often, aggregation is performed by applying the union operator on the output fuzzy sets. The next two defuzzification methods treat these output fuzzy sets differently.

Weighted average method calculates a representative scalar  $x^*$  of output fuzzy sets  $A_i$ ,  $i = 1, \dots, I$  by weighting each membership function  $\mu_{A_i}$  by its maximum membership degree. More precisely,

$$x^* = \frac{\sum_{i=1}^I x_i^* \cdot \mu_{A_i}(x^*)}{\sum_{i=1}^I \mu_{A_i}(x^*)} \quad (3.17)$$

where  $x_i^*$ ,  $i = 1, \dots, I$  is the mean of values in the support of fuzzy set  $A_i$  that obtain a maximum membership degree.

Centre of sums involves the algebraic sum of the output fuzzy sets  $A_i$ ,  $i = 1, \dots, I$  instead of their union. The defuzzified value  $x^*$  is calculated by the following equation:

$$x^* = \frac{\sum_{x \in X} x \cdot \left( \sum_{i=1}^I \mu_{A_i}(x) \right)}{\sum_{x \in X} \left( \sum_{i=1}^I \mu_{A_i}(x) \right)} \quad (3.18)$$

### 3.1.4. Extension Principle

Extension Principle is one of the most important principles in fuzzy sets theory since it allows the generalisation of classical mathematical concepts to the fuzzy framework



(Zadeh, 1965). It provides a means for any function  $f$  that maps a crisp set  $X$  to a crisp set  $Y$  to be generalised such that it maps a fuzzy set of  $X$  to a fuzzy set of  $Y$ . Formally, given a function  $f: X \rightarrow Y$  and a fuzzy set  $A$  of  $X$ ,  $A = \{(x, \mu_A(x)) \mid x \in X\}$ , a fuzzy set  $B$ ,  $B = \{(y, \mu_B(y)) \mid y \in Y\}$  is induced as:

$$\mu_B(y) = \begin{cases} \max_{x \in f^{-1}(y)} \mu_A(x) & \text{iff } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases} \quad (3.19)$$

where  $f^{-1}(y) = \{x \in X \mid f(x) = y\}$ .

The Extension Principle simply states that if only one element  $x \in X$  is mapped by function  $f$  to an element  $y \in Y$ , then the membership degree of  $y$  in fuzzy set  $B$  induced by fuzzy set  $A$  of  $X$  is equal to  $\mu_A(x)$ ; if there are more than one elements in  $X$  mapped to the same element  $y$ , then the maximum of the membership degrees of these elements in fuzzy set  $A$  is chosen as the membership degree of  $y$  in fuzzy set  $B$ ; if no element  $x$  is mapped to element  $y$ , then the membership degree of  $y$  in fuzzy set  $B$  is 0.

The Extension principle can be applied to a function  $f: X_1 \times X_2 \times \dots \times X_I \rightarrow Y$  in the following way. If fuzzy sets  $A_i$ ,  $i = 1, \dots, I$  are defined in  $X_i$ , function  $f$  induces a fuzzy set  $B$  of  $Y$ , with the membership function  $\mu_B(y)$ ,  $y \in Y$ :

$$\mu_B(y) = \begin{cases} \max_{(x_1, x_2, \dots, x_I) \in f^{-1}(y)} \min(\mu_{A_1}(x_1), \mu_{A_2}(x_2), \dots, \mu_{A_I}(x_I)) & \text{iff } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases} \quad (3.20)$$

where  $f^{-1}(y) = \{(x_1, x_2, \dots, x_I) \in X_1 \times X_2 \times \dots \times X_I \mid f(x_1, x_2, \dots, x_I) = y\}$ .

### 3.1.5. Fuzzy numbers and fuzzy arithmetic

Definition of a fuzzy number. Fuzzy number  $R$  is a fuzzy set defined on real numbers  $\mathcal{R}$  with a (1) convex, (2) normalised and (3) piecewise continuous membership function.

Very often, engineering and manufacturing applications deal with uncertain data specified to be *around* a certain value, *close* to a certain value or *approximately equal* to a certain value. They can be represented by fuzzy numbers with triangular, trapezoidal or bell-shaped membership function (Cox, 1994).

A triangular membership function is determined by three values  $[l, m, u]$ , where  $l$  represents the lower bound,  $m$  the mean value, and  $u$  the upper bound of the fuzzy number (see Figure 3.1. (a)). The membership degrees are 0 for values  $l$  and  $u$ , and it reaches 1 for value  $m$ . The larger the difference  $u-l$ , the greater the uncertainty in the represented data.

A triangular membership function can be extended to a trapezoidal membership function, depicted in Figure 3.1. (b). It is determined by 4 values  $(l, m_1, m_2, u)$ , where all the values in the interval  $[m_1, m_2]$  reach the maximum membership degree 1.

Three types of bell-shaped membership functions most often used to represent fuzzy numbers are the PI, the Beta and the Gaussian functions. They are all symmetrical around a central value, but they have different slopes and membership degrees at the end-points.

The PI membership function, given in Figure 3.1. (c), is defined by a central value  $\gamma$  and a parameter  $\beta$ . The membership degree of the central value  $\gamma$  is 1 and it smoothly decreases and reaches 0 at two end-points,  $\gamma - \beta$  and  $\gamma + \beta$ . The membership function is given as:

$$PI(x; \gamma, \beta) = \begin{cases} S(x; \gamma - \beta, \gamma - \beta/2, \gamma) & x \leq \gamma \\ 1 - S(x; \gamma, \gamma + \beta/2, \gamma + \beta) & x > \gamma \end{cases} \quad (3.21)$$

where  $S(x; \alpha, \beta, \gamma)$  denotes an  $S$ -curve, defined by three values:  $\alpha$  - a value with membership degree 0,  $\gamma$  - a value with membership degree 1 and  $\beta$  - a crossover value with membership degree 0.5:

$$S(x; \alpha, \beta, \gamma) = \begin{cases} 0 & \text{for } x \leq \alpha \\ 2 \cdot ((x - \alpha) / (\gamma - \alpha))^2 & \text{for } \alpha \leq x \leq \beta \\ 1 - 2 \cdot ((x - \gamma) / (\gamma - \alpha))^2 & \text{for } \beta \leq x \leq \gamma \\ 1 & \text{for } x \geq \gamma \end{cases} \quad (3.22)$$

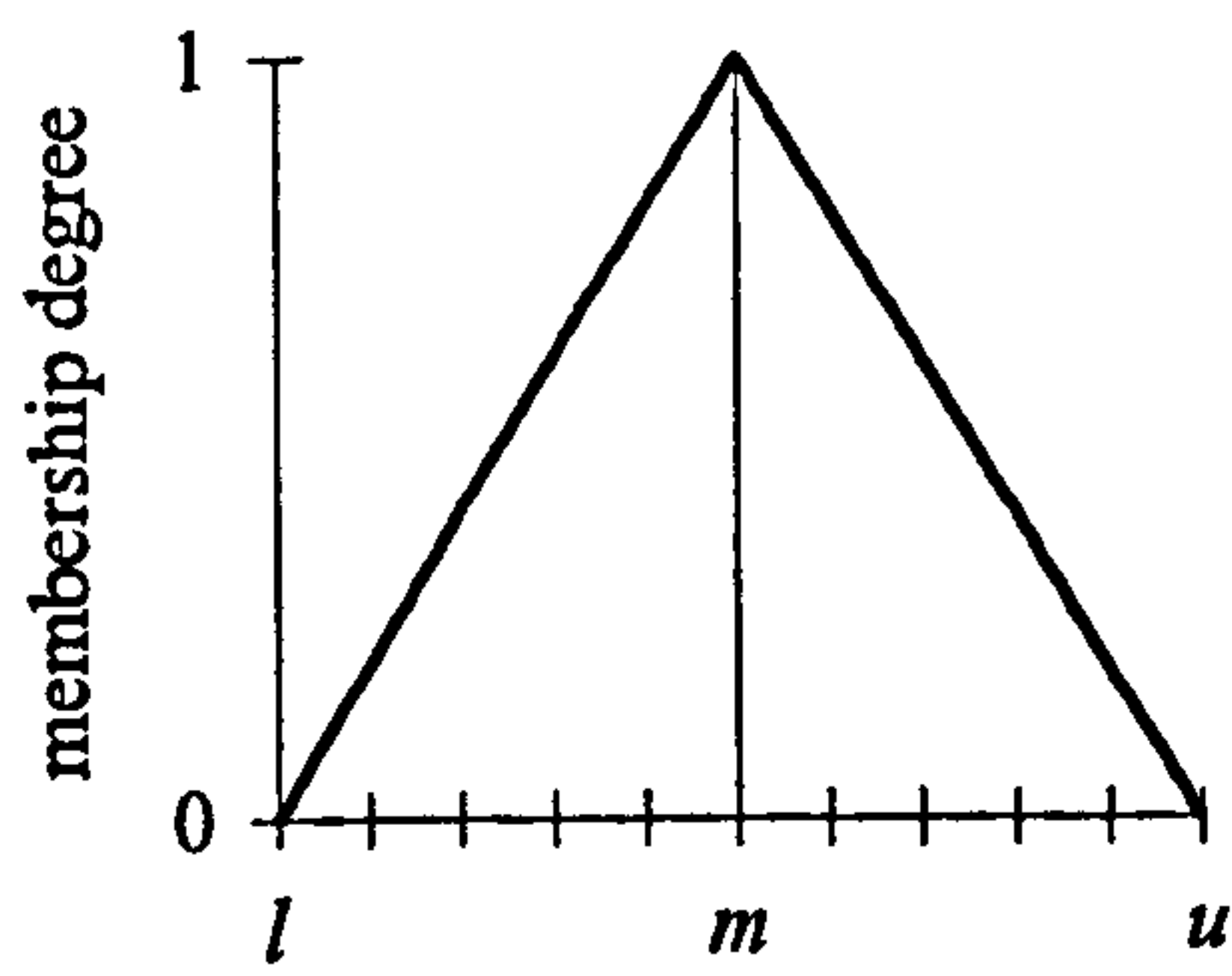
The Beta membership function is like the PI fuzzy number, defined by 2 parameters:  $\gamma$  - a central value and  $\beta$  - the width between the central and a crossover value. However, as illustrated in Figure 3.1. (d), the Beta function reaches 0 at infinity. The shape and the scope of the membership function are determined by parameter  $\beta$ . The larger  $\beta$ , the wider the membership function curve. The Beta membership function is defined by the following formula:

$$B(x; \gamma, \beta) = \frac{1}{1 + \frac{(x - \gamma)^2}{\beta^2}} \quad (3.23)$$

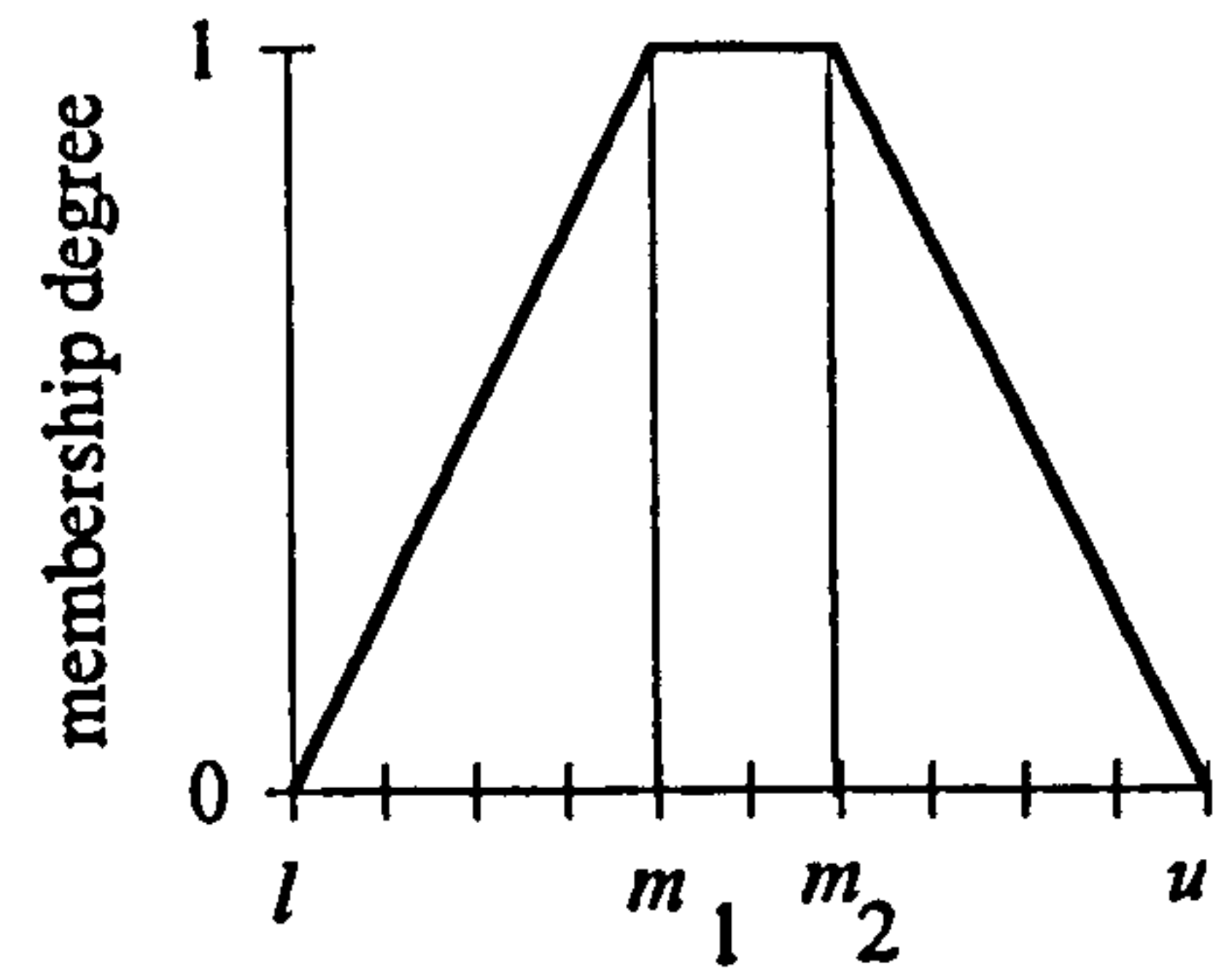
The Gaussian membership function is defined by two parameters  $\gamma$  and  $k$ , that have the same meanings as the parameters  $\gamma$  and  $\beta$  of the Beta function. The two functions have similar curves, but different slopes. The Gaussian membership function, represented in Figure 3.1. (e), converge to 0 very quickly. It is given by:

$$G(x; \gamma, k) = e^{-1/k \cdot (\gamma - x)^2} \quad (3.24)$$

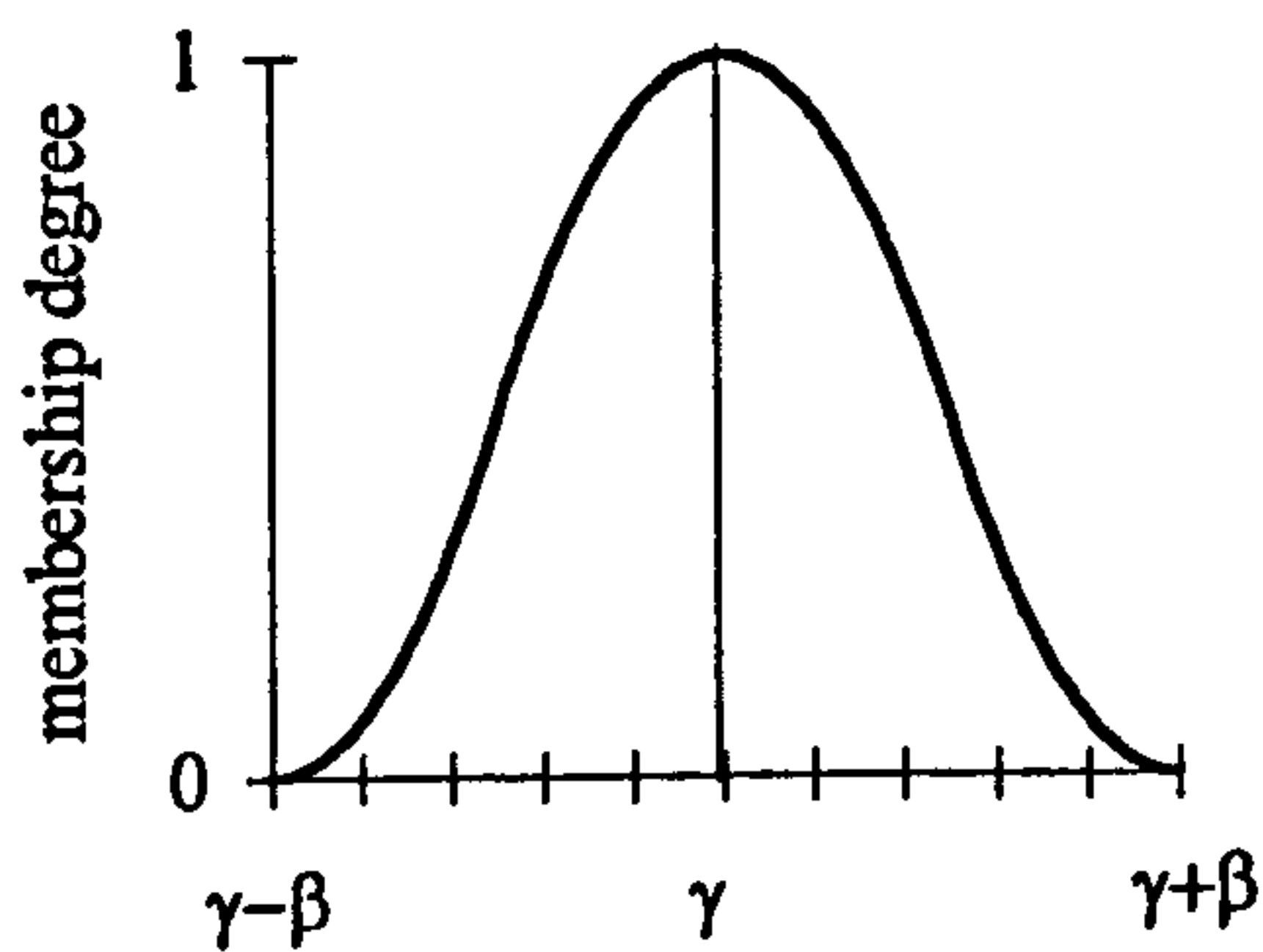




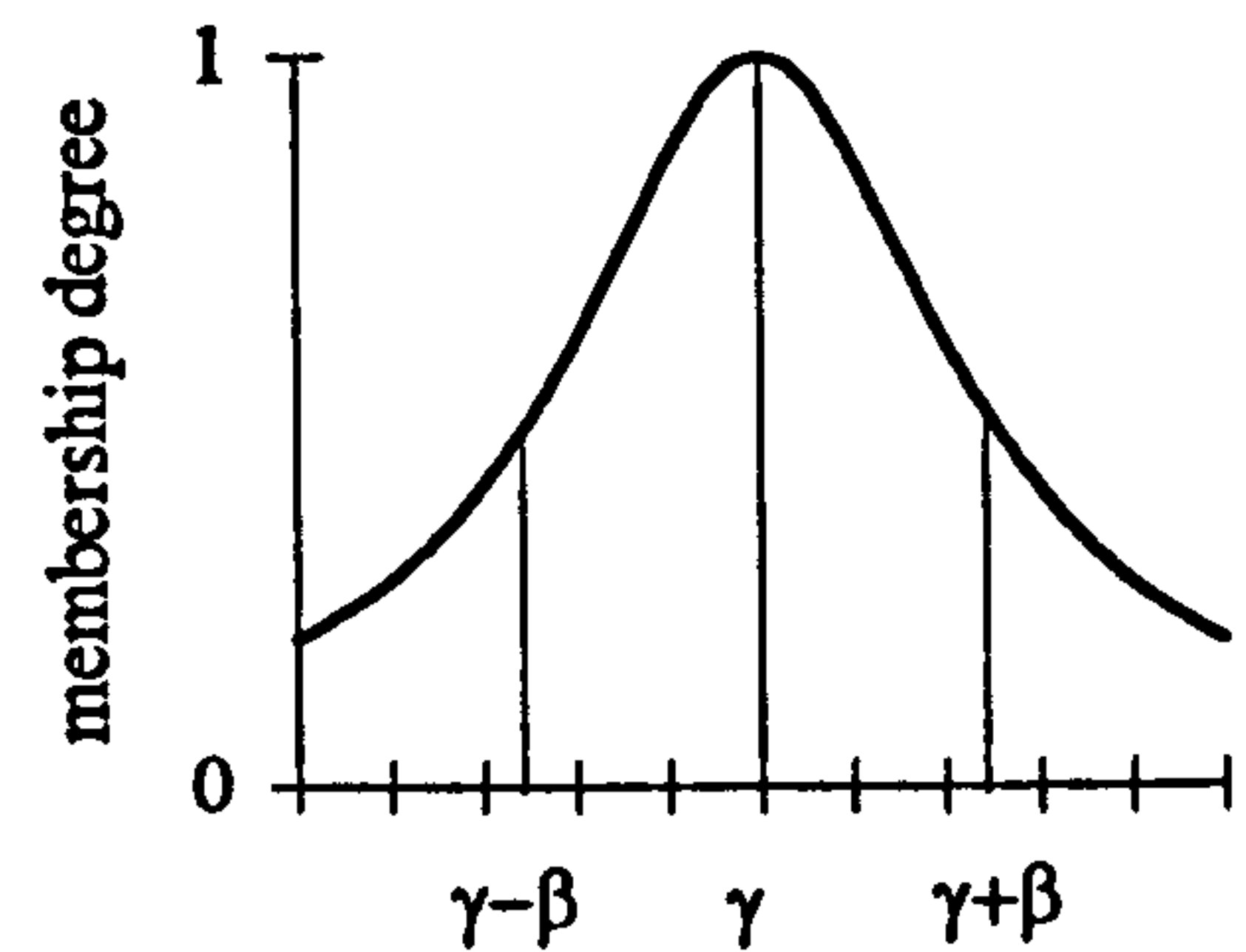
(a) triangular



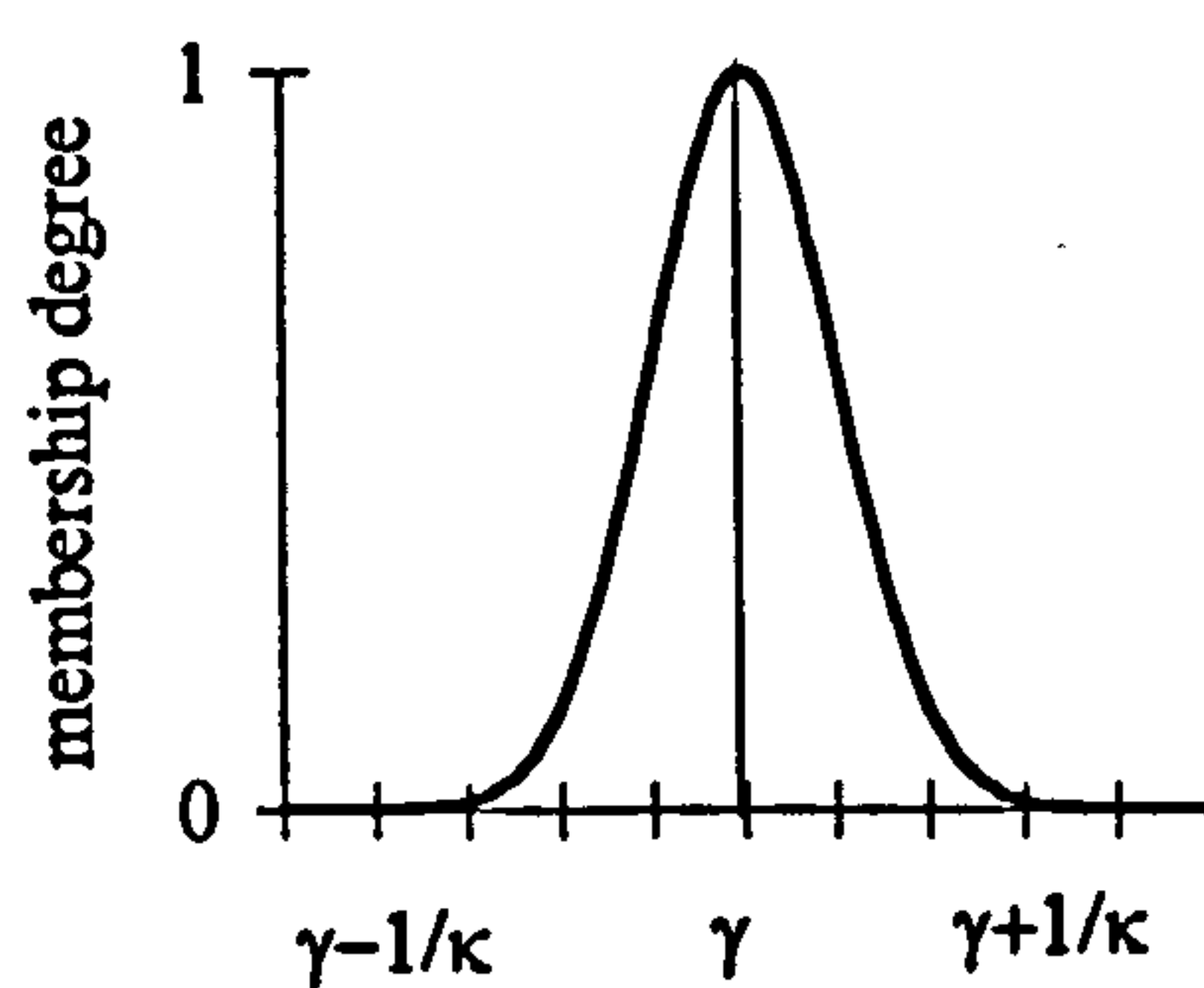
(b) trapezoidal



(c) PI



(d) Beta



(e) Gaussian

Figure 3.1. Membership functions of fuzzy numbers

Definition of a fuzzy arithmetic operation. Let  $R$  and  $P$  be fuzzy numbers with the membership functions  $\mu_R(r)$ ,  $r \in R$  and  $\mu_P(p)$ ,  $p \in R$ , respectively. Let  $*$  be a standard binary arithmetic operation. Operation  $*$  can be extended to the fuzzy domain by using the Extension Principle in the following way:

$$\mu_{R*P}(q) = \sup_{q=r*p} \min(\mu_R(r), \mu_P(p)) \quad (3.25)$$

Fuzzy arithmetic operations can involve rather extensive computations. Using fuzzy numbers with the membership functions defined previously, simplifies fuzzy arithmetic operations performed on them. In general, fuzzy numbers and arithmetic operations on them do not have the same properties as the standard real numbers and real number operations (Mizumoto and Tanaka, 1979).

### 3.1.6. Possibility theory

Possibility theory was developed to deal with imprecisions and vagueness inherent in a natural language (Zadeh, 1978). It offers a framework for representing a meaning of uncertain and imprecise information and reasoning on uncertain and imprecise premises. The mathematical apparatus of fuzzy sets theory provides a basis for possibility theory. A basic concept of the possibility theory is a possibility distribution. The possibility distribution is viewed as a fuzzy restriction on a variable and acts as an elastic constraint on the values that may be assigned to the variable.

Definition of a possibility distribution function. Let  $X$  be a variable which takes values in a universal set  $X$  and let  $F$  be a fuzzy set of  $X$  with a membership function  $\mu_F(x)$ , interpreted as the compatibility of  $x$  with the concept represented by  $F$ . The proposition  $X$  is  $F$  induces a possibility distribution function, denoted by  $\pi_X$ . The possibility distribution function is defined to be numerically equal to  $\mu_F$ :

$$\pi_X = \mu_F \quad (3.26)$$

In other words,  $\pi_X(x)$  represents the possibility that  $X$  takes value  $x$  and it is equal to membership degree  $\mu_F(x)$ . Thus,  $X$  becomes a fuzzy variable which is associated with the possibility distribution in a similar way as a random variable is associated with a probability distribution.

Possibility theory uses the standard definitions of the fuzzy operators: complement, union and intersection given by (3.5)-(3.7). Based on the link given by (3.26), possibility theory is also referred to as fuzzy set theory that uses the standard definitions of the fuzzy set operators. In this thesis, the terms a fuzzy set and a possibility distribution function are used interchangeably with the understanding that the possibility distribution is induced by the fuzzy set.

Definition of possibility measure. A possibility measure on an universal set  $X$  is a set function  $\Pi$  from the set of all subsets of  $X$ ,  $\mathcal{P}(X)$ , to  $[0,1]$ , which satisfies the following axioms (Dubois and Prade, 1984):

1.  $\Pi(\emptyset) = 0$
  2.  $\Pi(X) = 1$
  3.  $\Pi(A \cup B) = \max(\Pi(A), \Pi(B)) \quad A, B \in \mathcal{P}(X)$
- (3.27)

A consequence of these axioms is:

$$\max(\Pi(A), \Pi(A^c)) = 1 \quad (3.28)$$

If universal set  $X$  is considered to be a set of elementary events, the relation (3.28) ensures that for two opposite events, at least one has a possibility equal to 1. However, the fact that an event has a possibility equal to 1 does not prevent the opposite event to have a non-zero possibility.

Possibility theory introduces a measure dual to the possibility measure, called the necessity measure  $N$ . Necessity of an event measures the impossibility of the opposite event. This is expressed by:

$$N(A) = 1 - \Pi(A^c), A \in \mathcal{P}(X) \quad (3.29)$$

There is a link between a possibility measure and a possibility distribution function. A possibility measure  $\Pi$  can be derived from a possibility distribution function  $\pi_X$  associated with a variable  $X$  in an universal set  $X$  in the following way:



$$\Pi(A) = \sup_{x \in A} \pi_X(x) \quad A \in \mathcal{P}(X) \quad (3.30)$$

where  $\Pi(A)$  represents a possibility that fuzzy variable  $X$  takes a value in the given set  $A$  of  $X$ .

### 3.2. Possibility vs probability

Questions concerning the relationship between fuzzy set theory and probability theory have been frequently raised. Two special issues of the journals *IEEE Expert* 9(4), 1994 and *IEEE Transactions on Fuzzy Systems* 2(1), 1994, devoted to these questions prove their challenge and actuality. The comparison between the concepts of possibility and probability has been made at different levels: mathematical, semantic, philosophical and others. Various points of view have been debated, from advocating negative attitudes to fuzzy sets theory, its axiomatic mathematical background and applications, to claiming superiority of fuzzy sets theory over probability theory in treating all types of uncertainty. A standpoint accepted in this thesis is that different types of uncertainty exist in real-world problems, and that fuzzy sets theory and the probability theory should be used accordingly, as complementary rather than competitive methods.

First, two of the aspects from which fuzzy sets theory and probability theory can be differentiated are briefly discussed, such as different types of uncertainty that the two theories treat and their different mathematical properties.

Traditionally, probability theory has been based on the concept of randomness, i.e., a random process whose outcomes are strictly a matter of chance. However, not all uncertainty is random in nature. Fuzzy sets theory offers a framework for modelling a different type of uncertainty associated with vagueness, imprecision or ambiguity.

Despite the fact that a possibility distribution (i.e., a fuzzy set membership function) and a probability distribution can take values from the same interval  $[0,1]$ , the membership degrees are not probabilities at all. Their mathematical properties are quite

different. One apparent difference is that the summation of probabilities over an universal set equals 1, while there is no such requirement for membership degrees.

Regarding a possibility and a probability of occurrence of events, one of the most important difference is in treating the union of events. The probability is an additive measure, i.e.,

$$P(A \cup B) = P(A) + P(B), \text{ where } A \text{ and } B \text{ are two mutually exclusive events,} \quad (3.31)$$

while the possibility of the union of the events is equal to:

$$\Pi(A \cup B) = \max (\Pi(A), \Pi(B)) \quad (3.32)$$

This generates a difference in relating possibilities and probabilities of an event and its complement. There is a very-well known formula that links a probability of an event and a probability of its complement:

$$P(A) + P(A^c) = 1 \quad (3.33)$$

On the other hand, deciding about a possibility of occurrence of an event does not constraint the possibility of occurrence of its complement. There is only a requirement that at least one, an event or its complement, must be possible. The following relation holds:

$$\Pi(A) + \Pi(A^c) \geq 1 \quad (3.34)$$

There are many other differences between the two theories. Ways for matching and combining the possibility and the probability concepts have also been analysed. Zadeh (1978) stated the possibility/probability consistency principle that refers to a variable associated with both a possibility and a probability distribution. The principle expresses the heuristic connection between possibilities and probabilities, based on common sense rather than a relation intrinsic in the possibility and probability concepts. It is stated that if an event is probable it is also possible, and in addition, reducing the possibility of occurrence of an event also reduces its probability. The following

inequalities mathematically express the common sense requirement; the more necessary an event, the more probable it is, and also, the more probable the event, the more possible it is:

$$N(A) \leq P(A) \leq \Pi(A) \quad (3.35)$$

The link between the possibility and the probability concepts can be summarised as follows: (1) the possibility and the probability measures must satisfy the requirements of mutual consistency, and (2) a possibility of an event can be related to the frequency of its occurrence. However, probability theory involves a richer formal structure based on additivity, expressed by (3.31), than possibility theory based on comparability expressed by (3.32).

Naturally, the question arises on how probability distribution can be matched with a related possibility distribution, i.e., how to derive possibility degrees if statistical data exist. Formally, the problem is to find a mapping from a discrete probability distribution to a discrete possibility distribution and vice versa. Both distributions are defined on a finite set  $X$  composed of  $I$  elementary events,  $X = \{x_1, x_2, \dots, x_I\}$ . A simple linear transformation of a probability distribution defined by  $p_i = P(\{x_i\})$ ,  $i = 1, \dots, I$  to a possibility distribution defined by  $\pi_i = \Pi(\{x_i\})$ , given by

$$\pi_i = k \cdot p_i \quad (3.36)$$

where  $k = 1 / \max_i p_i$ , does not preserve the required inequalities (3.35), as proved in (Dubois and Prade, 1980).

Dubois and Prade (1983) suggested a mapping that keeps the required inequalities between a probability and the derived possibility distribution. Let the elementary events  $x_i$ ,  $i = 1, \dots, I$  be reordered in such a way that  $p_1 \geq p_2 \geq \dots \geq p_I$ . A mapping from the probability distribution to the possibility distribution is defined by the following formulae:



$$\begin{aligned}
\pi_1 &= \sum_{k=1}^I p_k = 1 \\
\pi_i &= i \cdot p_i + \sum_{k=i+1}^I p_k, \quad i = 2, \dots, I-1 \\
\pi_I &= I \cdot p_I
\end{aligned} \tag{3.37}$$

Conversely, a possibility distribution determines a unique probability distribution in the following way:

$$p_i = \sum_{k=i}^I \frac{1}{k} \cdot (\pi_k - \pi_{k+1}), \quad i = 1, \dots, I \tag{3.38}$$

where  $\pi_{I+1} = 0$ .

Actually, a mapping from a possibility distribution into a probability distribution describes a way in which a random experiment with a fuzzy set can be performed. For example, the mapping given by formula (3.38) can be interpreted and justified as follows.

Assume that  $A$  is a fuzzy set on  $X$ , with a membership function  $\mu_A$  such that

$$\mu_A(x_i) = \pi_i \tag{3.39}$$

where  $x_i, i=1, \dots, I$  are reordered in such a way that  $\pi_1 \geq \pi_2 \geq \dots \geq \pi_I$ .

The problem we are focused on is how to choose an element from  $X$  which would be a prototype of an imprecise data described by fuzzy set  $A$ . The method of selecting the prototype element proceeds in two steps.

Step 1. Choose an  $\alpha$ -cut of fuzzy set  $A$  at random.

For a level  $\alpha \in (\pi_{k+1}, \pi_k]$ ,  $k = 1, \dots, I$ , where  $\pi_{I+1}=0$ ,  $\alpha$ -cut is the crisp set, denoted by  $A_k$ :

$$A_k = \{x_1, x_2, \dots, x_k\} \tag{3.40}$$

due to the ordered possibilities  $\pi_1 \geq \pi_2 \geq \dots \geq \pi_I$ .

The probability of choosing level  $\alpha$ , i.e.,  $\alpha$ -cut  $A_k$  at random is

$$P(A_k) = \pi_k - \pi_{k+1} \quad (3.41)$$

Step 2. Choose an element from  $\alpha$ -cut  $A_k$ , generated in Step 1, at random.

The probability of choosing an element  $x_i$ ,  $i = 1, \dots, I$  from  $A_k$  at random is

$$\Pr(x_i | A_k) = \begin{cases} \frac{1}{|A_k|} = \frac{1}{|\{x_1, x_2, \dots, x_k\}|} = \frac{1}{k} & \text{for } i \leq k \\ 0 & \text{otherwise} \end{cases} \quad (3.42)$$

where  $|\cdot|$  denotes the cardinality, i.e., the number of elements in a crisp set.

Finally, the probability of choosing an element  $x_i$ ,  $i = 1, \dots, I$  which would be a prototype of fuzzy set  $A$  is

$$\begin{aligned} \Pr(x_i) &= \sum_{k=1}^I \Pr(x_i | A_k) \cdot \Pr(A_k) \\ &= \sum_{k=i}^I \frac{\pi_k - \pi_{k+1}}{k} \end{aligned} \quad (3.43)$$

The formula derived is the same as formula (3.38). In this way it is proved that the mapping of a possibility distribution to a probability distribution suggested by (3.38) corresponds to the two step procedure: first, choosing an  $\alpha$ -cut of a fuzzy set at random and then choosing an element from the selected  $\alpha$ -cut at random.

### 3.3. Measures of uncertainty

The term uncertainty has a broad semantic meaning, and it makes the problem of measuring uncertainty very difficult. Various facets of uncertainty can be measured in different ways. The question on how to measure uncertainty represented by a fuzzy set has been one of the important issues in the development of fuzzy sets theory. Two

approaches are described in the two sections that follow. They generate two classes of fuzzy set assessment functions: (1) a measure of fuzziness and (2) a measure of non-specificity. However, only recently an attempt has been made to provide a common framework for all different fuzzy set assessment functions (Pal and Bezdek, 1994, Wierman, 1996). Establishing a link between measures of fuzziness and a measure of non-specificity requires further research. Also, one can notice that there is no practical guideline on which of these different measures of uncertainty to use in real-world applications and particularly in SC considerations.

### 3.3.1. Measures of fuzziness

In order to capture the meaning of intuitively acceptable characterisation of the concept of degree of vagueness or fuzziness, De Luca and Termini (1972) postulated three axiomatic requirements that a measure of fuzziness  $f$  must satisfy:

1.  $f(A) = 0$  iff  $A$  is a crisp set.
2. If a fuzzy set  $A$  is *sharper* (i.e., *less fuzzy*) than a fuzzy set  $B$  then  $f(A) \leq f(B)$ .
3.  $f(A)$  assumes the maximum value iff a fuzzy set  $A$  is *maximally fuzzy*.

Obviously, the relations *sharper than* and *maximally fuzzy* can be defined in many ways that lead to different measures of fuzziness. Three of them are briefly introduced here to illustrate variety of approaches in measuring the degree of fuzziness. They are:

1. a measure of fuzziness based on the entropy of a fuzzy set,
2. an index of fuzziness based on a distance of a fuzzy set from the nearest crisp set,
3. a measure of fuzziness based on the degree of distinction between a fuzzy set and its complement.



## 1. Entropy of a fuzzy set

This measure of fuzziness is based on the following definitions of the relations *sharper than* and *maximally fuzzy*, referred to in Axioms 2 and 3:

- Fuzzy set  $A$  is *sharper than* a fuzzy set  $B$  if  $\mu_A(x) \leq \mu_B(x)$  for  $\mu_B(x) \leq 0.5$  and  $\mu_A(x) \geq \mu_B(x)$  for  $\mu_B(x) \geq 0.5$ , for all  $x \in X$ .
- The *maximally fuzzy* is a fuzzy set with all the membership degrees equal to 0.5.

The corresponding measure of fuzziness is defined by the function:

$$f(A) = - \sum_{x \in X} [\mu_A(x) \cdot \log_2 \mu_A(x) + (1 - \mu_A(x)) \cdot \log_2 (1 - \mu_A(x))] \quad (3.44)$$

This measure, defined and analysed by De Luca and Termini (1972), is also called the entropy of a fuzzy set due to its similarity with the Shannon entropy function. However, as discussed in (Klir and Folger, 1988), the two functions measure fundamentally different types of uncertainties.

## 2. Index of fuzziness.

Another class of measures of fuzziness, based on the same definitions of the relations *sharper than* and *maximally fuzzy* previously introduced, is called an index of fuzziness (Kaufmann, 1975). It is defined as the distance of a fuzzy set  $A$  from any of the nearest crisp set  $C$ . For example, the nearest crisp set  $C$  to fuzzy set  $A$  can be defined by  $\mu_C(x) = 0$  if  $\mu_A(x) \leq 0.5$  and  $\mu_C(x) = 1$  if  $\mu_A(x) > 0.5$ .

The index of fuzziness depends on a metric distance used. For example, Minkowski class of distances yields the following class of index of fuzziness:

$$f_w(A) = \left( \sum_{x \in X} |\mu_A(x) - \mu_C(x)|^w \right)^{1/w}, \quad w \in [1, \infty) \quad (3.45)$$

Special cases of this class of index of fuzziness are derived by setting parameter  $w$  to  $w = 1$ , or  $w = 2$ . The indices of fuzziness become

$$f(A) = \sum_{x \in X} |\mu_A(x) - \mu_{A^c}(x)|, \text{ for } w = 1 \quad (3.46)$$

$$f(A) = \left( \sum_{x \in X} |\mu_A(x) - \mu_{A^c}(x)|^2 \right)^{1/2}, \text{ for } w = 2 \quad (3.47)$$

### 3. Distinction between a fuzzy set and its complement

This class of measures of fuzziness takes into account the degree of distinction between a fuzzy set and its complement (Yager, 1979). Actually, the lack of distinction between a fuzzy set and its complement is one of the major properties that distinguishes a fuzzy set from a crisp set. The less a fuzzy set differs from its complement, the fuzzier it is.

This measure utilises the following definitions of the relations *sharper than* and *maximally fuzzy*:

- A fuzzy set  $A$  is *sharper than* a fuzzy set  $B$  iff  $|\mu_A(x) - \mu_{A^c}(x)| \geq |\mu_B(x) - \mu_{B^c}(x)|$ , for all  $x \in X$ .
- The *maximally fuzzy* is a fuzzy set  $A$  whose membership function satisfies the condition  $\mu_A(x) = \mu_{A^c}(x)$ , for all  $x \in X$ .

A measure of fuzziness  $f_{c,d}$  of a fuzzy set  $A$  based on this approach has the form:

$$f_{c,d}(A) = d_c(Z, Z^c) - d_c(A, A^c) \quad (3.48)$$

where  $d_c$  denotes a distance between a set and its complement and  $Z$  is any arbitrary crisp set of  $X$  such that the distance between  $Z$  and its complement  $Z^c$  is the largest distance that can be achieved.

Obviously, the definition of a measure of fuzziness in this class depends on the metric distance  $d$  and fuzzy complement  $c$  used. For example, using any metric distance from the Minkowski class, the distance  $d_{c,w}(A, A^c)$  between a fuzzy set  $A$  and its complement  $A^c$  is expressed by:

$$d_{c,w}(A, A^c) = \left( \sum_{x \in X} |\mu_A(x) - \mu_{A^c}(x)|^w \right)^{1/w}, \quad w \in [1, \infty) \quad (3.49)$$

If the standard definition of the fuzzy set complement is used, where  $\mu_{A^c}(x) = 1 - \mu_A(x)$ ,  $x \in X$ , then the distance between fuzzy set  $A$  and its complement  $A^c$  becomes:

$$d_{c,w}(A, A^c) = \left( \sum_{x \in X} |2 \cdot \mu_A(x) - 1|^w \right)^{1/w}, \quad w \in [1, \infty) \quad (3.50)$$

In this case, the largest possible distance is achieved when the crisp set  $Z$  contains all elements of  $X$ :

$$d_{c,w}(Z, Z^c) = |X|^{1/w} \quad (3.51)$$

and the measure of fuzziness of fuzzy set  $A$  is then

$$f_{c,w}(A) = |X|^{1/w} - \left( \sum_{x \in X} |2 \cdot \mu_A(x) - 1|^w \right)^{1/w}, \quad w \in [1, \infty) \quad (3.52)$$

### 3.3.2. Measure of non-specificity

The measures of fuzziness defined in the previous section assess a fuzzy set based on its membership degrees only. In addition to it, a measure of non-specificity takes into consideration the number of elements in the support of a fuzzy set. Generally, non-specificity of a fuzzy set is large when the information represented by it is spread over many elements (alternatives). Non-specificity is calculated as (Higashi and Klir, 1983):

$$U(A) = \int_0^1 \log_2 |A_\alpha| d\alpha \quad (3.53)$$



where  $A_\alpha$  denotes an  $\alpha$ -cut of fuzzy set  $A$ . The non-specificity of a fuzzy set  $A$  of  $X$  ranges from 0 to  $\log_2|X|$  and increases with increasing number of elements with high membership degrees.

The measures of fuzziness and the measure of non-specificity assess two different facets of uncertainty, both modelled by fuzzy sets. The fundamental difference between them can be illustrated using as an example a fuzzy set whose elements all have the membership degree 1. The degree of fuzziness, calculated using any of the measures of fuzziness, defined in Section 3.3.1., is 0 indicating that there is no fuzziness associated with this fuzzy set. On the other hand, the non-specificity of the fuzzy set is greater than 0 and depends on the number of elements with membership degree 1. The more of these elements, the higher the non-specificity of the fuzzy set.

### 3.4. Use of fuzzy concepts in production management

Fuzzy sets theory has been applied in very diverse and widespread areas encompassing both natural and social sciences, engineering, manufacturing, medicine, management and decision making, computer science and others. This section is focused on the applications of the fuzzy methodology in production management in general, with a special emphasis on fuzzy sets applications in inventory management and control problems.

Several reasons for the use of fuzzy sets in production management can be outlined (Karwowski and Evans, 1985, Turksen, 1985):

- For many production management problems two distinct forms of knowledge exist that supplement each other: (1) objective knowledge, which is well structured and expressed by standard mathematical models and (2) subjective knowledge, which

involves the experience and managerial judgement and is usually impossible to quantify using standard mathematical methods.

- Many of the decisions in production management are made in a fuzzy environment where objectives and constraints are not completely defined or cannot be precisely measured.
- There is a lack of understanding of some phenomena in production management problems that limits the application of standard models.
- In many cases the appropriate statistical data are lacking.

Despite the large amount of work published on fuzzy sets theory and its applications, one can notice that surprisingly small number of applications that utilise fuzzy methodology was developed in production management by the mid of eighties. Karwowski and Evans (1986) reviewed a few applications developed in various areas of production management, and discussed further potential usage of fuzzy sets theory in areas such as new product development, facilities planning, human-production management, industrial process control and production planning and inventory control. In the last decade, there has been a rapid growth of interest in developing fuzzy set applications dealing with production management, particularly in the domain of industrial process control (Hirota and Sugeno, 1995).

Inventory management and control problems face different types of uncertainty which can be modelled by fuzzy sets. However, it seems that suitability and usefulness of applying fuzzy sets to these problems have still not been investigated thoroughly and requires further research. The next section reviews fuzzy inventory models developed so far and published in the literature.

### 3.4.1. Fuzzy inventory models

There are only a small number of fuzzy inventory models presented in the literature. One can notice that most of them treat simple production and inventory control problems, but utilise various fuzzy methods and techniques. These models show that there are many sources of uncertainty inherent in production and inventory control problems that can be successfully modelled by fuzzy sets and that there are many ways in which fuzzy sets theory can be applied in solving these problems. The fuzzy inventory models developed so far can be classified into two groups: (1) numerical fuzzy inventory models based on fuzzy arithmetic and fuzzy optimisation methods, and (2) fuzzy inventory models based on approximate reasoning, i.e., on methods by which possible imprecise conclusions are deduced from imprecise premises.

One of the simplest ways to introduce uncertainty in an inventory control problem is "to fuzzify" an existing deterministic inventory model. The "fuzzification" can be performed by replacing deterministic numerical data by fuzzy numbers and performing calculations on them using fuzzy arithmetic rules. For example, Park (1987) used the very-well known economic-order-quantity (EOQ) formula to determine optimal quantities to be ordered when the cost of placing an order and annual holding cost are fuzzy; the fuzzy costs are represented by trapezoidal fuzzy numbers. Similarly, in (Chen, Wang and Ramer, 1996) optimal economic-order-quantity is calculated in the case when shortages are permitted and backordered in a fuzzy environment where customer demand, order cost, inventory cost and backordering cost are fuzzy. Wang and Chen (Wang and Chen, 1995) give a fuzzy interpretation of a standard economic-production-quantity (EPQ) formula; an optimal economic production quantity is determined when demand, production rate, production cost, setup cost and holding cost are fuzzy. In (Vujosevic, Petrovic and Petrovic, 1996) different ways of applying the EOQ formula in a fuzzy environment are examined. All the papers mentioned above showed that standard deterministic models



can be successfully extended to the corresponding fuzzy models. However, these inventory models use fuzzy methodology in a rather simplified way. They mainly consider computational aspects of the fuzzy formulae, but do not shed new light on the complex problems of inventory control in the presence of uncertainty.

There are other fuzzy inventory models which involve more sophisticated fuzzy methods and techniques. Sommer (1981) uses the fuzzy dynamic programming method to solve a very specific production scheduling and inventory problem. The objective is to determine production levels for each time period during a planning horizon in order to satisfy deterministic demand. There is an additional requirement to stop production at the end of the planning horizon. Two imprecise objectives can be stated in order to satisfy this requirement: (1) production should decrease *as continuously as possible* from one to the succeeding time period, and (2) inventory should be *at best zero* at the end of the planning horizon. Naturally, the two objectives expressed by imprecise linguistic terms, such as *as continuously as possible* and *at best zero* are represented by fuzzy sets. Dynamic programming method (Bellman and Zadeh, 1970) is used to find the production levels over time. It is also shown by an example that a solution found by applying the proposed method depends on the definition of the fuzzy set operators used. This raises a complex question on how to select fuzzy set operators that will generate an optimal solution.

Kacprzyk and Staniewski (1982) proposed an interesting decision-making model for replenishing an inventory over an infinite planning horizon. The inventory problem is formulated as the following optimisation problem: find an optimal time-invariant strategy for replenishing an inventory over an infinite time horizon when demand at each time instance is imprecisely specified. A fuzzy goal of inventory control is imprecisely stated as a desired inventory level to be maintained at each time instance. Fuzzy constraints are imposed on replenishments in such a way that for each possible replenishment quantity a preference expressed by a degree of membership is given. The inventory under control is viewed as a dynamic fuzzy system in which a

fuzzy inventory level at time  $t+1$  depends on the fuzzy inventory level, fuzzy replenishment and fuzzy demand at time  $t$ . Since the number of distinct fuzzy sets representing the fuzzy inventory levels and fuzzy replenishments may be very high, in order to improve the algorithm's efficiency sets of reference fuzzy inventory levels and fuzzy replenishments are predetermined. Any fuzzy value that occurs in finding the optimal replenishment strategy is approximated by an appropriate fuzzy value from the reference sets. An optimal time-invariant strategy is represented in the form of fuzzy conditional statements that relate the optimal replenishment at each time instance to the current inventory level:

IF  $Z = z_1$  THEN  $R = r_1$  ELSE  
 IF  $Z = z_2$  THEN  $R = r_2$  ELSE ...  
 IF  $Z = z_I$  THEN  $R = r_I$ .

where  $Z$  is the current inventory level,  $R$  is the replenishment quantity,  $z_i, i = 1, \dots, I$  is a reference fuzzy inventory level and  $r_i, i = 1, \dots, I$  is a reference fuzzy replenishment.

A few comments can be made on the model. First, fuzziness of different costs such as carrying or out-of-stock costs are modelled only implicitly, through the fuzzy inventory level desired and the fuzzy constraints imposed on replenishments. However, it seems that the link between the costs and inventory levels and replenishments is not clearly defined and it needs further explanation and analysis. Second, the algorithm developed is very complex. It involves and combines variety of fuzzy techniques: algebraic operations on fuzzy numbers, fuzzy decision-making and techniques for approximation by a reference fuzzy set.

The newsboy problem in a fuzzy environment is treated in (Petrovic, Petrovic and Vujosevic, 1996). It is assumed that uncertainty may appear in demand and in inventory costs, including the overage and the shortage costs. The fuzzy newsboy problem is to determine an optimal order quantity for a fixed time period that will minimise the possible total cost when fuzzy demand, precise or fuzzy unit overage and unit shortage cost and a precise unit purchasing cost are given. The fuzzy newsboy

model is compared with the standard stochastic newsboy model that assumes random demand. In the case when only demand is uncertain, the two models generate similar results, despite the different interpretation they have. The fuzzy newsboy model presented could be modified for solving similar inventory problems.

Rinks's model (Rinks, 1981) is one of the earliest fuzzy production and inventory models based on an approximate reasoning method. It uses fuzzy sets to represent heuristic decision rules for determining production and work force levels. Three variables are considered to be important for determining production level  $P_t$  and a change in work force level  $\Delta W_t$  in period  $t$ . They are:

$FS_t$  - sales (demand) forecast for period  $t$ ,

$W_{t-1}$  - work force level in period  $t-1$ ,

$I_{t-1}$  - inventory level at the end of period  $t-1$ .

All these variables are represented as fuzzy variables that can take predefined imprecise linguistic values, such as *very high*, *high*, *rather high*, *average*, *at least average*, *rather low*, etc. The relationships between the fuzzy variables are expressed by fuzzy conditional statements, i.e., fuzzy IF-THEN rules of the following form:

IF  $FS_t$  is  $fs$  AND  $I_{t-1}$  is  $i$  AND  $W_{t-1}$  is  $w$  THEN  $P_t$  is  $p$

IF  $FS_t$  is  $fs$  AND  $I_{t-1}$  is  $i$  AND  $W_{t-1}$  is  $w$  THEN  $\Delta W_t$  is  $\Delta w$

where  $fs$ ,  $i$ ,  $w$ ,  $p$  and  $\Delta w$  are the predefined linguistic values modelled by fuzzy sets. The rules are defined based just on managerial experience and judgement, with no claim that they are optimal. The approximate reasoning method performed on the fuzzy IF-THEN rules is based on the standard compositional rule of inference (Zadeh, 1973). The fuzzy model is compared with one standard deterministic linear production scheduling model that minimises the total production cost involved. Interesting results are observed proving that using a fuzzy heuristic (judgmental) model rather than an optimisation model may be fully warranted. First, from the production cost point of view the fuzzy model leads to the slightly more expensive solution that the



optimisation method; the excess cost is considered to be acceptable due to the facts that the fuzzy model uses a shorter forecasting horizon, it is computationally more efficient and it does not impose restrictive assumptions regarding a specific cost structure. Second, the period by period comparisons of the production and work force decisions reveal an overall similarity between the optimal results and those achieved by the heuristic.

Turksen (Turksen, 1988) developed a new approximate reasoning method to deal with a production planning problem. This method involves interval valued fuzzy sets. New definitions of fuzzy set operators that generate interval-valued fuzzy sets are proposed. Using the Rink's fuzzy IF-THEN rules for production and work force scheduling, Turksen shows that the approximate reasoning method based on interval-valued fuzzy sets generate results nearer to the optimal solution than the method based on the compositional rule of inference used in (Rinks, 1981). In addition, Turksen reduces the set of predefined linguistic values that are associated with the fuzzy variables to only three values, namely *low*, *average* and *high*. Consequently, the number of fuzzy IF-THEN rules is decreased, but yet valuable results can be obtained. Two important questions are opened for further research: (1) How to decide on *the most suitable* fuzzy set operators for a given domain specific problem? and (2) How to determine *a minimum set* of fuzzy rules for a domain specific problem?

Fuzzy heuristic rules for estimating quantity to be ordered for inventory replenishment are proposed and examined in (Petrovic and Sweeney, 1994). Order quantity is determined in the presence of imprecise demand, imprecise actual inventory level and imprecise lead time. Dominance between demand and actual inventory level, linguistically described, is calculated as the degree to which demand is higher than the inventory level. Fuzzy rules relate dominance between demand and actual inventory level, and imprecise lead time to the recommended order quantity, and have the following form:



IF dominance between demand and actual inventory level is  $d$  AND lead time is  $l$   
 THEN order  $o$  item quantity

where  $d$ ,  $l$  and  $o$  are predetermined imprecise linguistic values. An approximate method for reasoning on these fuzzy IF-THEN rules is proposed and verified.

Turksen and Berg (1991) give an interesting approach to synthesising a stochastic and a fuzzy model for planning inventory capacity, in order to bridge the communication gap between stochastic model developers and inventory managers. A relatively simplified production process is considered: a single machine produces items at a constant rate up to a maximum inventory capacity  $N$ . Production is stopped whenever the inventory level reaches maximum level  $N$  and it is resumed after demand arrives. Demand arrives according to a Poisson process at rate  $\lambda$  and demand size is exponentially distributed with a mean  $\mu^{-1}$ . Shortages are considered as lost sales. The machine failure rate and its repair time are exponentially distributed with parameter  $\theta$  and mean  $\sigma^{-1}$ , respectively. The fuzzy rules that link these parameters are in the following form:

IF  $\lambda$  is  $l$  AND  $\mu^{-1}$  is  $m$  AND  $\theta$  is  $t$  AND  $\sigma^{-1}$  is  $s$  AND  $\rho$  is *high*  
 THEN take action  $a$  with respect to  $N$

where  $l$ ,  $m$ ,  $t$ , and  $s$  are predetermined imprecise linguistic values such as *low*, *medium* and *high*, and  $\rho$  is a performance criterion expressed by a fraction of satisfied customer demand. An approximate action represented by linguistic term  $a$  in the THEN part of the fuzzy rule is determined based on the stochastic model. In this way, the behaviour of the production/inventory system described by the stochastic model is expressed in what is believed to be a more natural way, using linguistic terms.

Yager (1984) considers an interesting problem of measuring a quality of a forecast which is expressed linguistically. The use of fuzzy sets for representing linguistic forecasts is described. A measure of quality of linguistic forecasts is proposed which takes into account both the validity and the specificity of the fuzzy forecasts. Validity, i.e., truthfulness of a forecast is determined after one realisation of the

forecast value and is equal to the degree of membership of the realised value in the fuzzy forecast. However, validity on its own is not a sufficient measure. By extending the range of the fuzzy forecast, its validity is increased. On the other hand, a large range of the fuzzy forecast might not provide useful information. Yager suggests that the specificity of the fuzzy forecast should also be used to measure its quality. The results of this work may be interesting for inventory control problems where forecasts of customer demand, supplier reliability or lead time can be expressed linguistically.

### 3.5. Summary

Fuzzy sets theory offers a wide range of concepts and techniques for treating different types of uncertainty, such as uncertainty in data values and relations between them; data which are difficult to measure precisely; vaguely and unclearly defined concepts; decision making that involves human reasoning and perception. The literature devoted to fuzzy sets theory and its application shows that fuzzy sets have been successfully applied in areas where intuition and judgement play an important role.

The potential of using fuzzy sets theory in treating different sources of uncertainty in various production management and control problems has been acknowledged in the literature. However, the literature search revealed that the fuzzy methodology has been utilised in only a small number of these problems. This is particularly apparent for the domain of inventory systems, where only a few results addressing simple isolated single stocking point inventory control problems, have been reported.

Linking production and inventory facilities into an SC increases the number of sources of uncertainty, and initiates propagation of uncertainty through the SC. Dealing with uncertainty becomes more complex. There has been no SC model reported in the literature which treats uncertainty in data that characterise an SC and its environment by means of fuzzy sets. Investigation of the potential for using fuzzy sets for modelling uncertainty in SC data is, therefore, undertaken in this research.

## **CHAPTER 4**

### **REPRESENTING UNCERTAINTIES IN A SUPPLY CHAIN USING FUZZY SETS**

There are many sources of uncertainty inherent in an environment in which an SC operates and in the SC processes themselves. Uncertainty propagates through the SC network and affects its performance. Uncertainties which exist in many inter-related factors that may have an impact on strategic SC planning and operational SC management and control, vary in their nature. There are economic, technological, even social and political factors. State of an economy in general, market growth, market instability and other factors cause uncertainty in demand patterns, availability of raw materials, competitors' behaviour, order cancellations, delays in supply, machine failures, willingness of a customer to wait for delivery in the stock-out case, etc. Obviously, it is not easy even to recognise and to list all of the cause-effects relations.

Complexity appears in both describing and measuring all relevant sources of uncertainty, and understanding and representing relationships between them. Recognising the complexity and interactions between these sources of uncertainty is essential in an attempt to assess and model uncertainty in a synthetic way.

In this chapter three basic sources of uncertainty inherent in an SC and its environment have been identified and a new approach to their representation and modelling using fuzzy sets has been described. They are: (1) customer demand, (2) supply, including external supply of raw material and supply from one to the succeeding facility along an SC and (3) lead times along an SC. The modelling of these parameters using fuzzy sets as non conventional descriptions of uncertainties is described in the sections that follow.



In a similar way, other parameters involved in operational inventory control and SC management and control problems, that may be difficult to specify precisely, can be represented by fuzzy sets; typical examples are unit shortage and unit holding costs. It is an aim to show in this chapter that fuzzy sets theory is an appropriate framework for representing vaguely and imprecisely specified data which offers conceptual simplicity and provides techniques for efficient fuzzy data manipulation and operations on them.

#### **4.1. Uncertainty in demand**

Customer demand is one of the key parameters in inventory and SC control problems. A lot of attention has been placed on customer demand modelling. In general, assumptions made about customer demand determine essentially the structure of an inventory or an SC model and its complexity. Customer demand is usually characterised by three important formal features: (1) it is a continuous or a time-discrete process, (2) it is time-stationary or non-stationary and (3) it is known in advance with certainty or there is uncertainty in demand.

Generally, customer demand can be treated as either a time-continuous or a time-discrete process. Time continuity of demand is usually an assumption in analytic approaches to inventory or SC control. It is a realistic assumption as long as demand at a time is small enough in comparison with the replenishment at a time and the total demand over a time horizon considered is sufficiently large. An alternative is the description of demand as a time-discrete process. It can be viewed either as an appropriate representation of actual customer demand or as an approximation in commonly used discrete type analysis of inventory or SC control problems. In this thesis, customer demand is modelled as a time-discrete process.

By examining isolated inventory control models one may conclude that a prevailing assumption in specifying customer demand is time-stationarity. On the other hand, it has been proved that non-stationary customer demand has an important impact



on SC operations. It causes demand amplifications along an SC and creates chain instability. This phenomenon is explained in Section 2.6. This thesis treats time-stationary customer demand.

As far as demand certainty is concerned, the simplest case involves deterministic customer demand, i.e., demand known in advance with certainty. It is appropriate to assume deterministic demand whenever variations of demand over an expected value are small during a time horizon considered. If this is not the case, uncertainty in customer demand has to be taken into account. Typically, uncertainty is associated with the time when demand occurs, and when it happens the number of items requested may be uncertain, too.

Traditionally in literature, uncertainty in customer demand in inventory and SC management and control problems has been treated as a stochastic process. It has been described by either a completely known probability distribution with parameters assumed to be known or by a known form of a probability distribution, with parameters initially assumed and corrected using Bayesian approach (Brown and Rogers, 1972).

A probability distribution is usually derived from evidences recorded in the past. This requires a valid hypothesis that evidences collected are complete and unbiased, and that the stochastic mechanism generating the data recorded continues in force on an unchanged basis. However, there are situations where all these requirements are not satisfied and, therefore, the standard probabilistic reasoning methods are not appropriate. For example, there may be a lack of evidences available or lack of certainty or confidence in evidences or simply evidences may not exist, as in the case of launching a new product. In these situations, uncertain demand can be specified based on the experience and managerial subjective judgement. Often, an expert may feel that a given demand is within a certain range and may even have an intuitive feel for the "most likely" value within that range.

One of the prime objectives of this section is to indicate that fuzzy sets theory provides the appropriate framework to describe and treat uncertainty in customer

demand related to imprecisions. Customer demand in a time unit can be vaguely expressed by different terms, such as: (a) "demand in a time unit is *about*  $d_m$ , but definitely not less than  $d_l$  and not greater than  $d_u$ ", (b) "demand in a time unit is *much larger* than  $d_l$ " or in a more complex form, (c) "demand in a time unit will be in the interval  $[d_l', d_u']$  with a *high* degree of possibility, but there is a *moderate* degree of possibility that demand will be zero". The approximate qualifiers that correspond to such natural language expressions may be represented by fuzzy sets with the membership functions shown in Figure 4.1. Each of the expressions induces a possibility distribution which is numerically equal to the corresponding membership function, as explained in Section 3.1.6. Generally, a range of possible demand values can be either continuous or discretised.

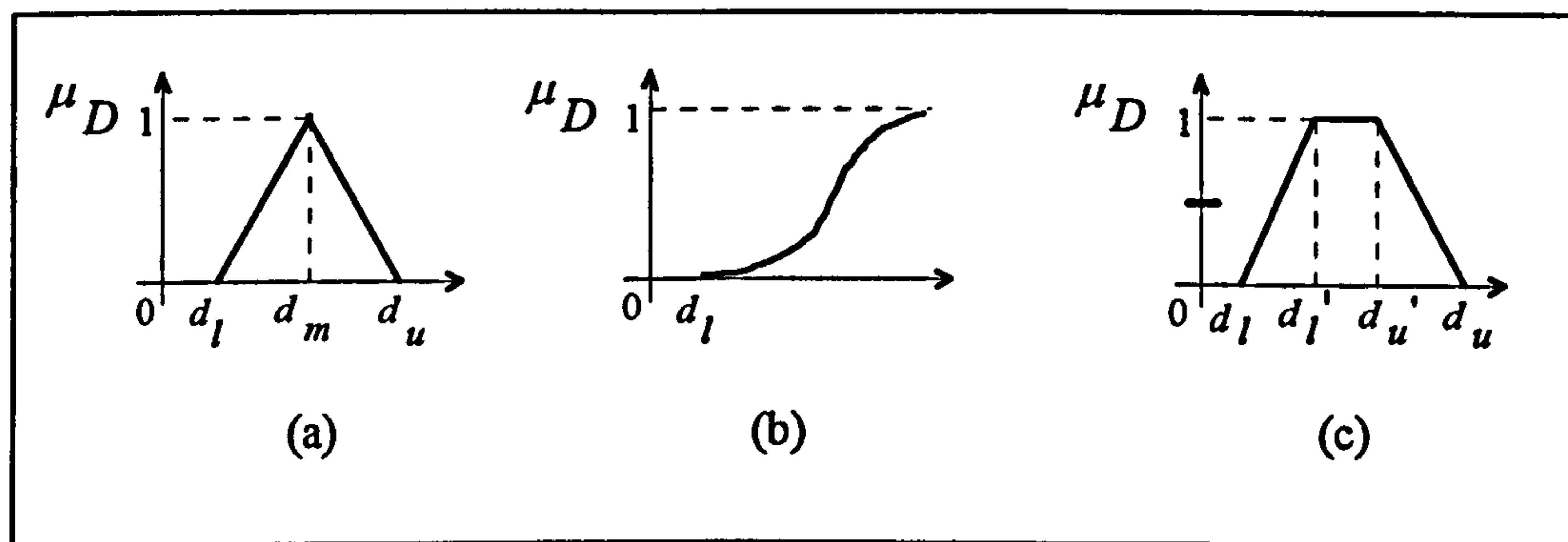


Figure 4.1. Typical fuzzy sets which represent uncertainty in demand

A possibility distribution of fuzzy customer demand can be derived either from subjective belief or from a probability distribution, if it exists. An approach to transforming a discrete probability distribution into the corresponding discrete possibility distribution is given in Section 3.2. Possibility distributions and probability distributions are quite different - in principle and in practice.

Suitability of using fuzzy sets to describe customer demand is demonstrated by a simple example. Consider customer demand as in Figure 4.1. (a). Suppose that circumstances have brought into existence a strong belief that customer demand can

take a value outside the interval  $[d_l, d_u]$  with possibility 1. In such a case it is easy to modify the existing possibility distribution by simply adding a new possible value of demand with no other changes of the distribution. Let us notice that such an intervention, having a probability distribution, is not straightforward at all.

Choosing fuzzy sets to describe imprecise demand usually means that it is done before seeing any data. As the solution to the problem progresses with the acquisition of real data about an SC, one can begin to model these values as relative frequencies and probability distributions.

It is clear that uncertainty in customer demand causes uncertainty in internal demand placed from one to the preceding facility along an SC. Customer demand and internal demand possibility distributions need not have the same shape. Customer demand may be composed of firm orders and imprecise forecasts and, consequently, customer demand and internal demands are derived as sums of fuzzy and/or crisp values, obtained by using fuzzy arithmetic rules.

In this thesis, uncertainty in customer demand and internal demands are modelled by discrete, normalised fuzzy sets.

## 4.2. Uncertainty in supply

An SC is linked with an uncertain external environment by customer demand from one side and a raw material supplier from the other side. Traditionally, attention has been focused on uncertainty in customer demand. However, uncertainty is inherent in the supply side, also. This means that a quantity and quality of raw material delivered from an external supplier may differ from those requested.

In a new approach to modelling uncertainty in SC data, developed in this research, external supplier reliability is described using imprecise linguistic terms, such as "*reliable* supplier", "*moderately reliable* supplier" or "*unreliable* supplier", etc. Naturally, the meaning of imprecise linguistic terms that describe external supplier



reliability and the corresponding possibility distributions depend on the specific context in which the terms are used and on a subjective judgement. A few examples of defining a fuzzy set  $SP$  which represents imprecise external supplier reliability are illustrated in Figure 4.2. Uncertainty in supply is expressed by percentages of ordered raw material that can be delivered by the external supplier.

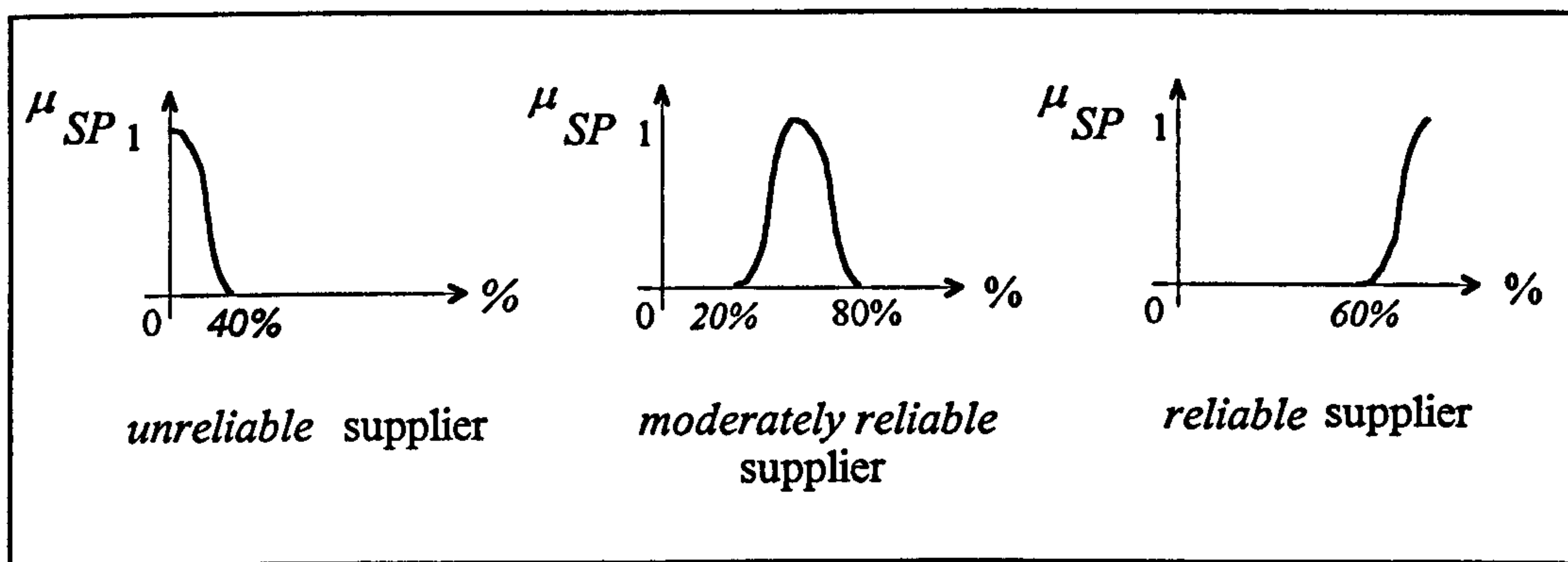


Figure 4.2. Typical fuzzy sets which represent uncertainty in supplier reliability

In addition to uncertainty in external supply of raw material at the very input of an SC, supply of items from one to the succeeding facility in the SC may be considered as a source of uncertainty, too. Supply along the chain may also be unreliable in the sense that not all the replenishment quantities ordered by an inventory can be received from the preceding facility. The reasons for unreliable supply are of different nature, such as uncertainty in production which is caused by machine breakdowns, quality problems and rejection rates or low inventory level of the preceding inventory stock in the SC. A new approach to modelling supply along an SC by a fuzzy set that represents uncertainty in quantity of items that is not delivered from the preceding facility in the SC is developed and used in this work. The corresponding possibility distribution of supply from the predecessor can be determined based on the maximum stock level at the preceding facility that supplies the facility under consideration, a possibility distribution of demand imposed on the preceding facility, a possibility



distribution of machine breakdowns at the production part of the facility, etc. A typical discretised fuzzy set  $SQ$  that represents an uncertainty in undelivered quantity from the preceding facility is illustrated in Figure 4.3.

This thesis treats both uncertainty in external supply of raw material and uncertainty in supply from one to the succeeding facility along an SC and represents them by discretised fuzzy sets.

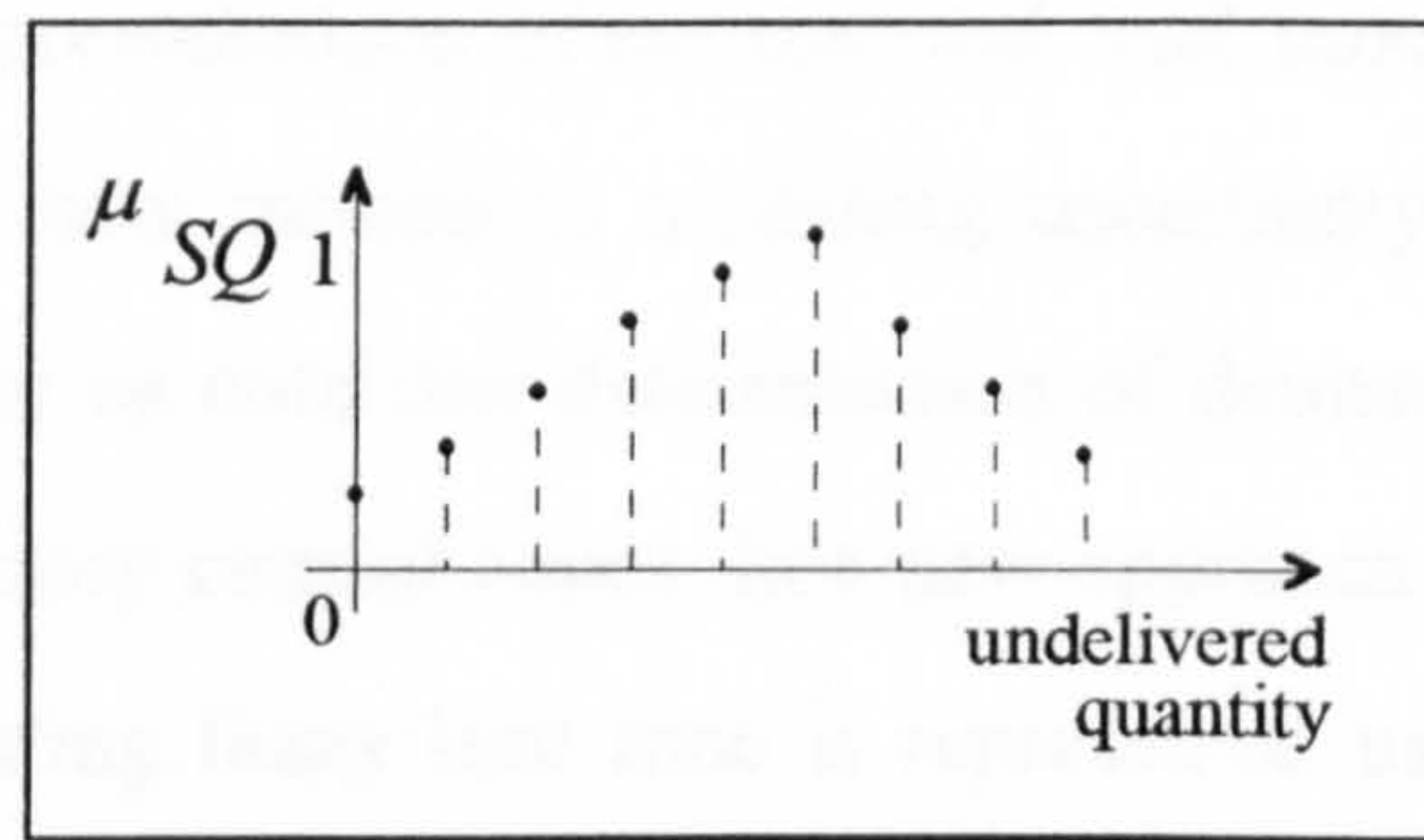


Figure 4.3. Typical fuzzy set which represents uncertainty in undelivered quantity

### 4.3. Uncertainty in lead time

A lead time to any facility in an SC includes a time necessary for order processing, a production time and/or a transportation time. Each of these is often difficult to specify accurately and hence, there is uncertainty associated with the lead time.

Along the lines of what has preceded, the uncertainty in lead time can be represented by a fuzzy set, too. For example, discrete fuzzy sets in Figure 4.4. represent uncertainty in a discrete lead time which can take two values,  $l_1$  and  $l_2$  time units, with the same possibilities, case (a), or with different possibilities, case (b). Generally, a range of possible lead time values can be either continuous or discretised.



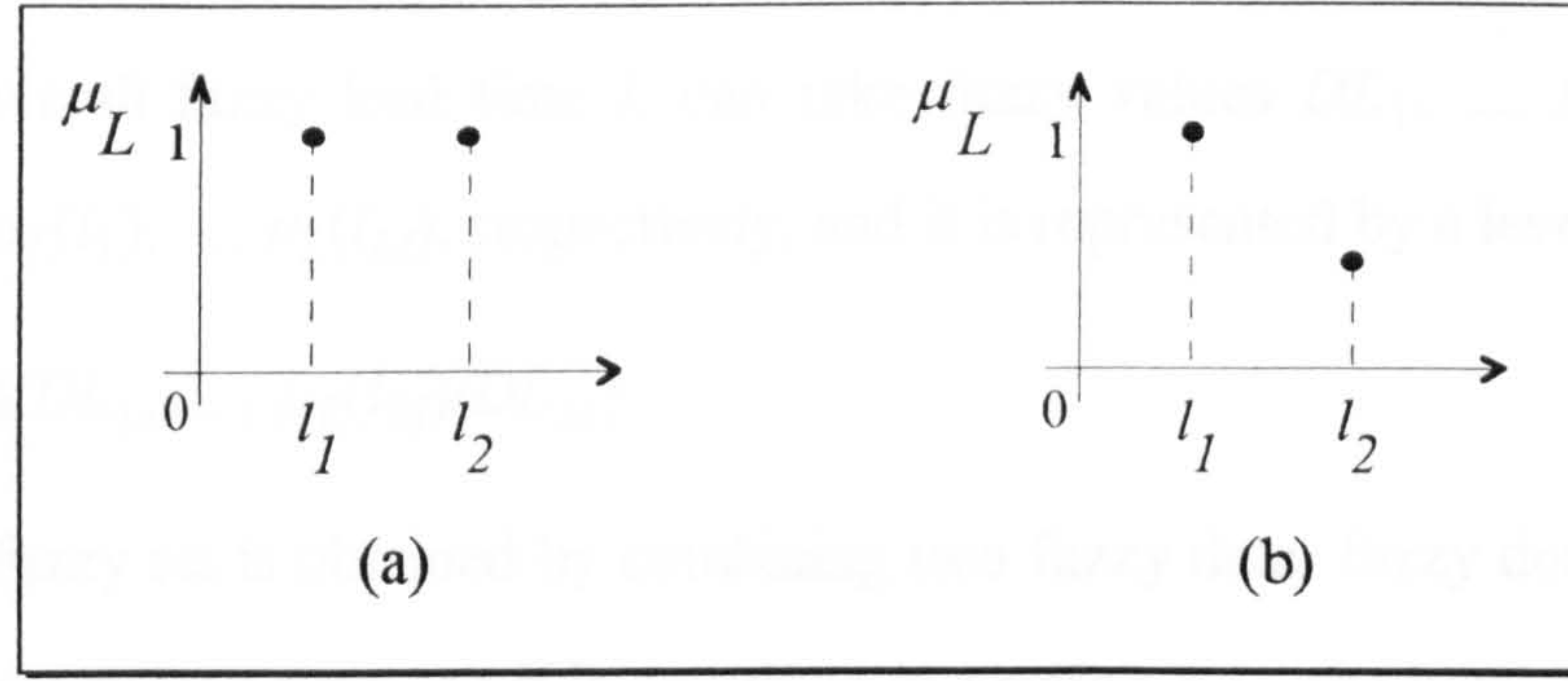


Figure 4.4. Typical fuzzy sets which represent uncertainty in lead time

Using fuzzy representations of demand and lead time described previously in this chapter leads to a new method to modelling uncertainty in total demand during uncertain lead time. Let us note that determination of demand during lead time is an important task in inventory control issues. In a new approach proposed by the author, fuzzy total demand during fuzzy lead time is represented using a level 2 fuzzy set, which is then transformed to an ordinary fuzzy set as follows.

Let us assume that a time horizon considered is homogeneously discretised and unit time intervals are indexed by  $t$ ,  $t = 1, 2, \dots$ . Fuzzy demand is specified for each time unit  $t$ . It is modelled by a discrete fuzzy set  $D_t$ , with membership function  $\mu_{D_t}(d_t)$ ,  $d_t \in \mathcal{D}_t$ . Let a fuzzy lead time be represented by a discrete fuzzy set  $L = \{\mu_L(l_1)/l_1, \dots, \mu_L(l_M)/l_M\}$ , where  $l_m$ ,  $m = 1, \dots, M$  represents the length of the lead time that can occur, expressed by the number of time units and  $\mu_L(l_m)$  is the possibility associated to it.

Let us focus on the case when the length of the lead time is  $l_m$  time units with the possibility  $\mu_L(l_m)$ . Fuzzy demand  $DL$  during lead time takes in this case a fuzzy value  $DL_m$ , obtained as a sum of fuzzy demands in the  $l_m$  time units within the lead time:

$$DL_m = D_1 + \dots + D_{l_m}$$

The possibility that fuzzy demand during lead time takes the value  $DL_m$  is equal to the possibility  $\mu_L(l_m)$  of the lead time being  $l_m$  time units. Accordingly, fuzzy demand  $DL$



during the overall fuzzy lead time  $L$  can take fuzzy values  $DL_1, \dots, DL_M$  with the possibilities  $\mu_L(l_1), \dots, \mu_L(l_M)$ , respectively, and it is represented by a level 2 fuzzy set:

$$DL = \{\mu_L(l_1)/DL_1, \dots, \mu_L(l_M)/DL_M\}$$

This level 2 fuzzy set is obtained by combining two fuzzy data, fuzzy demand per time unit and the fuzzy lead time. As it is described in Section 3.1., the level 2 fuzzy set  $DL$  can be transformed to an ordinary fuzzy set applying the s-fuzzification procedure. In this way, a uniform representation of different uncertainties in demands, such as uncertainty in demand during a precisely specified time period and uncertainty in demand during an uncertain time period, is obtained using standard fuzzy sets.

Let us note that the same approach to modelling fuzzy demand during fuzzy lead time can be used in the case of continuous demand per unit time which is described by a fuzzy set with any form of continuous membership function.

## **CHAPTER 5**

### **FUZZY MODELLING OF A SERIAL SUPPLY CHAIN**

In this chapter a new original approach to modelling SCs in uncertain environments is presented. The chapter is organised in the following way.

In Section 5.1., the structure of an SC under consideration, the basic assumptions concerning the SC, operations and control rules are defined.

New SC fuzzy analytical models developed by the author are presented in Section 5.2. In these fuzzy models optimal order-up-to levels and replenishment quantities over time are determined for all inventories along a serial SC. Development of the SC fuzzy models consists of two parts. In the first part, new fuzzy models for an isolated single stocking point inventory control have been developed. In the three fuzzy models, different sources of uncertainty, such as customer demand, supply and the supply lead time, inherent in an environment in which an inventory operates are included (Petrovic, 1995). In these models uncertain data are represented by fuzzy sets. An optimisation problem is stated where the objective is to minimise the inventory possible total cost. The optimisation method developed and applied is based on a simple one-dimensional searching. In each model two new algorithms have been developed: (1) an algorithm for determining inventory order-up-to levels during a finite time horizon, and (2) an algorithm for determining replenishment quantities to be ordered during the time horizon. In the second part of Section 5.2., the fuzzy models for an isolated inventory have been extended to the SC fuzzy models. Two important issues concerning SC modelling are discussed: (1) demand propagation along an SC

and (2) SC control strategies. Two control strategies have been formally defined: (1) decentralised control and (2) a new concept of partially coordinated control.

Illustrative numerical SC examples are given in Section 5.3.

Finally, the potential of using the fuzzy approach to model serial SCs in uncertain environments is summarised in Section 5.4.

### **5.1. Supply chain structure and operations**

The research is focused on an SC with all facilities in a serial link, including a raw material inventory, a number of in-process inventories and an end-product inventory and production facilities between them (Figure 5.1). Each facility along the SC encompasses a production facility and an inventory where the production output is stored, except the first facility in the SC which includes a raw material inventory only. Distribution and selling points are not treated as parts of the SC, but their functions are taken into consideration through customer demand. It is assumed that each facility in the SC adds value to the end-product or represents a buffer storage. The SC is linked with the external environment by customer demand from one side and a raw material supplier from the other side.

SC management and control cover the flow of goods from supplier through production facilities to the end-product inventory. Different types of management and control integration and, consequently, different control strategies can exist in an SC, as completely decentralised management and control, partial coordination to fully centralised management and control and total SC integration. Sometimes, all facilities in the SC may be "under the roof" of one company. However, parts of the SC, or even each facility in the SC may belong to a number of different companies; then, the succeeding facility in the SC may be viewed as its customer and the preceding facility in the SC as its external supplier.



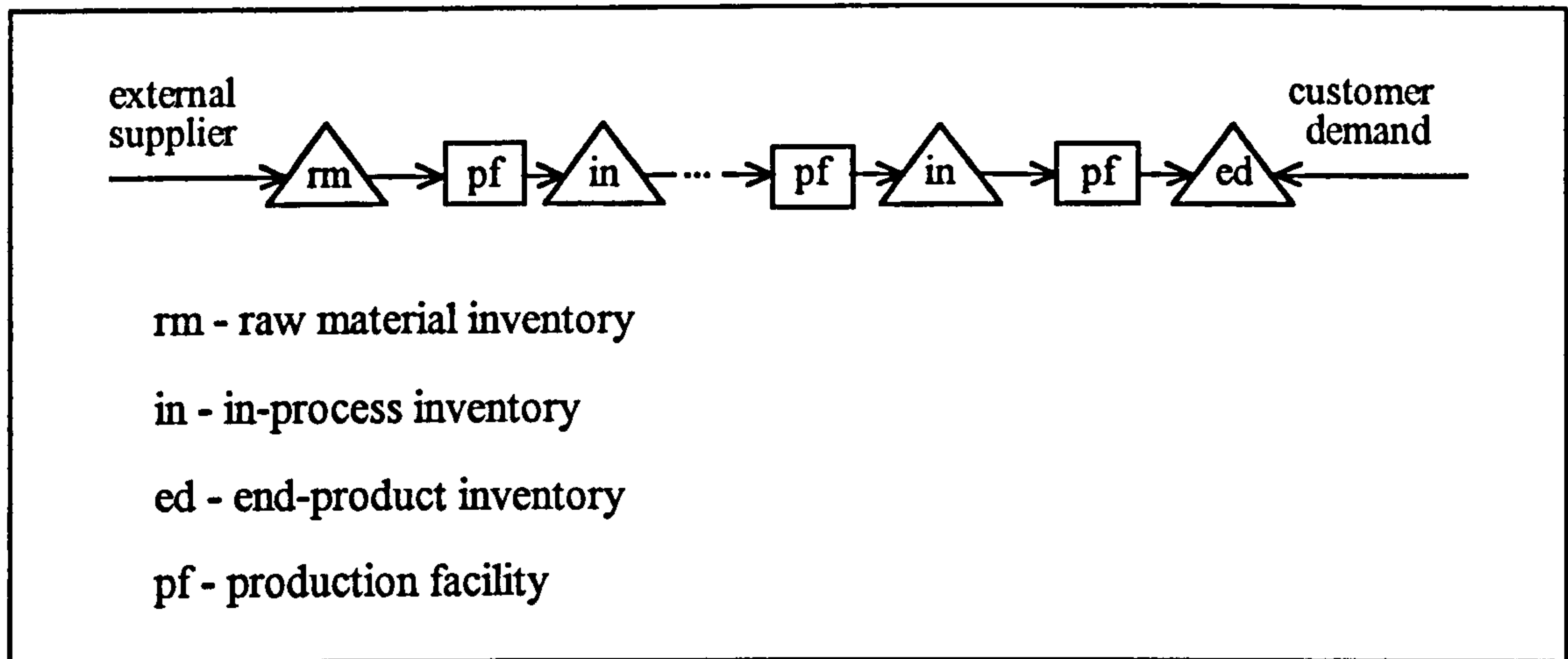


Figure 5.1. Schematic view of a serial SC

Assumptions concerning SC processes and SC management and control treated here are the following:

- Each customer demands a non-fixed number of end-products per time interval and demand is confined to a single type of the end-product.
- Each inventory in the SC is controlled based on a periodic review policy. This includes the regular review of stock levels with fixed intervals between orders. An order quantity is determined in such a way as to replenish inventory to a certain predetermined level, known as the order-up-to level.
- Customer demand is fulfilled from the end-product inventory, only. When demand exceeds the end-product stock, unmet demand is backordered and delivered to customers as soon as it becomes available in stock.
- Each production facility replenishes the succeeding inventory in the SC, where items are stored until they are consumed by the next succeeding facility, or by external customers in the case of the end-product inventory. Each production facility places orders on the preceding facility in the SC. Complete back-ordering is assumed and the policy adopted is to satisfy an order as much as possible. If an

order from the production facility exceeds the stock of the preceding inventory, the order is only partially filled and unmet quantity is backordered. When a part or the whole backordered quantity becomes available in stock, it is sent to the production facility in the next delivery period. This delivery process continues until the whole quantity ordered from the production facility is delivered by the preceding inventory in the SC.

- The raw material inventory is supplied from an external source.
- The production facilities have unlimited capacities.
- Replenishment quantities for each inventory are received with a lead time. This means that there is a time period that elapses from the moment when a replenishment order is placed until it arrives, provided that the preceding facility has a sufficient stock level. The lead time includes the time necessary for order processing, production time and/or transportation time.

## 5.2. Supply chain fuzzy models

Consider an SC in an uncertain environment with  $N$  facilities in a series during a finite time horizon  $T_h$ , taking into account all the assumptions given in the previous section. The structure of the  $N$ -serial SC is depicted in Figure 5.2. The inventories along the SC are identified by upper-scripts; the upper-script for the end-product inventory is 1 and the upper-script for the raw material inventory is  $N$ .

The aims of SC fuzzy models are to determine an order-up-to level for each review period during the time horizon  $T_h$  for each inventory in the  $N$ -serial SC and replenishment quantities to be ordered over the time, periodically, by each inventory, in order to obtain an acceptable service level of the SC at a reasonable SC total cost. The SC service level and the SC total cost over  $T_h$  will be defined in Section 5.3. The first

step towards SC fuzzy modelling is the development of fuzzy control models for an isolated single stocking point inventory.

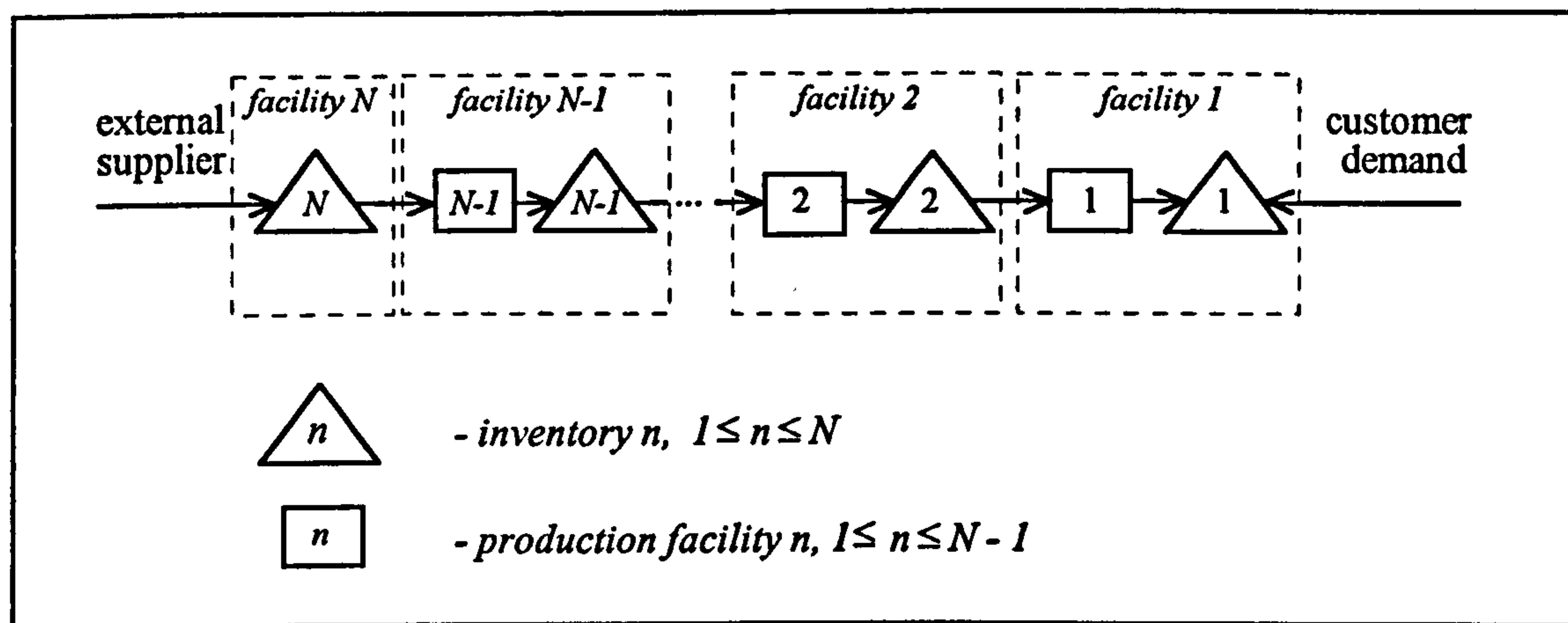


Figure 5.2.  $N$ -serial SC

### 5.2.1. Fuzzy models of an isolated inventory

Let us focus on inventory  $n$ ,  $1 \leq n \leq N$  in the SC and consider it isolated from the rest of the SC facilities. In order to simplify the notation, the superscript  $n$ , used to identify the inventory is not necessary and, therefore, it is omitted in this Section.

The problem is to find order-up-to levels for an inventory over the finite time horizon  $T_h$ . Assume that the time horizon  $T_h$  is homogeneously discretised into  $T$  unit time intervals of equal length. They are indexed by an index  $t$ ,  $t = 1, \dots, T$ . It is assumed that review periods are fixed over time and  $R$  time units long. The review periods within  $T_h$  are indexed by an index  $k$ ,  $k = 1, \dots, K$ . The replenishment quantity ordered at the beginning of review period  $k$  is received after a given fixed lead time. Complete back-ordering is assumed, i.e., in the stock-out situation unmet demand is backordered and filled as soon as a replenishment quantity arrives in stock. If the replenishment quantity received is large enough, unmet demand is fully satisfied. Otherwise, the



quantity available in stock is sent to the customers and the rest of the backorder is filled when the necessary quantity becomes available in stock.

Two problems are considered:

1. find the inventory order-up-to level for each review period  $k$  within  $T_h$ , to minimise the total inventory cost incurred during  $T_h$ ,
2. calculate the quantity that should be ordered at the beginning of each review period.

The total inventory cost is calculated as a sum of: (1) holding cost, linearly dependent on the inventory carried and (2) shortage cost, linearly dependent on customer demand that has not been satisfied promptly from the shelf; it is assumed that the length of the time the customer has been kept waiting for unmet demand has no influence on the shortage cost. The holding cost per unit stored per time interval is a constant  $c_h$  and the shortage cost per unit undelivered on request is a constant  $c_s$ .

There is another approach to formulate the problem of finding an inventory order-up-to level, which is widely used in traditional inventory control models. The order-up-to level is determined to achieve some service target, such as the fill rate, at the minimum holding cost. In this research, the approach to minimise the total inventory cost is applied. This is motivated by the following reason. A model for an isolated inventory stocking point to be developed is a building block for the whole SC model. However, the specification of target fill rates at different inventories in an SC to support a desired overall SC fill rate is generally a complex and analytically intractable task. Instead, the concept of shortage cost is used. Let us notice that in this approach, the inventory fill rate is implicitly controlled through the mechanism of unit shortage cost; increasing the unit shortage cost causes higher fill rate.

Three fuzzy models of an isolated inventory which treat uncertainty in customer demand, supply and lead time, respectively, are presented in the sections that follow. Each model includes two algorithms: an algorithm for determining order-up-to levels for the inventory during a finite time horizon and an algorithm for determining replenishment quantities during the time horizon.

### 5.2.1.1. Fuzzy model 1: Fuzzy demand

This fuzzy model assumes that uncertainty exists in customer demand only. Supplier lead time is fixed and the length is  $L$  time units. Once, the order is placed to the supplier, no other orders can be received within the period of  $L+R$  units. It is assumed that all replenishment quantities ordered by the inventory will be received from a supplier.

Customer demand during time unit  $t$  within the time horizon under consideration is imprecisely specified and represented by a discrete fuzzy set  $D_t$ , i.e., by an induced discrete possibility distribution  $\mu_{D_t}(d_t)$ ,  $d_t \in \mathcal{A}_t$ , where  $d_t$  is the possible discrete demand value in time unit  $t$  and  $\mu_{D_t}(d_t)$  is the associated possibility of demand taking the value  $d_t$ .

Figure 5.3. illustrates dynamic changes of the inventory levels in two cases: (a) when customer demand appears in each time unit, as in the case of an end-product inventory in a serial SC, and (b) when demand appears periodically and the period between two successive demands includes more than one time units; this corresponds to an in-process or a raw material inventory in a serial SC; namely, regular demand placed on an end-product inventory is converted into demand periodically placed on an in-process or a raw material inventory, applying the periodic review policy. In Figure 5.3. the inventory level during each time unit  $t$ ,  $t = 1, 2, \dots$ , is approximated to be equal to the inventory level recorded at the end of the time unit and Figure 5.3.(b) shows the case when demand on the inventory is placed at the beginning of each review period.

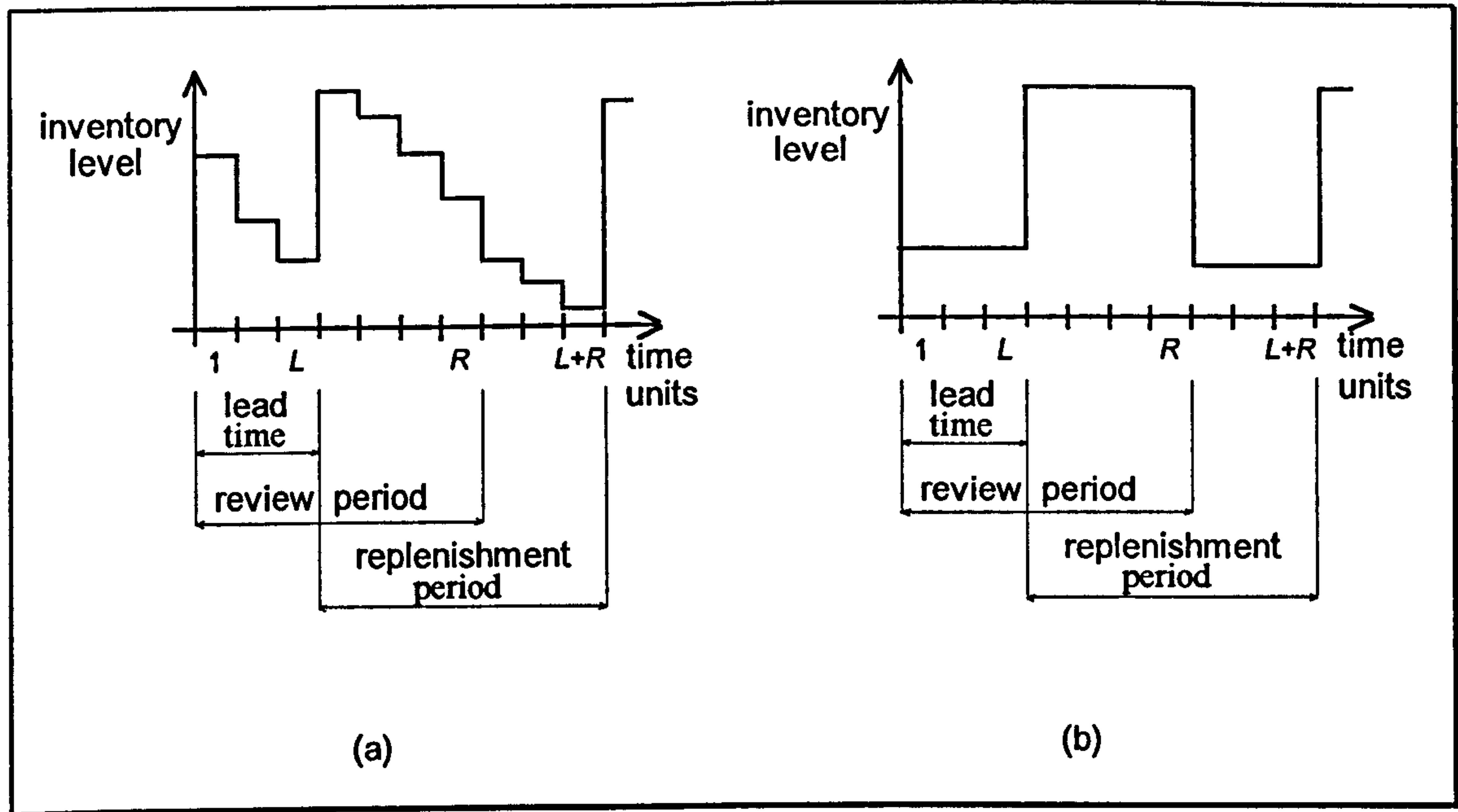


Figure 5.3. Dynamic changes of inventory levels when

(a) demand appears in each time unit

(b) demand appears periodically, every  $R$  time units

#### Algorithm for determining inventory order-up-to levels during the finite time horizon

The problem of determining inventory order-up-to levels during the finite time horizon  $T_h$  is decomposed into a sequence of subproblems of determining an order-up-to level for each review period within  $T_h$  independently. The order-up-to level  $S_{d,k}$  for review period  $k$ ,  $k = 1, \dots, K$  within  $T_h$  is determined to minimise the possible total cost incurred during the corresponding replenishment period  $k$ . Order-up-to levels  $S_{d,k}$ ,  $k = 1, \dots, K$  determined in this way lead to the minimum possible total cost incurred during the whole time horizon  $T_h$  due to the assumptions about SC operations, such as the constant unit holding and unit shortage cost, unlimited productions and backordering.

To simplify notation, the algorithm for determining the order up-to level  $S_{d,1}$  for the first review period  $k = 1$  is presented, only.  $S_{d,1}$  is calculated to minimise the possible total cost incurred during the first replenishment period, i.e., during the time



period that elapses between the first and the second inventory replenishment. Formally, the first replenishment period comprises time units  $L+1, L+2, \dots, L+R$ . The algorithm for determining  $S_{d,1}$  contains four steps.

### Step 1. Demand forecast

Fuzzy demand  $DR_1$  during the first replenishment period is forecast using the arithmetic rule for fuzzy number addition, given in (3.25):

$$\begin{aligned} DR_1 &= D_{L+1} + D_{L+2} + \dots + D_{L+R} \quad \text{and} \\ \mu_{DR_1}(dr_1) &= \max \min(\mu_{D_{L+1}}(d_{L+1}), \mu_{D_{L+2}}(d_{L+2}), \dots, \mu_{D_{L+R}}(d_{L+R})) \\ dr_1 &= d_{L+1} + d_{L+2} + \dots + d_{L+R} \\ d_{L+1} &\in \mathcal{D}_{L+1}, d_{L+2} \in \mathcal{D}_{L+2}, \dots, d_{L+R} \in \mathcal{D}_{L+R}, \quad dr_1 \in \mathcal{D}_1 \end{aligned} \quad (5.1)$$

Step 2 and Step 3 are then repeated sequentially, for each potential inventory order-up-to level  $S \in \mathcal{S}_1$ .

### Step 2. Calculation of possible holding and possible shortage costs

For  $S \in \mathcal{S}_1$ , the possible holding and the possible shortage costs incurred during the first replenishment period are calculated as follows.

Fuzzy demand  $DR_1$  causes the fuzzy holding cost  $FR_{h,1}(S)$  and the fuzzy shortage cost  $FR_{s,1}(S)$ . The possible holding cost  $FR'_{h,1}(S, dr_1)$  incurred when demand is  $dr_1 \in \mathcal{D}_1$  is:

$$\begin{aligned} FR'_{h,1}(S, dr_1) &= c_h \cdot \{(S - d'_{L+1})^+ + (S - d'_{L+1} - d'_{L+2})^+ + \dots + \\ &\quad (S - d'_{L+1} - d'_{L+2} - \dots - d'_{L+R})^+\} \end{aligned} \quad (5.2)$$

where  $(a)^+ = \max(a, 0)$ ,  $d'_{L+1} \in \mathcal{D}_{L+1}$ ,  $d'_{L+2} \in \mathcal{D}_{L+2}$ ,  $\dots$ ,  $d'_{L+R} \in \mathcal{D}_{L+R}$  are the possible demands and  $(S - d'_{L+1})^+$ ,  $(S - d'_{L+1} - d'_{L+2})^+$ ,  $\dots$ ,  $(S - d'_{L+1} - d'_{L+2} - \dots - d'_{L+R})^+$  are the corresponding inventory levels in time units  $L+1, L+2, \dots, L+R$ , respectively.

Demands  $d'_{L+1}, d'_{L+2}, \dots, d'_{L+R}$  are selected in such a way as to satisfy two conditions:

$$dr_1 = d'_{L+1} + d'_{L+2} + \dots + d'_{L+R} \quad (5.3)$$

and the term

$$\min(\mu_{D_{L+1}}(d_{L+1}), \mu_{D_{L+2}}(d_{L+2}), \dots, \mu_{D_{L+R}}(d_{L+R})) \quad (5.4)$$

reaches the maximum for these particular demands, i.e., when  $d_{L+1} = d'_{L+1}$ ,  $d_{L+2} = d'_{L+2}$ ,  $\dots$ ,  $d_{L+R} = d'_{L+R}$ . If there are more than one demand combinations that satisfy (5.3) and (5.4), the demand combination that generates the highest holding cost is taken into account. In other words, in the calculation of the holding cost only the most possible combination of demands in time units that generate particular demand during the replenishment period and the corresponding most possible inventory levels are taken into account. It is interesting to notice that in this step a difference between the fuzzy and stochastic approach to calculating holding cost is clearly demonstrated. In contrast to the simple averaging of inventory levels during the replenishment period applied in a stochastic approach, the notion of the most possible inventory levels is introduced and used to calculate the possible holding cost.

The possible shortage cost  $FR'_{s,1}(S, dr_1)$  charged for failures to meet demand  $dr_1 \in \mathcal{R}_1$  during the first replenishment period is equal to

$$FR'_{s,1}(S, dr_1) = c_s \cdot (dr_1 - S)^+ \quad (5.5)$$

The possibilities of the holding and shortage costs are determined based on the demand possibility distribution. Applying the Extension Principle, given in Section 3.1., both the possibility of the holding cost being  $FR'_{h,1}(S, dr_1)$  and the possibility of the shortage cost being  $FR'_{s,1}(S, dr_1)$  are equal to the possibility of demand being  $dr_1$ :

$$\mu_{FR_{h,1}}(FR'_{h,1}(S, dr_1)) = \mu_{DR_1}(dr_1), \quad dr_1 \in \mathcal{R}_1 \quad (5.6)$$

$$\mu_{FR_{s,1}}(FR'_{s,1}(S, dr_1)) = \mu_{DR_1}(dr_1), \quad dr_1 \in \mathcal{DR}_1 \quad (5.7)$$

### Step 3. Calculation of the total cost

The possible total cost  $FR_1(S)$  incurred during the first replenishment period with order-up-to level  $S \in \mathcal{SR}_1$  is obtained by translating the fuzzy holding and the fuzzy shortage costs into scalars which most properly represents the costs:

$$\begin{aligned} FR_1(S) &= defuzz(FR_{h,1}(S)) + defuzz(FR_{s,1}(S)) \\ &= \frac{\sum_{dr_1 \in \mathcal{DR}_1} FR'_{h,1}(S, dr_1) \cdot \mu_{FR_{h,1}}(FR'_{h,1}(S, dr_1))}{\sum_{dr_1 \in \mathcal{DR}_1} \mu_{FR_{h,1}}(FR'_{h,1}(S, dr_1))} \\ &\quad + \frac{\sum_{dr_1 \in \mathcal{DR}_1} FR'_{s,1}(S, dr_1) \cdot \mu_{FR_{s,1}}(FR'_{s,1}(S, dr_1))}{\sum_{dr_1 \in \mathcal{DR}_1} \mu_{FR_{s,1}}(FR'_{s,1}(S, dr_1))} \\ &= \frac{\sum_{dr_1 \in \mathcal{DR}_1} FR'_{h,1}(S, dr_1) \cdot \mu_{DR_1}(dr_1)}{\sum_{dr_1 \in \mathcal{DR}_1} \mu_{DR_1}(dr_1)} + \frac{\sum_{dr_1 \in \mathcal{DR}_1} FR'_{s,1}(S, dr_1) \cdot \mu_{DR_1}(dr_1)}{\sum_{dr_1 \in \mathcal{DR}_1} \mu_{DR_1}(dr_1)} \end{aligned} \quad (5.8)$$

where the operator *defuzz* denotes arithmetic defuzzification based on the moment rule which is defined by formula (3.16). The moment rule takes as a fuzzy set representative a scalar value in the domain at which a line perpendicular to the axes would pass through the centre of the fuzzy set area.

Alternatively, the possible total cost  $FR_1(S)$ ,  $S \in \mathcal{SR}_1$  can be calculated as:

$$FR_1(S) = defuzz(FR_{h,1}(S) + FR_{s,1}(S)) \quad (5.9)$$

It can be proved that the two ways of calculating the total cost  $FR_1(S)$ , given in (5.8) and (5.9) are equivalent:



$$\text{defuzz}(FR_{h,1}(S)) + \text{defuzz}(FR_{s,1}(S))$$

$$= \frac{\sum_{dr_1 \in \mathcal{DR}_1} FR'_{h,1}(S, dr_1) \cdot \mu_{DR_1}(dr_1)}{\sum_{dr_1 \in \mathcal{DR}_1} \mu_{DR_1}(dr_1)} + \frac{\sum_{dr_1 \in \mathcal{DR}_1} FR'_{s,1}(S, dr_1) \cdot \mu_{DR_1}(dr_1)}{\sum_{dr_1 \in \mathcal{DR}_1} \mu_{DR_1}(dr_1)}$$

$$= \frac{\sum_{dr_1 \in \mathcal{DR}_1} \{FR'_{h,1}(S, dr_1) + FR'_{s,1}(S, dr_1)\} \cdot \mu_{DR_1}(dr_1)}{\sum_{dr_1 \in \mathcal{DR}_1} \mu_{DR_1}(dr_1)}$$

$$= \text{defuzz}(FR_{h,1}(S) + FR_{s,1}(S))$$

#### Step 4. Determining the optimal order-up-to level

Optimal order-up-to level  $S$  for the first review period that minimises  $FR_1(S)$  in (5.8) is determined by a simple one-dimensional search through the set  $\mathcal{DR}_1$  and it is denoted by  $S_{d,1}$ .

#### Additional remarks pertaining to the algorithm

The following remarks give more insight into the function of possible total cost  $FR_1(S)$  given by formula (5.8) and point out at some important characteristics of the optimal order-up-to level calculated using the algorithm proposed.

Remark 1. The total cost  $FR_1(S)$ ,  $S \in \mathcal{DR}_1$ , attains a single minimum and it is proved in the following way. By analysing the total cost function one can find that the increment  $\delta FR_1(S) = FR_1(S+\delta) - FR_1(S)$  is negative for  $S = 0$  and positive for  $S \rightarrow \infty$ . Generally, keeping no inventory ( $S = 0$ ) eliminates holding cost and generates only shortage cost. In this case  $FR_1(0)$  includes shortage cost only. Increasing the order-up-to level by  $\delta$ , from  $S = 0$  to  $S = \delta$ , decreases the total cost  $FR_1(\delta)$  in comparison with  $FR_1(0)$ ; the shortage cost is decreased by  $c_s \cdot \delta$  and the holding cost per unit time interval is increased by  $c_h \cdot \delta$ . Since it is reasonable to assume that  $c_s$  is greater or even much

greater than  $c_h$ ,  $\delta FR_1(0)$  becomes negative. On the other hand, raising  $S$  to levels greater than the maximum possible demand ( $S \rightarrow \infty$ ) increases the holding cost, while shortage cost remains zero. This means that  $\delta FR_1(S)$  is positive for  $S \rightarrow \infty$ . Due to the fact that  $\delta FR_1(S)$  is negative for  $S = 0$  and positive for  $S \rightarrow \infty$  and that the cost increment function  $\delta FR_1(S)$  is monotone, a single minimum is guaranteed, while  $S$  is passing from 0 to  $\infty$ .

**Remark 2.** Searching for the optimal order-up-to level  $S_{d,1}$  is confined initially to the set  $\mathcal{M}_1$  which contains possible demand values during the first replenishment period. It is interesting to analyse two boundary cases, when the optimal order-up-to level selected from this set is equal to the maximum or the minimum possible demand value. Let us first examine the case when the order-up-to level is equal to the maximum possible demand value  $dr_{1,\max}$ . Setting the order-up-to level to a value higher than  $dr_{1,\max}$  will result in a cost higher than the cost incurred when the order-up-to level is equal to  $dr_{1,\max}$ . This is explained in Remark 1, for  $S \rightarrow \infty$ . This means that it is not necessary to examine order-up-to levels greater than  $dr_{1,\max}$ . On the other hand, if the selected order-up-to level is equal to the minimum possible demand value  $dr_{1,\min}$ , it is worth continuing the searching process because further reduction of the order-up-to level can lead to a smaller cost than the cost achieved with setting the order-up-to level to  $dr_{1,\min}$ . In this case, initially determined order-up-to level should be decreased iteratively, until the next iteration leads to a higher possible total inventory cost or the order-up-to level is reduced to 0.

**Remark 3.** The algorithm proposed can be applied to inventories with different dynamic changes of the stock levels. For example, if demand appears periodically, every  $R$  time units as illustrated in Figure 5.3. (b), customer demand in time unit  $t$  is represented by a fuzzy set  $D_t$ , where  $t = 1, R+1, 2R+1, \dots$ , and it is 0 in the rest of the time units. Demand during the first replenishment period, using formula (5.1), becomes:

$$DR_1 = D_1, \text{ and} \quad \mu_{DR_1}(dr_1) = \mu_{D_1}(d_1), \quad \mathcal{DR}_1 = \mathcal{D}_1, \quad dr_1, d_1 \in \mathcal{D}_1 \quad (5.10)$$

Also, the calculation of the possible holding cost  $FR'_{h,1}(S, dr_1)$  during the replenishment period using formula (5.2) becomes simpler. During the first  $R-L$  time units within the replenishment period, the inventory level is unchanged, equal to  $S$ , and then, when demand  $dr_1$  is placed it drops to  $S-dr_1$  and remains unchanged during the lead time of  $L$  time units. Formally,

$$FR'_{h,1}(S, dr_1) = c_h \cdot \{ S \cdot (R - L) + (S - dr_1)^+ \cdot L \} \quad (5.11)$$

Remark 4. It should be noted that the optimal inventory order-up-to level, determined using the algorithm described, depends on the ratio of the unit shortage cost to the unit holding cost, not on their absolute values. This can be proved in the following way.

The possible total cost incurred when inventory level is  $S$  and demand is  $dr_1$ , is equal to the sum of the possible holding cost  $FR'_{h,1}(S, dr_1)$  and the possible shortage cost  $FR'_{s,1}(S, dr_1)$ . Using (5.2) and (5.5), the sum of these is:

$$\begin{aligned} FR'_{h,1}(S, dr_1) + FR'_{s,1}(S, dr_1) &= c_h \cdot C1(S, dr_1) + c_s \cdot C2(S, dr_1) \\ &= c_h \cdot \left\{ C1(S, dr_1) + \frac{c_s}{c_h} \cdot C2(S, dr_1) \right\} \end{aligned} \quad (5.12)$$

where

$$C1(S, dr_1) = (S - d'_{L+1})^+ + (S - d'_{L+1} - d'_{L+2})^+ + \dots + (S - d'_{L+1} - d'_{L+2} - \dots - d'_{L+R})^+ \text{ and} \\ C2(S, dr_1) = (dr_1 - S)^+.$$

Then, the possible total cost  $FR_1(S)$  calculated using (5.9) becomes:

$$FR_1(S) = \frac{\sum_{dr_1 \in \mathcal{DR}_1} \left\{ C1(S, dr_1) + \frac{c_s}{c_h} \cdot C2(S, dr_1) \right\} \cdot \mu_{DR_1}(dr_1)}{\sum_{dr_1 \in \mathcal{DR}_1} \mu_{DR_1}(dr_1)} \cdot c_h \quad (5.13)$$



One can see that the relative magnitude of the possible total costs  $FR_1(S)$  calculated for different order-up-to levels depends on the ratio  $\frac{c_s}{c_h}$ , only.

Consequently, inventories that have the same ratio of unit shortage to unit holding cost and with the same demand during a replenishment period have the same optimal order-up-to levels.

#### Algorithm for determining replenishment quantities during the finite time horizon

Replenishment quantity  $O_{d,k}$  that has to be ordered at the beginning of review period  $k$ ,  $k = 1, \dots, K$  is determined in such a way as to bring the inventory level after the replenishment to the pre-selected level  $S_{d,k}$ . Two approaches to determining  $O_{d,k}$  are analysed: (1) the first approach is to choose them simultaneously and (2) the second approach is to choose them one at a time.

1. Replenishment quantities  $O_{d,k}$ ,  $k = 1, \dots, K$  have to be predetermined for all the review periods within the finite time horizon  $T_h$ . In this case uncertainty associated with possible inventory levels is propagated from one to the succeeding review period. The future is not precisely determined by the present. Starting with initial level  $I_0$  precisely known, the inventory level at the beginning of the second review period,  $k = 2$ , becomes uncertain due to uncertain demand, and so on, for  $k = 3, \dots, K$ . The number of possible inventory levels is increasing over  $T_h$ . Fuzziness is accumulated and in the last review period,  $k = K$ , the fuzzy representation of possible inventory levels might become considerably blurred. A replenishment quantity that has to be ordered at the beginning of a review period depends on possible inventory levels at that moment. Consequently, the simultaneous calculation of replenishment quantities over  $T_h$  is too difficult. It involves more and more uncertainty and the algorithm becomes more and more computationally extensive.

2. The whole problem is decomposed into a sequence of independent subproblems, each of them for one review period. Replenishment quantities  $O_{d,k}$ ,  $k = 1, \dots, K$  are calculated one at a time, using information about the actual current inventory level. In this way, a feedback control concept is involved. After determining the order quantity for the first review period,  $O_{d,1}$ , and applying it, the inventory level at the beginning of the second review period,  $k = 2$ , is precisely known. The algorithm for determining replenishment quantity  $O_{d,2}$  is repeated for the current inventory level, and so on, until the last replenishment quantity  $O_{d,K}$ .

The second approach which is actually easier to utilise from the stand point of formulation and numerical evaluation is applied here. First, the replenishment quantity  $O_{d,1}$ , when the initial inventory level is  $I_0$ , is calculated at the beginning of the first review period,  $k = 1$ . The calculation is performed through three steps.

#### Step 1. Calculation of demand during lead time

Uncertain demand  $DL$  during the lead time of  $L$  time units is determined using the arithmetic rule for addition of fuzzy numbers:

$$\begin{aligned}
 DL &= D_1 + D_2 + \dots + D_L \quad \text{and} \\
 \mu_{DL}(dl) &= \sup \min(\mu_{D_1}(d_1), \mu_{D_2}(d_2), \dots, \mu_{D_L}(d_L)) \\
 dl &= d_1 + d_2 + \dots + d_L \\
 d_1 &\in \mathcal{D}_1, d_2 \in \mathcal{D}_2, \dots, d_L \in \mathcal{D}_L, \quad dl \in \mathcal{DL}
 \end{aligned} \tag{5.14}$$

#### Step 2. Calculation of the most possible inventory level before replenishment

Due to uncertain demand  $DL$  the inventory level before replenishment cannot be determined precisely. It is also fuzzy. The most possible inventory level before replenishment is equal to  $I_0 - dl^*$ , where  $dl^*$  is the most possible demand during the lead time  $L$ :

$$\mu_{DL}(dl') = \max_{dl \in \mathcal{L}} \mu_{DL}(dl) \quad (5.15)$$

It should be noted that the inventory level before a replenishment can be negative, indicating that a shortage appeared in the previous replenishment period.

### Step 3. Calculation of replenishment quantity

The replenishment quantity  $O_{d,1}$  at the beginning of the first review period, that will most possibly bring the inventory level after replenishment to the optimal level  $S_{d,1}$  is

$$O_{d,1} = \max [S_{d,1} - (I_0 - dl'), 0] \quad (5.16)$$

This algorithm is repeated at the beginning of the second, third and all other review periods when the current inventory levels become precisely known. It is important to emphasise that the concept of a replenishment policy as a function of the current stock is a necessity in the study of inventory control with uncertain demand.

#### **5.2.1.2. Fuzzy model 2: Fuzzy demand and fuzzy supply**

In this fuzzy model it is assumed that uncertainty exists in both demand and supply of items. Uncertain demands in time units  $t = 1, \dots, T+L$  are represented in the same way as in Fuzzy model 1. Let us note that the demand estimations in time units  $T+1, \dots, T+L$  which do not belong to the specified time horizon  $T_h$  are required and used in the calculation of the optimal order-up-to level for the last replenishment period within  $T_h$ . Uncertain supply during  $T_h$  is represented by a discrete fuzzy set  $SQ = \{\mu_{SQ}(s_1)/s_1, \dots, \mu_{SQ}(s_U)/s_U\}$ , where  $s_1, \dots, s_U$  are the undelivered quantities of items from the supplier and  $\mu_{SQ}(s_1), \dots, \mu_{SQ}(s_U)$  are the associated possibilities.



### Algorithm for determining inventory order-up-to levels during the finite time horizon

As in the case of Fuzzy model 1, inventory order-up-to levels  $S_{p,k}$ , for review periods  $k = 1, \dots, K$  are determined separately. The algorithm is presented for  $k = 1$ , i.e., for determining  $S_{p,1}$  for the first review period.

Let  $S_{d,1}$  denote the optimal order-up-to level for the first review period, determined using Fuzzy model 1, which considers uncertainty in customer demand only. In other words,  $S_{d,1}$  is selected so as to minimise the possible total cost during the first replenishment period  $FR_1(S)$ . However,  $S_{d,1}$  leads to the minimum cost under the assumption that all the quantity ordered by the inventory will be received from the supplier. In order to include supplier uncertainty, if it exists,  $S_{d,1}$  has to be modified. It is apparent that an increase of the order-up-to level is required to protect against this additional uncertainty. It is clear that the increase in the order-up-to level must be chosen so as to minimise the possible inventory total cost when supply is uncertain. The algorithm contains four steps.

#### Step 1. Calculation of possible inventory levels after replenishment

Due to uncertain supply, represented by fuzzy set  $SQ$ , the inventory level after the first replenishment will not necessarily raise to the level  $S_{d,1}$ . It becomes fuzzy too. The inventory levels that can be achieved after the first replenishment are:  $S_{d,1} - s_1, \dots, S_{d,1} - s_U$  with the possibilities  $\mu_{SQ}(s_1), \dots, \mu_{SQ}(s_U)$ , respectively.

#### Step 2. Cost calculation

The inventory level after replenishment is a fuzzy value and, consequently, the total cost during the replenishment period becomes fuzzy too. To each possible inventory level  $S_{d,1} - s_u$ ,  $u = 1, \dots, U$ , the possible total cost  $FR_1((S_{d,1} - s_u)^+)$  incurred during the first replenishment period is associated, where  $(a)^+ = \max(a, 0)$ . The

possible total cost  $FR_1((S_{d,1}-s_u)^+)$  is calculated using formula (5.8) and the corresponding possibility is  $\mu_{SQ}(s_u)$ .

### Step 3. Cost defuzzification

The defuzzified possible total cost,  $FRS_1(S_{d,1})$ , incurred when the order-up-to level is  $S_{d,1}$ , is calculated using the defuzzification formula:

$$FRS_1(S_{d,1}) = \frac{\sum_{u=1}^U FR_1((S_{d,1}-s_u)^+) \cdot \mu_{SQ}(s_u)}{\sum_{u=1}^U \mu_{SQ}(s_u)} \quad (5.17)$$

Let us notice that as a consequence of uncertainty in supply, the possible total cost incurred during the first replenishment period, when the order-up-to level is set to  $S_{d,1}$ , is increased from  $FR_1(S_{d,1})$  to  $FRS_1(S_{d,1})$ , given by (5.8) and (5.17), respectively. This is analytically proved here as follows. The possible total cost  $FR_1(S)$  attains its minimum for  $S = S_{d,1}$  and, consequently,  $FR_1((S_{d,1}-s_u)^+) \geq FR_1(S_{d,1})$ ,  $u = 1, \dots, U$ .

Therefore

$$FRS_1(S_{d,1}) \geq \frac{\sum_{u=1}^U FR_1(S_{d,1}) \cdot \mu_{SQ}(s_u)}{\sum_{u=1}^U \mu_{SQ}(s_u)} \quad (5.18)$$

i.e.,  $FRS_1(S_{d,1}) \geq FR_1(S_{d,1})$ .

### Step 4. Cost minimisation

The characteristics of the cost function  $FRS_1$  are similar to the characteristics of the cost function  $FR_1$ , analysed in the previous section. In addition, increasing the order-up-to level  $S_{d,1}$  by one unit decreases the possible total cost. The author gives the following proof:

$$\begin{aligned}
 & FRS_1(S_{d,1}+1) - FRS_1(S_{d,1}) \\
 &= \frac{\sum_{u=1}^U FR_1((S_{d,1}+1-s_u)^+) \cdot \mu_{SQ}(s_u)}{\sum_{u=1}^U \mu_{SQ}(s_u)} - \frac{\sum_{u=1}^U FR_1((S_{d,1}-s_u)^+) \cdot \mu_{SQ}(s_u)}{\sum_{u=1}^U \mu_{SQ}(s_u)} \\
 &= \frac{\sum_{u=1}^U [FR_1((S_{d,1}+1-s_u)^+) - FR_1((S_{d,1}-s_u)^+)] \cdot \mu_{SQ}(s_u)}{\sum_{u=1}^U \mu_{SQ}(s_u)} \\
 &\leq 0, \text{ because } FR_1((S_{d,1}+1-s_u)^+) \leq FR_1((S_{d,1}-s_u)^+)
 \end{aligned} \tag{5.19}$$

The cost function  $FRS_1(S)$ ,  $S \in \mathcal{N}_1$ , has a minimum that can be found by iteratively repeating Step 1 - Step 3, starting from  $S = S_{d,1}$  and incrementally increasing  $S_{d,1}$ , until the order-up-to level  $S_{p,1}$  is found, at which  $FRS_1$  attains the minimum.

It is interesting to notice that the order quantity increases which is a normal reaction to provide local protection from unreliable supply.

#### Algorithm for determining replenishment quantities during the finite time horizon

The algorithm for determining replenishment quantities  $O_{p,k}$ ,  $k = 1, \dots, K$ , when both demand and supply are fuzzy, is very much the same as the corresponding algorithm described in the previous section. Obviously, in this case the inventory order-up-to level at the beginning of each review period  $k$  is set to  $S_{p,k}$ .

##### **5.2.1.3. Fuzzy model 3: Fuzzy demand and fuzzy lead time**

Calculation of the inventory order-up-to levels and the order quantities for replenishment has been, so far, based on a precisely known delivery lead time. In Fuzzy model 3 in addition to uncertainty in customer demand, the delivery lead time is considered as a source of uncertainty, too.



The fuzzy representation of demands during the finite time horizon  $T_h$  is similar to the representation in two previous fuzzy models. The lead time is represented by a discrete fuzzy set  $\{\mu_L(l_1)/l_1, \dots, \mu_L(l_M)/l_M\}$ , where  $l_m$ ,  $m = 1, \dots, M$  represents the length of the lead time expressed in time units and  $\mu_L(l_m)$  is the associated possibility.

#### Algorithm for determining inventory order-up-to levels during the finite time horizon

The optimal inventory order-up-to level  $S_{l,k}$  for each review period  $k$ ,  $k = 1, \dots, K$  is calculated separately. The algorithm is similar to the corresponding one described in Fuzzy model 1. Modifications are made in Step 1 only, to include the fuzziness of the lead time in demand calculation. Without loss of generality, the algorithm is presented for  $k = 1$ .

#### Step 1. Demand calculation

Due to the fuzzy lead time the length of the first replenishment period,  $P_1$ , i.e., the number of the time units that elapse between the first and the second replenishment, is not necessarily equal to the length  $R$  of the fixed review period. The length of the first replenishment period  $P_1$  is a fuzzy number. It depends on the fuzzy lead times for the first and for the second delivery and the length of the review period  $R$ .

Let  $l' \in \{l_1, \dots, l_M\}$  and  $l'' \in \{l_1, \dots, l_M\}$  denote possible lengths of the lead times for the first and the second delivery, respectively, as illustrated in Figure 5.4. Then, the possible length of the first replenishment period is  $p_i$  time units where

$$p_i = R - l' + l'', \quad l', l'' \in \{l_1, \dots, l_M\}, i = 1, \dots, I \quad (5.20)$$

The possibility distribution of length  $P_1$  of the first replenishment period is:

$$\begin{aligned} \mu_{P_i}(p_i) &= \max \min(\mu_L(l'), \mu_L(l'')) \quad i = 1, \dots, I \\ p_i &= R - l' + l'' \\ l', l'' &\in \{l_1, \dots, l_M\} \end{aligned} \quad (5.21)$$

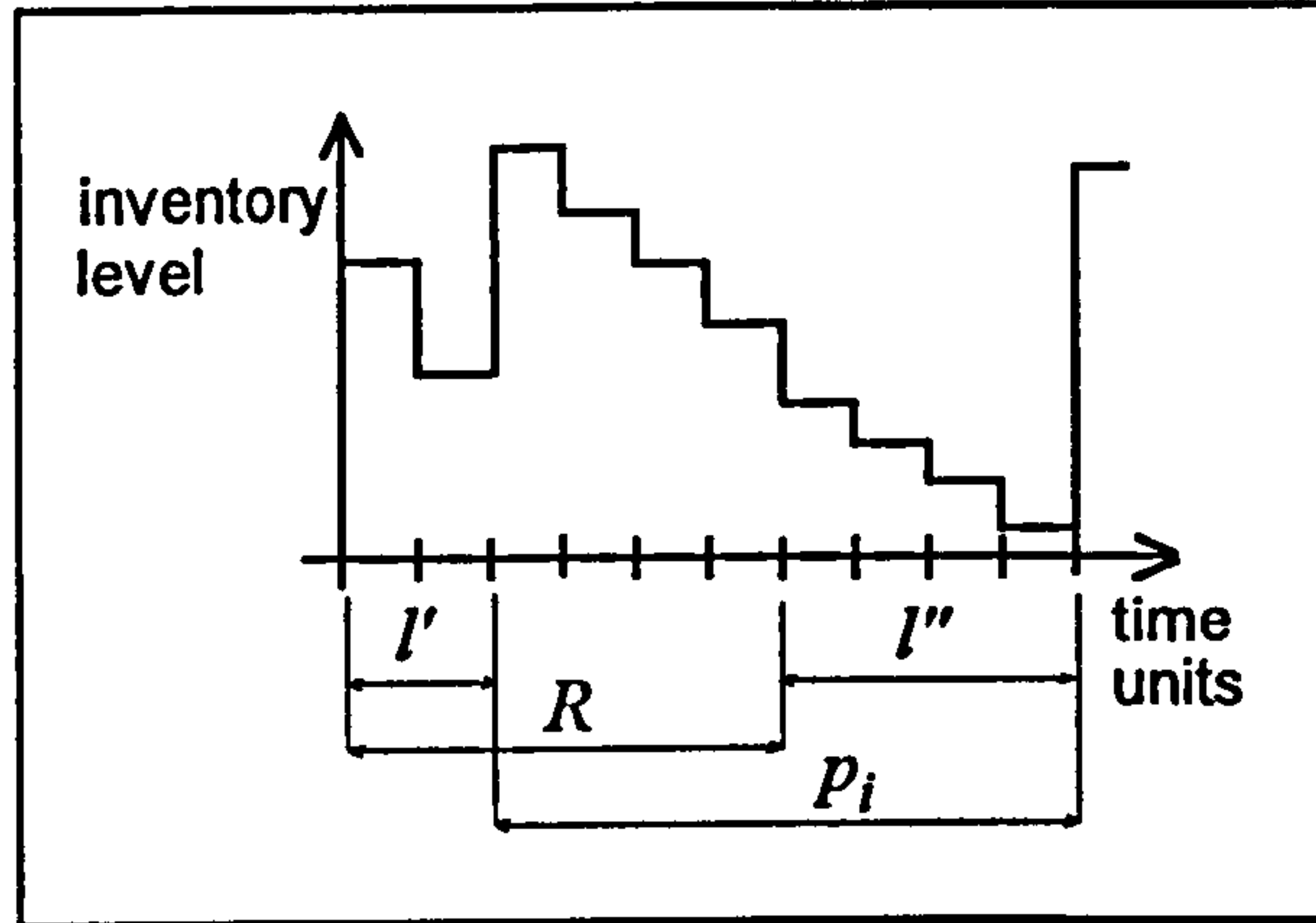


Figure 5.4. Fixed review policy under uncertainty in lead time

In a new approach developed in this research, fuzzy demand during the fuzzy replenishment period is represented by a level 2 fuzzy set and transformed into an ordinary fuzzy set as follows. Demand during the first replenishment period,  $DR_1$ , depends on both the fuzzy length of the replenishment period  $P_1$  and fuzzy demand in each time unit within the replenishment period. When the replenishment period includes  $p_i$ ,  $i = 1, \dots, I$ , time units,  $DR_1$  is approximated by the fuzzy set  $DR_{i,1}$ . It is calculated using the rule for adding up fuzzy numbers:

$$\begin{aligned} DR_{i,1} &= D_{l'+1} + \dots + D_{l'+p_i} \quad \text{and} \\ \mu_{DR_{i,1}}(dr_1) &= \max \min(\mu_{D_{l'+1}}(d_{l'+1}), \dots, \mu_{D_{l'+p_i}}(d_{l'+p_i})) \\ dr_1 &= d_{l'+1} + \dots + d_{l'+p_i} \\ d_{l'+1} &\in \mathcal{D}_{l'+1}, \dots, d_{l'+p_i} \in \mathcal{D}_{l'+p_i}, \quad dr_1 \in \mathcal{DR}_1 \end{aligned} \quad (5.22)$$

where  $l'$  is the possible lead time for the first delivery, such that  $l'$  and  $l''$  is the most possible combination of the lead times for the first and second delivery which gives the

length of replenishment period  $p_i = R - l' + l''$ . According to the Extension Principle, defined in Section 3.1., the possibility of  $DR_1$  taking the fuzzy value  $DR_{i,1}$  is equal to the possibility  $\mu_{P_1}(p_i)$ , specified by formula (5.21).

In other words, demand  $DR_1$  during the first replenishment period has the fuzzy values  $DR_{i,1}$ ,  $i = 1, \dots, I$  with the possibilities  $\mu_{P_1}(p_i)$ . Each fuzzy value  $DR_{i,1}$  is represented by  $\mu_{DR_{i,1}}(dr_1)$ ,  $dr_1 \in \mathcal{DR}_1$ . This means that  $DR_1$  is a level 2 fuzzy set.

The level 2 fuzzy set  $DR_1$  is reduced to an ordinary fuzzy set through the process of s-fuzzification, defined in Section 3.1. An ordinary fuzzy set that represents demand during the first replenishment period,  $s\text{-fuzzif}(DR_1)$ , obtained by s-fuzzification of the level 2 fuzzy set  $DR_1$  is:

$$\mu_{s\text{-fuzzif}(DR_1)}(dr_1) = \sup_{i=1, \dots, I} \mu_{P_1}(p_i) \cdot \mu_{DR_{i,1}}(dr_1) \quad (5.23)$$

where  $dr_1$  is a real number from the set  $\mathcal{DR}_1$ .

Once demand during the first replenishment period is represented by the ordinary fuzzy set  $s\text{-fuzzif}(DR_1)$ , the algorithm for determining the optimal order-up-to level  $S_{l,1}$  continues, following Step 2 to Step 4, defined in the corresponding algorithm in Section 5.2.1.1.

#### Algorithm for determining replenishment quantities during the finite time horizon

The algorithm for determining the replenishment quantity  $O_{l,k}$  at the beginning of review period  $k$ ,  $k = 1, \dots, K$  is very similar to the corresponding algorithm given in Fuzzy model 1. A difference exists in Step 1 only, in which demand during the lead time is calculated. This is illustrated by determining the replenishment quantity  $O_{l,1}$ , to be ordered at the beginning of the first review period, when the initial inventory level is  $I_0$ .



### Step 1. Calculation of demand during fuzzy lead time

Due to the fuzzy lead time, demand  $DL$  during the lead time is represented by a level 2 fuzzy set. When the length of the lead time is  $l_m$  time units,  $m = 1, \dots, M$ , fuzzy demand  $DL$  takes a fuzzy value  $DL_m$ :

$$\begin{aligned}
 DL_m &= D_1 + \dots + D_{l_m} \quad \text{and} \\
 \mu_{DL_m}(dl) &= \max \min(\mu_{D_1}(d_1), \dots, \mu_{D_{l_m}}(d_{l_m})) \\
 dl &= d_1 + \dots + d_{l_m} \\
 d_1 &\in \mathcal{A}_1, \dots, d_{l_m} \in \mathcal{A}_{l_m}, \quad dl \in \mathcal{B}
 \end{aligned} \tag{5.24}$$

The possibility of fuzzy demand  $DL$  taking the fuzzy value  $DL_m$ ,  $m = 1, \dots, M$  is equal to the possibility  $\mu_L(l_m)$  of the lead time being  $l_m$  time units.

Fuzzy demand  $DL$  during the fuzzy lead time is therefore modelled by the level 2 fuzzy set. Applying the procedure of s-fuzzification, it is represented by an ordinary fuzzy set  $s\text{-fuzzif}(DL)$ . The ordinary fuzzy set  $s\text{-fuzzif}(DL)$  is:

$$\mu_{s\text{-fuzzif}(DL)}(dl) = \sup_{m=1, \dots, M} \mu_L(l_m) \cdot \mu_{DL_m}(dl) \tag{5.25}$$

where  $dl$  is a real number from the set  $\mathcal{B}$ .

The algorithm continues, following exactly the same Step 2 and Step 3 of the corresponding algorithm described in Section 5.2.1.1.

### **5.2.2. Extensions to supply chain fuzzy models**

Let us conceive now that  $N$  inventories, initially considered isolated, are connected and form the  $N$ -serial SC, depicted in Figure 5.2. An SC control problem is focused on determining stock and replenishment policies for all the inventories along the chain. It is supposed that production control realises all the conditions for the production

facilities in the SC to be able to execute orders according to the inventory control instructions.

New original SC fuzzy models developed in this research are based on the fuzzy models for isolated single stocking point inventory control described previously. By defining units of items in a suitable way, it is assumed that one unit of the item stored at certain facility is used for producing one unit of the item at the immediate downstream facility. Production units are included in the consideration by their lead times.

In order to determine stock and replenishment policies for inventories along the  $N$ -serial SC during the finite time horizon, i.e., to calculate order-up-to levels of the inventories and replenishment quantities to be ordered from one to the preceding facility over time, the following characteristics of the SC and its environment are considered.

- **Demand propagation along the SC.** Uncertainty in customer demand on the end-product inventory causes uncertainty in demand transmitted along the SC. Uncertain customer demand, in combination with the stock policies applied along the SC, determines internal demand, i.e., demand imposed from one to the preceding facility. An approach to deriving an internal demand forecast based on a customer demand forecast is described in Section 5.2.2.1.
- **SC control strategies.** Different SC control strategies can be applied, reflecting various levels of SC integration. Two SC control strategies are defined and analysed in Section 5.2.2.2.: (1) fully decentralised SC control and (2) a new SC control strategy developed in this research called partially coordinated control .
- **Sources of uncertainty.** Sources of uncertainty that can exist in the environment in which the SC operates, influence the selection of order-up-to levels of the inventories and calculation of the replenishment quantities to be ordered along the SC. The SC fuzzy models developed include uncertainty in customer demand, supply from one to

the succeeding facility and lead times along the chain, and treat all these sources of uncertainty simultaneously.

- **Customer demand characteristics.** Customer demand characteristics, such as stationarity or non-stationarity determine the characteristics of the order-up-to levels. For example, if customer demand is forecast in advance for the whole time horizon and internal demand forecasts are based on the customer demand forecast, the order-up-to levels of all the inventories can be calculated simultaneously. If, in addition, customer demand is stationary, i.e., time-independent, internal demands are also stationary. In this case, the order-up-to levels of any inventory are also stationary, i.e., they are the same for all the inventory review periods within the time horizon under consideration.

#### 5.2.2.1. Demand propagation

In this work, it is assumed that internal demand forecast is based on customer demand forecast. This approach to internal demand forecasting is applicable when the whole SC is "under one roof" and all facts, knowledge and forecast of customer demand are available to all the facilities in the SC.

Customer demand is forecast for the whole time horizon. It is based on subjective managerial judgement and represented by discrete fuzzy set  $D_t$ ,  $t = 1, \dots, T+L^1$ . Let us note that in addition to the demand estimations in time units  $1, \dots, T$ , the demand estimations in time units  $T+1, \dots, T+L^1$  are required, and used in the calculation of the optimal end-product inventory order-up-to level for the last review period within  $T_h$ .

Internal demand on inventory  $n$ ,  $n = 2, \dots, N$ , is forecast for a time unit which coincides with the beginning of a review period of the succeeding inventory  $n-1$ . An estimation of internal demand on inventory  $n$  is equal to the estimation of demand on inventory  $n-1$  during its review period. It is represented by a fuzzy set  $D_t^n$ . For all



other time units which do not coincide with the beginning of any review period of the successor, internal demand is 0. In this way a recursive relation is formed which relates the estimation of demand on one inventory to the estimation of demand on its successor. The notation  $D_t^n$  can also be used for end-product inventory  $n = 1$ , where  $D_t^1$  is equal to customer demand estimation  $D_t$ ,  $t = 1, \dots, T+L^1$ .

Forecast of demand  $D_t^n$ ,  $n = 1, \dots, N$ ,  $t = 1, \dots, T+L^n$  is input to the SC fuzzy models.

#### 5.2.2.2. Supply chain control strategies

Two SC control strategies, decentralised control and partially coordinated control, are built into the SC fuzzy models.

##### Decentralised control

Each inventory  $n$ ,  $n = 1, \dots, N$  in the SC is controlled independently, assuming that the preceding facility in the chain will fulfill any order imposed. If there is uncertainty in customer demand and, consequently, internal demands only, Fuzzy model 1, described in Section 5.2.1.1., is used for determining the order-up-to level  $S_{d,k^n}^n$ , for each inventory  $n$ ,  $n = 1, \dots, N$ , for each review period  $k^n$ ,  $k^n = 1, \dots, K^n$ . If there is uncertainty in both customer demand and lead times from one to the succeeding facility, the order-up-to levels  $S_{l,k^n}^n$  of the inventories are calculated, based on Fuzzy model 3, described in Section 5.2.1.3. The order-up-to levels,  $S_{d,k^n}^n$  or  $S_{l,k^n}^n$ , are determined starting from the end-product inventory ( $n=1$ ), and moving towards the raw material inventory ( $n = N$ ).

The order-up-to levels determined in this way are just locally optimal. It is clear that decentralised stock control does not guarantee satisfactory control of the SC as a whole. However, the overall SC optimisation, particularly with a large number of

inventories in an SC, becomes very complex, analytically intractable and a computationally extensive task. Instead, in order to simplify the problem, a new concept of partially coordinated control is proposed and developed.

### Partially coordinated control

Along the lines of what has preceded, partially coordinated control imposes coordination of each of the two neighboring inventories along an SC. The coordination is performed, starting from the raw material inventory ( $n = N$ ) and its successor ( $n = N-1$ ), continues with inventories  $N-1$  and  $N-2$ , towards the end-product inventory ( $n = 1$ ) that is coordinated with the preceding in-process inventory ( $n = 2$ ). The order-up-to levels  $S_{d,k^n}^n$  or  $S_{l,k^n}^n$ , determined initially in a fully decentralised manner, are modified taking into account uncertainty of delivery performance of the predecessor facility. The partially coordinated order-up-to levels  $S_{p,k^n}^n$  are calculated using Fuzzy model 2, described in Section 5.2.1.2., except the order-up-to levels of the raw material inventory which remain unchanged.

## 5.3. Numerical examples

The determination of order-up-to levels of inventories and the calculation of replenishment quantities is shown by four numerical examples which considers:

- Example 1: Decentralised control with fuzzy customer demand,
- Example 2: Decentralised control with fuzzy customer demand and fuzzy lead times to facilities,
- Example 3: Partially coordinated control with fuzzy customer demand and fuzzy supply along an SC,
- Example 4: Partially coordinated control with fuzzy customer demand, fuzzy supply along an SC, and fuzzy lead times to the facilities.

### 5.3.1. Example 1: Decentralised control with fuzzy customer demand

An SC under consideration, denoted by *SC 1*, consist of five inventories in a serial link ( $N = 5$ ): an end-product inventory, three in-process inventories, a raw material inventory and production facilities between them. (Figure 5.5.)

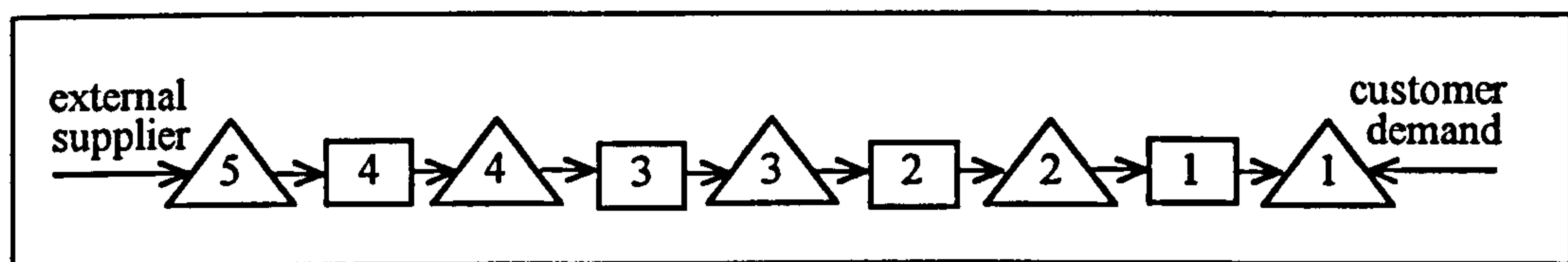


Figure 5.5. Structure of *SC 1*

Specifications of *SC 1* and an environment in which it operates are the following:

- time horizon  $T_h$  is 52 weeks,
- unit time interval  $t$  is 1 week,  $t = 1, \dots, 52$ ,
- customer demand is stationary and uncertain, forecast by a linguistic expression "about 10 end-products per week"; it is defined by a fuzzy set with a triangular membership function, as illustrated in Figure 5.6.,  
 $D_1 = \dots = D_{53} = \{0/6, 0.25/7, 0.5/8, 0.75/9, 1/10, 0.75/11, 0.5/12, 0.25/13, 0/14\}$ ,
- each inventory in *SC 1* has information about customer demand forecast and uses it,
- each inventory applies fixed review period order-up-to level policy,
- beginnings of the review periods of all the inventories coincide,
- the length of review period of each inventory  $n$ ,  $n = 1, \dots, 5$  is 4 weeks ( $R^n = 4$ ),
- external supplier is absolutely reliable,



- raw material delivery time is 1 week, ( $L^5 = 1$ ),
- lead time from facility  $n+1$  to facility  $n$ ,  $n = 1, \dots, 4$  is 1 week, ( $L^n = 1$ )
- unit holding costs are:  $c_h^1 = 1.5$ ,  $c_h^2 = 1$ ,  $c_h^3 = 0.5$ ,  $c_h^4 = 0.3$ ,  $c_h^5 = 0.1$ ,
- unit shortage costs are:  $c_s^1 = 6.5$ ,  $c_s^2 = 4.3$ ,  $c_s^3 = 2.15$ ,  $c_s^4 = 1.29$ ,  $c_s^5 = 0.43$ .

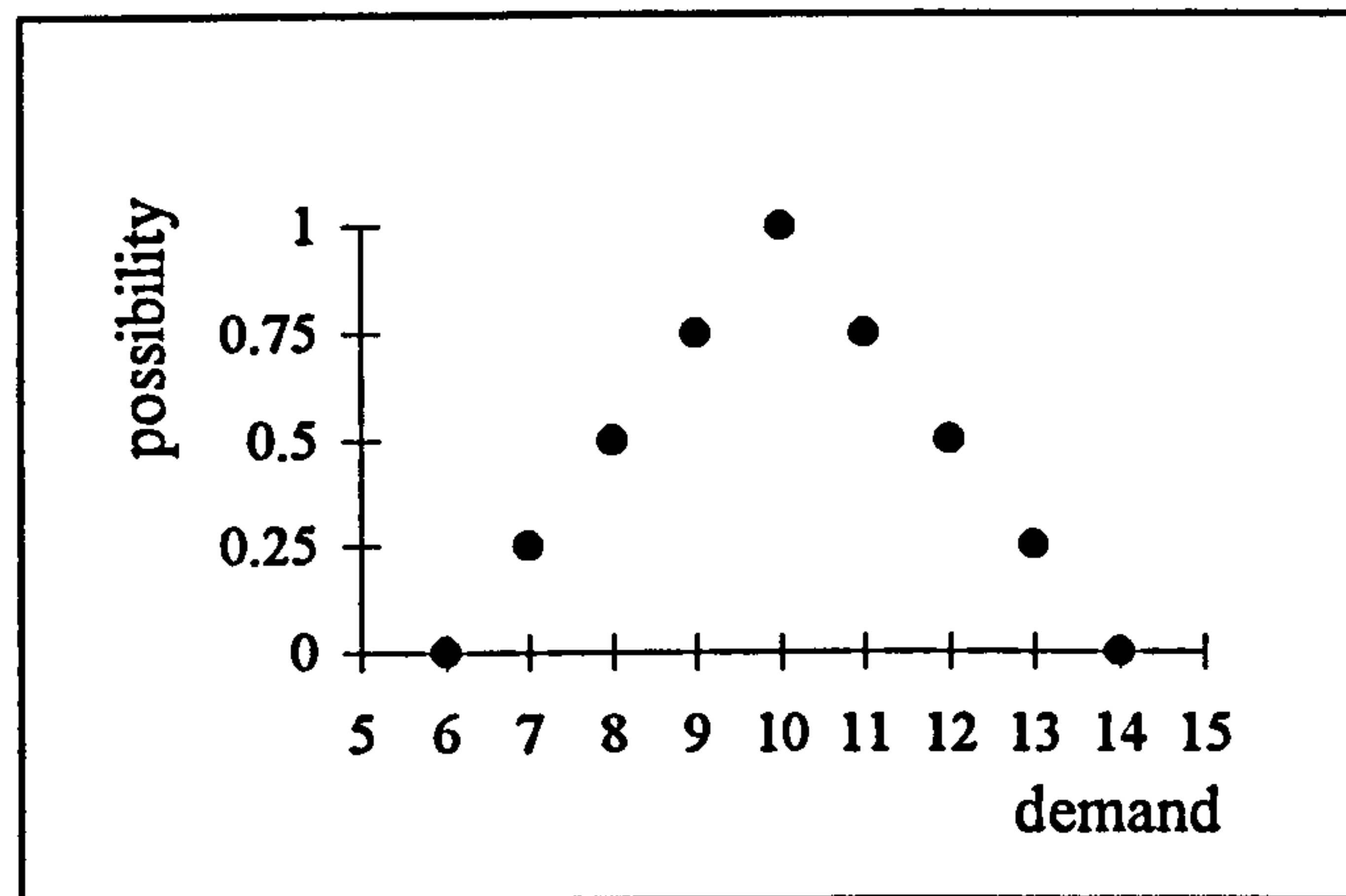


Figure 5.6. Fuzzy customer demand on SC 1

Decentralised control of SC 1 is performed in the following way. First, the order-up-to levels over the whole time horizon are calculated for the end-product inventory ( $n = 1$ ), using Fuzzy model 1. Due to stationary customer demand, order-up-to levels  $S_{d,k^1}^1$ , are the same for all review periods  $k^1 = 1, \dots, 13$ , in the 52 weeks, and therefore the subscript  $k^1$  for the review periods can be omitted.

Fuzzy customer demand  $DR^1$  on the end-product inventory during each replenishment period of 4 weeks is calculated using formula (5.1). It is the sum of the fuzzy numbers *about 10 + about 10 + about 10 + about 10*:

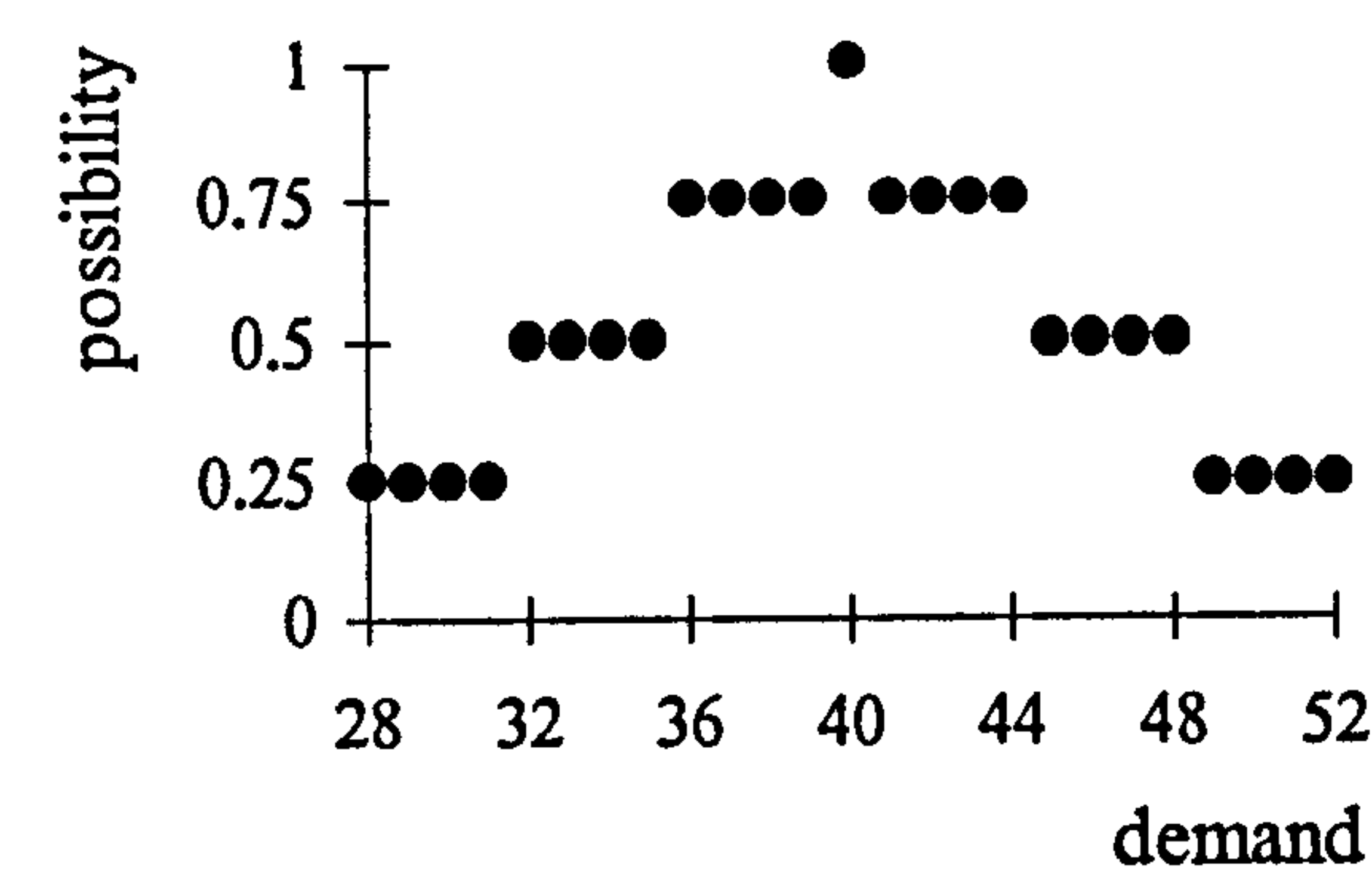
$$DR^1 = \{0/27, 0.25/28-31, 0.5/32-35, 0.75/36-39, 1/40, 0.75/41-44, 0.5/45-48, 0.25/49-52, 0/53\}.$$

The optimal order-up-to level  $S_d^1$  is searched through the set of possible values of demand during replenishment period,  $\mathcal{DR}^1 = \{28, 29, \dots, 52\}$ . For each potential order-up-to level, the corresponding possible total inventory cost is calculated. To illustrate this procedure, set the order-up-to level to, for example,  $S = 36$ , and calculate the corresponding possible total cost  $FR^1(S)$ . Fuzzy demand  $DR^1$  determines the fuzzy holding cost  $FR_h^1$  and the fuzzy shortage cost  $FR_s^1$  during a replenishment period. Each possible demand  $dr^1 \in \mathcal{DR}^1$  determines the possible holding cost  $FR_h'^1$  and the possible shortage cost  $FR_s'^1$ . For example, one of the possible customer demand  $dr^1$  is 29. It can occur when customer demand sequences in the 4 time units within the replenishment period are 7,7,7,8, or 7,7,8,7 or 7,8,7,7 or 8,7,7,7. The possibility of each of these demand sequences occurring is equal to the minimum of possibilities of customer demand being 7 or 8 end-products per week and it is 0.25. The customer demand sequence that would cause the highest holding cost is 7,7,7,8 and the holding cost charged in this case when  $S = 36$  and  $dr^1 = 29$  is:

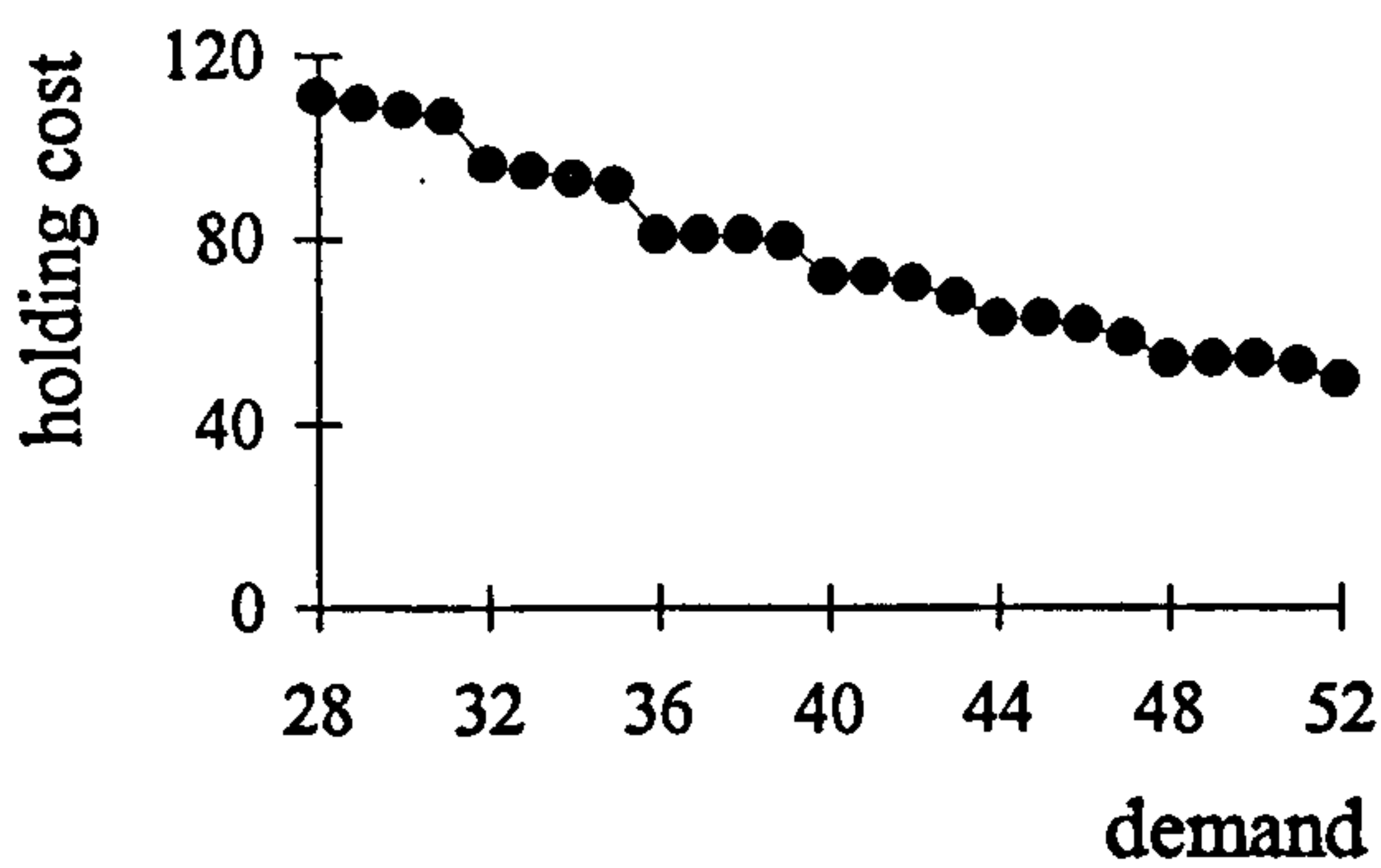
$$FR_h'^1(S, dr^1) = c_h^1 \cdot \{(S-7) + (S-7-7) + (S-7-7-7) + (S-7-7-7-8)\} = 109.5$$

The shortage cost  $FR_s'^1(S, dr^1)$ , charged when  $S = 36$  and  $dr^1 = 29$  is 0. The possibility associated with both costs  $FR_h'^1(S, dr^1)$  and  $FR_s'^1(S, dr^1)$  is 0.25.

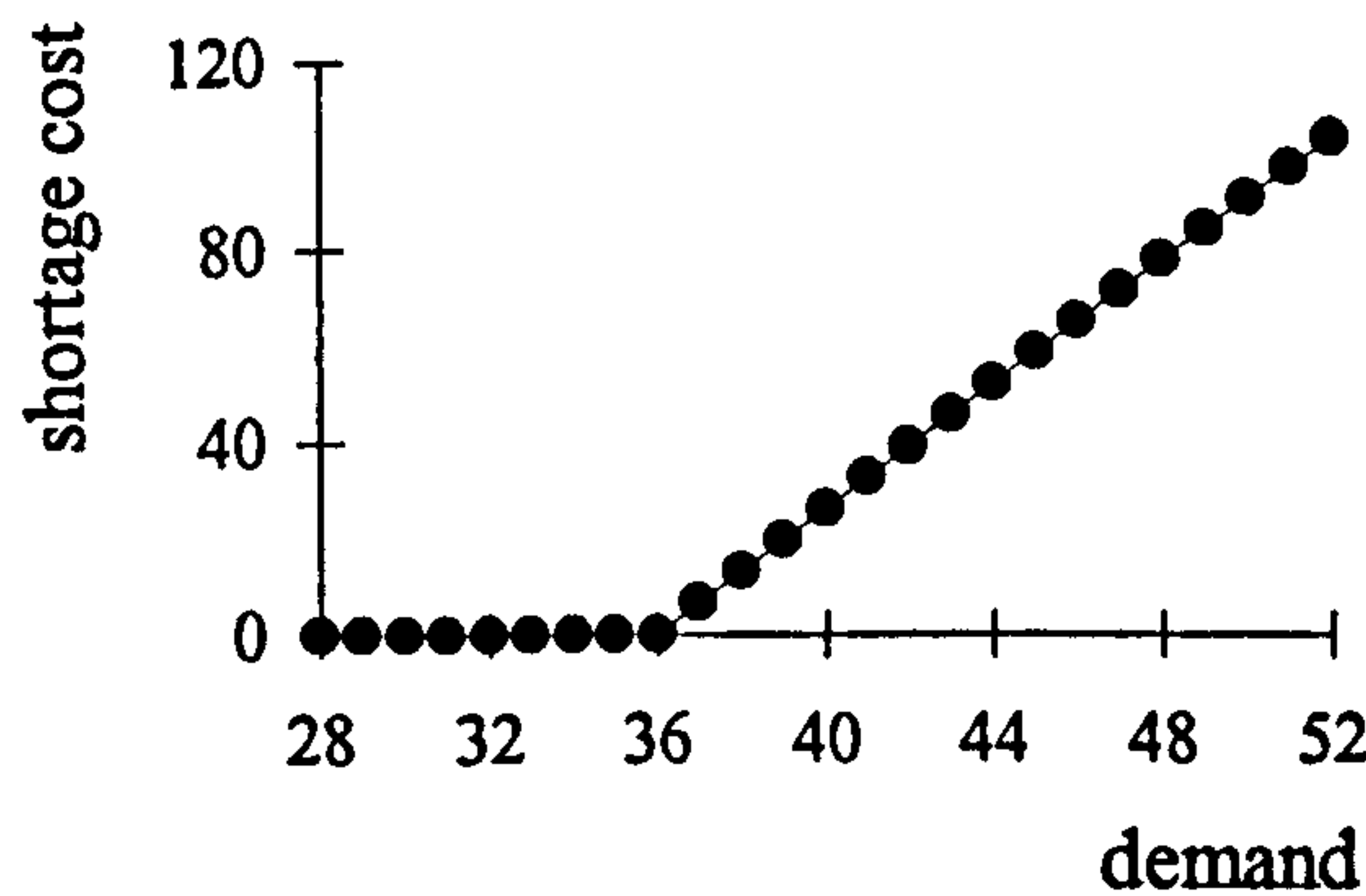
Similarly,  $FR_h'^1(S, dr^1)$  and  $FR_s'^1(S, dr^1)$  for fixed order-up-to level  $S = 36$  are calculated for all other possible demands  $dr^1 \in \mathcal{DR}^1$ . The possibility distribution of fuzzy customer demand  $DR^1$  during the replenishment period, the fuzzy holding cost  $FR_h^1$  and the fuzzy shortage cost  $FR_s^1$ , incurred for all possible demands  $dr^1 \in \mathcal{DR}^1$ , when  $S = 36$ , are shown in Figure 5.7.



(a)



(b)



(c)

Figure 5.7. (a) Fuzzy customer demand during the replenishment period,  
(b) Possible holding cost of the end-product inventory  
when the order-up-to level is 36,  
(c) Possible shortage costs of the end-product inventory  
when the order-up-to level is 36



Finally, the possible total cost  $FR^1(S)$ , when  $S = 36$  is:

$$FR^1(S) = defuzz (FR_h^1(S)) + defuzz (FR_s^1(S)) = 107.59.$$

The possible total costs  $FR^1(S)$  incurred during a replenishment period at the end-product inventory for different order-up-to levels  $S \in \mathcal{SR}^1$  are illustrated in Figure 5.8. Accordingly, the order-up-to level  $S_d^1$  that causes the minimum possible total cost is 36.

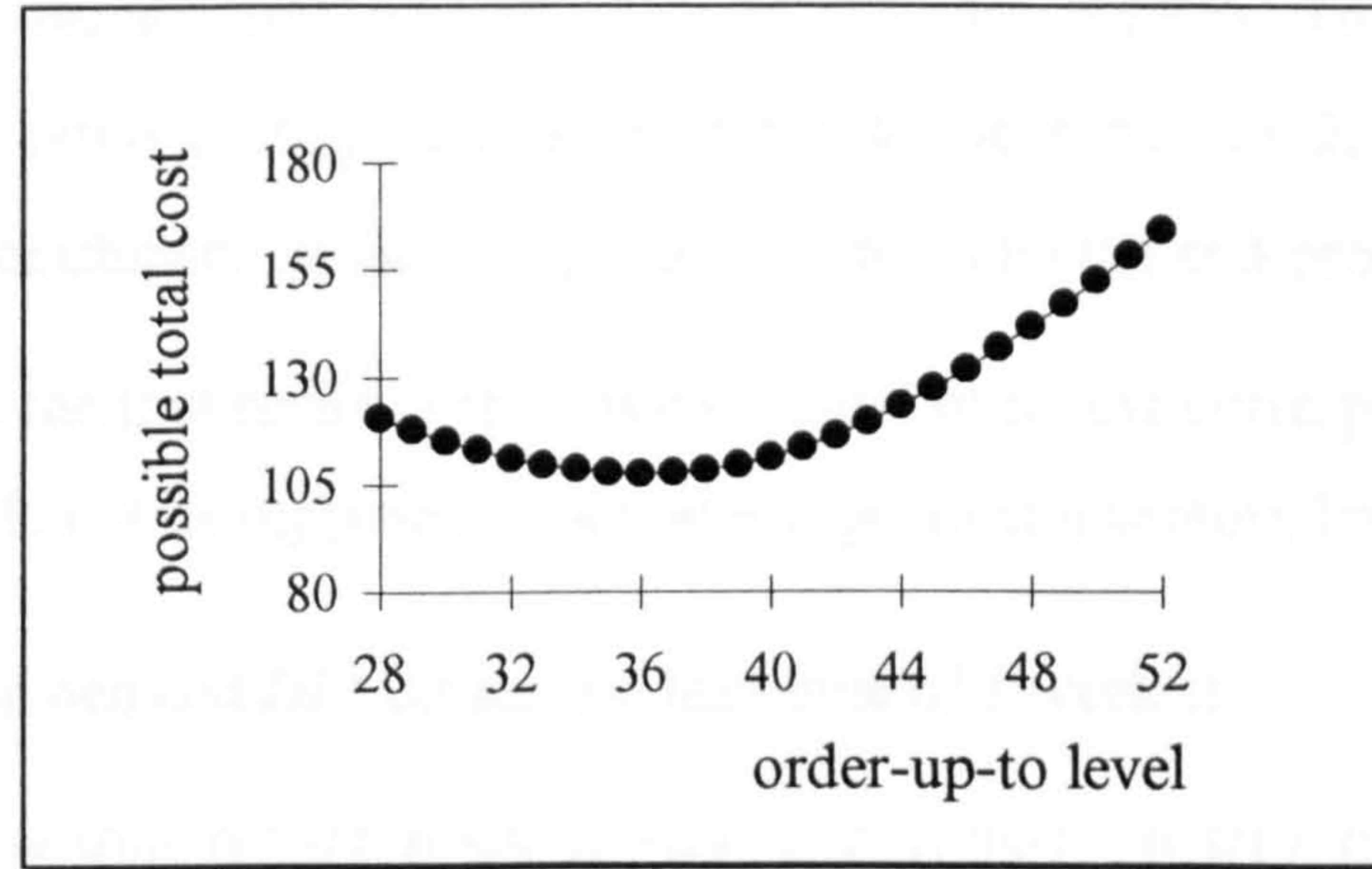


Figure 5.8. Possible total cost of the end-product inventory for different order-up-to levels

In order to use Fuzzy model 1 and the corresponding algorithm to determine the order-up-to level  $S_d^n$  of inventory  $n$ ,  $n = 2, \dots, 5$ , internal demand  $D_t^n$  imposed from facility  $n-1$  on facility  $n$  in time units  $t$ ,  $t = 1, \dots, 53$ , has to be determined. Note that stationary customer demand provides stationarity of internal demand, during the replenishment periods. Consequently, the order-up-to levels of inventory  $n$ ,  $n = 2, \dots, 5$  are the same for all the review periods.

Internal demand  $D_t^n$  is:

$$D_t^n = \text{about } 10 + \text{about } 10 + \text{about } 10 + \text{about } 10 =$$

$$\{0/27, 0.25/28-31, 0.5/32-35, 0.75/36-39, 1/40, 0.75/41-44, 0.5/45-48, \\ 0.25/49-52, 0/53\},$$

when the beginning of time unit  $t$  coincides with the beginning of a review period, i.e.,  $t = 1, 5, \dots, 49$ ,

$$D_t^n = 0, \text{ otherwise.}$$

The algorithm for determining the order-up-to level  $S_d^n$  is then applied for each inventory  $n$ ,  $n = 2, \dots, 5$ , independently. The optimal order-up-to levels selected in a decentralised manner are:  $S_d^2=36$ ,  $S_d^3=36$ ,  $S_d^4=36$ ,  $S_d^5=36$ . They are the same because the cost ratios  $c_s^n / c_h^n$ , are the same for all inventories  $n = 2, \dots, 5$ .

The replenishment quantity  $O_{d,1}^1$  to be ordered by the end-product inventory at the beginning of the first review period is calculated using the corresponding algorithm in Fuzzy model 1. Let us suppose that initial end-product inventory level is  $I_0^1 = 10$ .

Uncertain demand  $DL^1$  during the lead time of 1 week is:

$$DL^1 = \text{about } 10 = \{0/6, 0.25/7, 0.5/8, 0.75/9, 1/10, 0.75/11, 0.5/12, 0.25/13, 0/14\}.$$

Hence, the most possible end-product inventory level before replenishment is 0, and the quantity  $O_{d,1}^1 = 36$  will most possibly bring the inventory level after replenishment to the optimal level  $S_d^1 = 36$ .

The replenishment quantity  $O_{d,1}^n$  to be ordered by inventory  $n$ ,  $n = 2, \dots, 5$  is calculated in a very similar way.

The replenishment quantities to be ordered at the beginning of all the remaining review periods within the time horizon are calculated one at a time, as the inventory levels along SC 1 become precisely known.



### 5.3.2. Example 2: Decentralised control with fuzzy customer demand and fuzzy lead times to facilities

Let us consider the same chain, named *SC 1*, as in the previous example. Let us assume that *SC 1* operates in an environment in which in addition to uncertainty in customer demand, there is uncertainty in lead times to the facilities in the chain. Customer demand is described in the same way as in the previous example, i.e., it is judged to be “*about 10 end-products per week*”. Let us at the moment focus on the lead time to the end-product inventory ( $n=1$ ). Assume that the lead time is judged to be 1 or 2 weeks with possibilities 1 and 0.5, respectively. Accordingly, the lead time is modelled by a discrete fuzzy set  $L^1 = \{1/1, 0.5/2\}$ . Note that the very simple description of the lead time is chosen in this example just to reduce the required calculus. Of course, in the SC fuzzy models developed, lead times can be given by more complex forms.

Decentralised control is performed in *SC 1*, starting from the end-product inventory. The order-up-to levels for the end-product inventory over the specified time horizon of 52 weeks are calculated using Fuzzy model 2 as follows. Due to stationary character of customer demand, order-up-to levels  $S_{l,k^n}^n$ , are the same for all review periods  $k^1 = 1, \dots, 13$ , in the 52 weeks, and therefore the subscript  $k^1$  denoting the review periods can be omitted.

Let us recall that the review periods of the end-product inventory are 4 weeks. Due to the fuzzy lead time, the length  $P^1$  of a replenishment period of the end-product inventory is not necessarily 4 weeks. It also becomes fuzzy. The possibility distribution of the length of a replenishment period is calculated using formula (5.21). The calculation involves the length of the lead time prior to replenishment, length of the subsequent lead time and their possibilities, as given in Table 5.1.



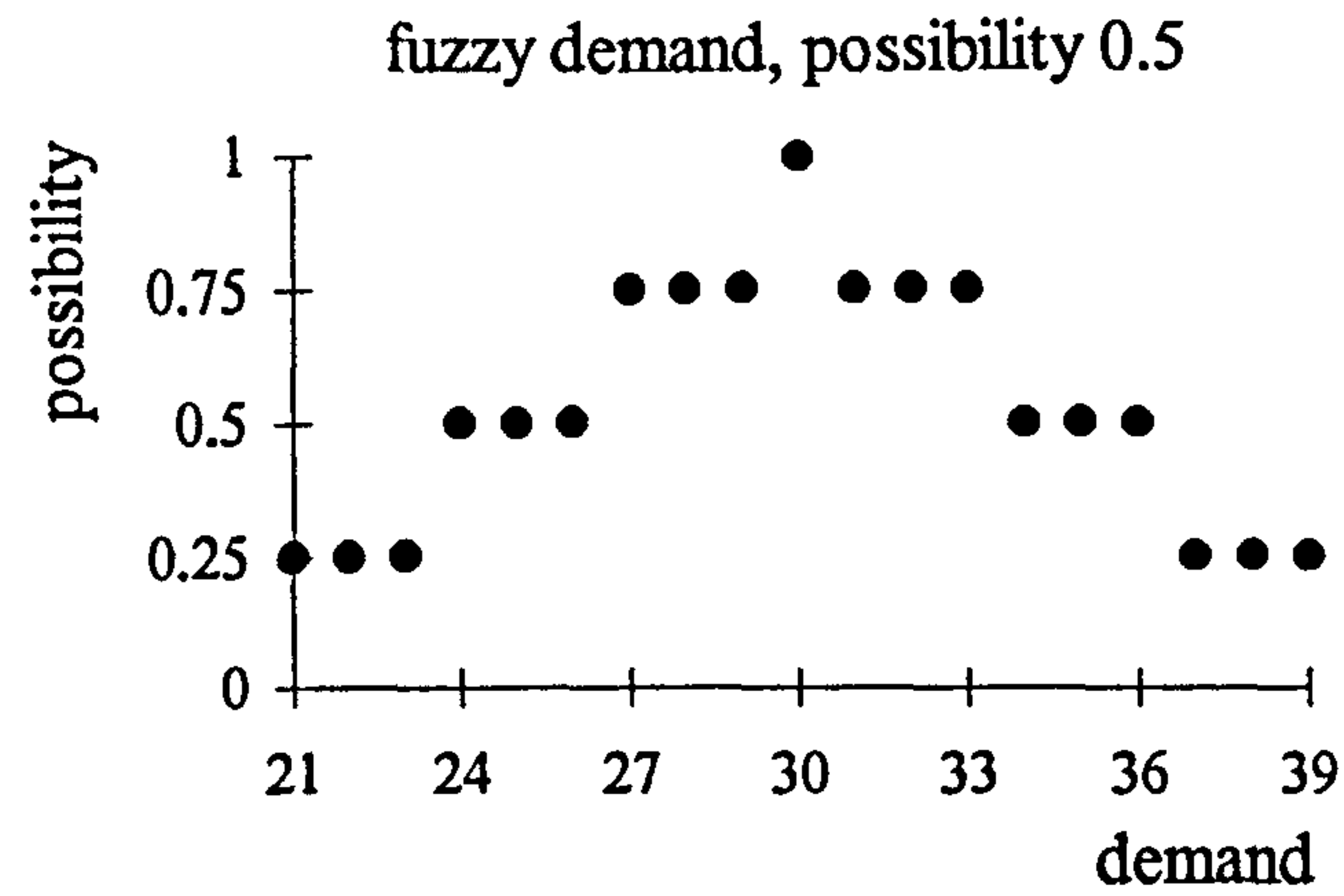
Table 5.1. The possibility distribution of the length of a replenishment period

Length of the lead time prior to repl.		Length of the subsequent lead time		Length of the repl. period	Possibility
1	1	1	1	4	1
1	1	2	0.5	5	0.5
2	0.5	1	1	3	0.5
2	0.5	2	0.5	4	0.5

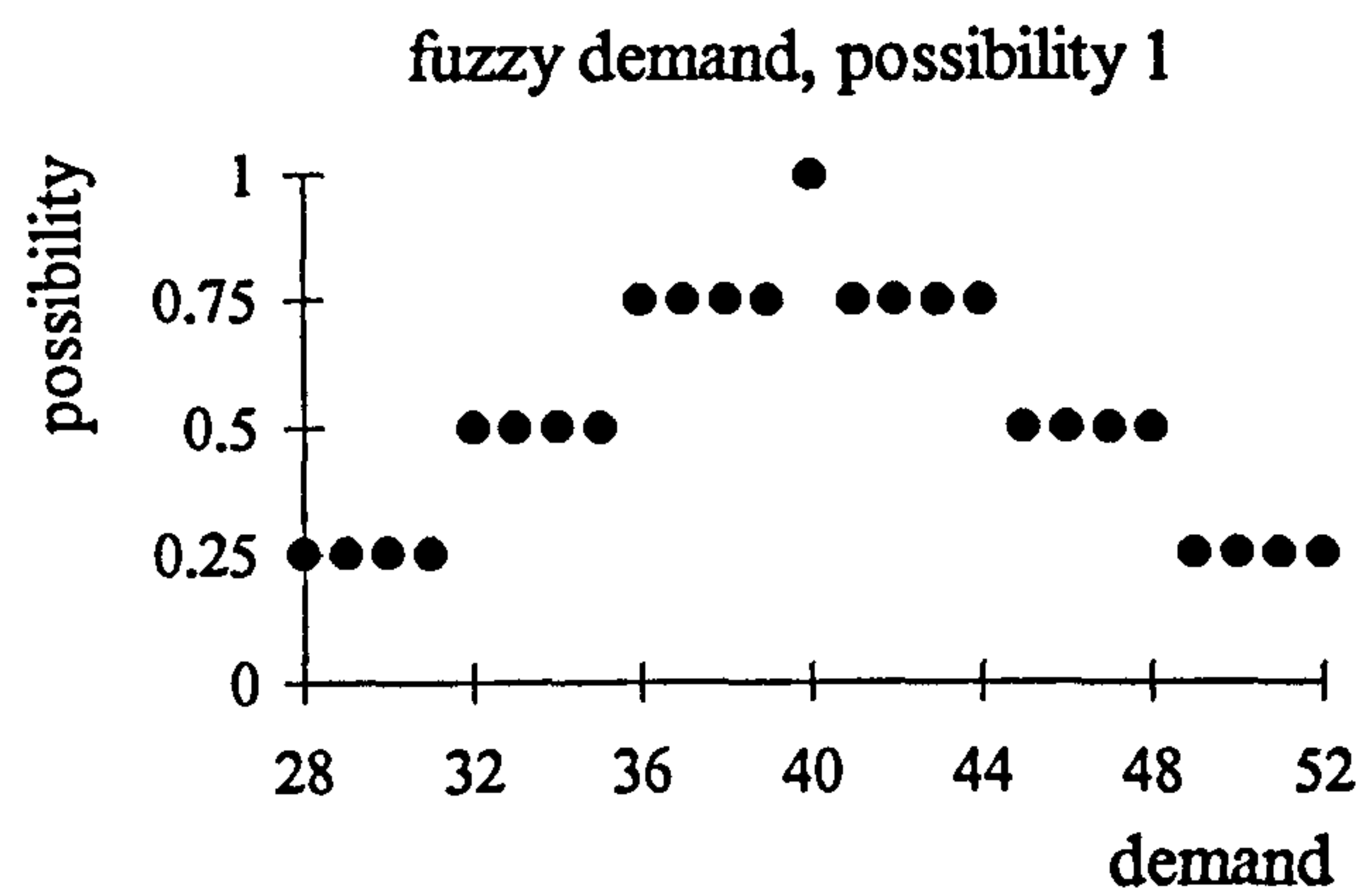
Therefore, the length of a replenishment period is a fuzzy number  $P^1 = \{0.5/3, 1/4, 0.5/5\}$ . Demand during a replenishment period  $DR^1$  takes fuzzy values with associated possibilities, as follows:

- $\{0.25/21-23, 0.5/24-26, 0.75/27-29, 1/30, 0.75/31-33, 0.5/34-36, 0.25/37-39\}$ , with possibility 0.5, derived as the sum of the fuzzy numbers *about 10 + about 10 + about 10* end-products (in the case when the replenishment period is 3 weeks),
- $\{0.25/28-31, 0.5/32-35, 0.75/36-39, 1/40, 0.75/41-44, 0.5/45-48, 0.25/49-52\}$ , with possibility 1, derived as the sum of the fuzzy numbers *about 10 + about 10 + about 10 + about 10* end-products (in the case when the replenishment period is 4 weeks),
- $\{0.25/35-39, 0.5/40-44, 0.75/45-49, 1/50, 0.75/51-55, 0.5/56-60, 0.25/61-65\}$ , with possibility 0.5, derived as the sum of the fuzzy numbers *about 10 + about 10 + about 10 + about 10 + about 10* end-products (in the case when the replenishment period is 5 weeks).

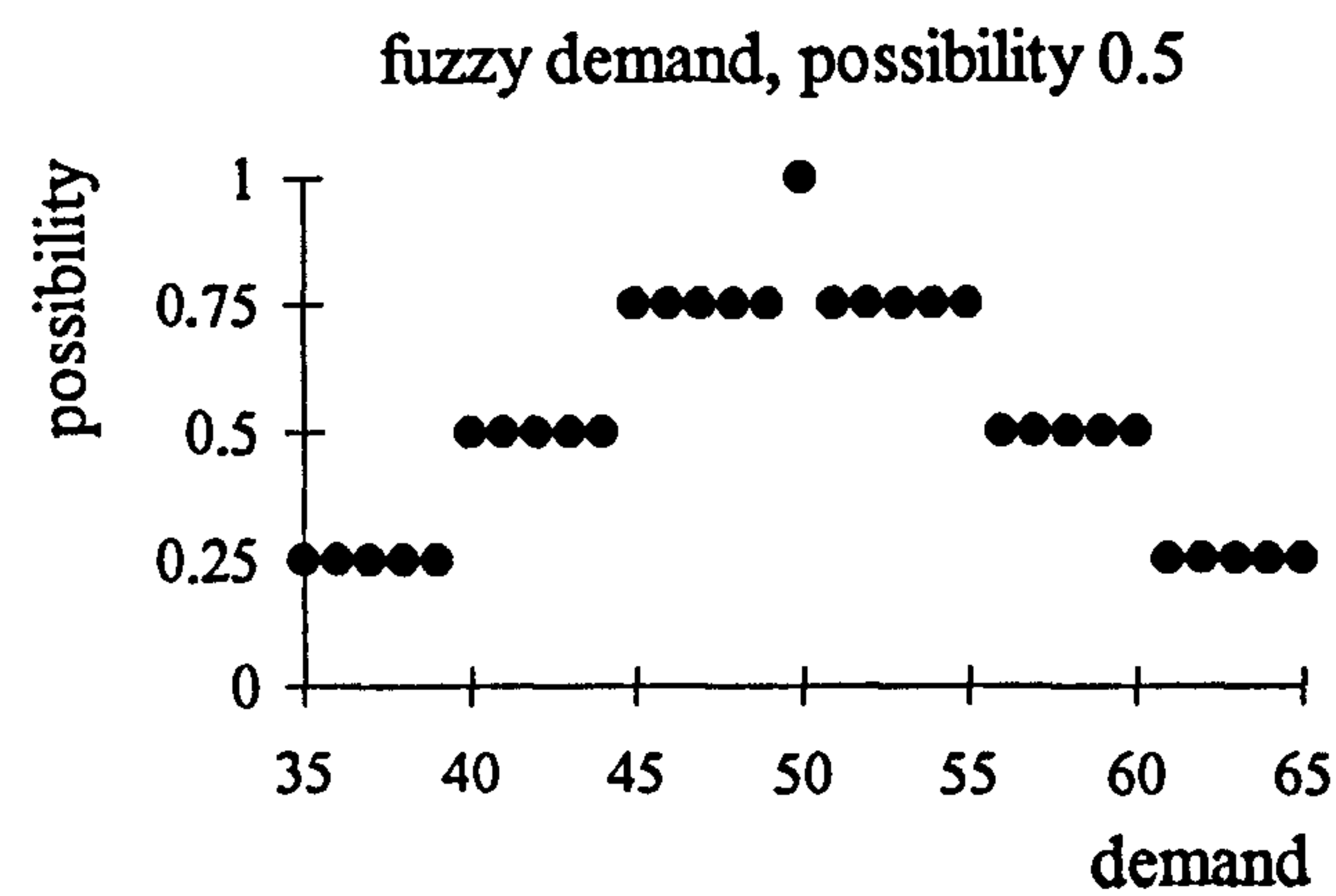
The three fuzzy values of demand  $DR^1$  during the replenishment period and associated possibilities are illustrated in Figure 5.9.



(a)



(b)



(c)

Figure 5.9. The fuzzy values of demand during the replenishment period and the associated possibilities

This implies that  $DR^1$  is represented as a level 2 fuzzy set. The level 2 fuzzy set  $DR^1$  is reduced to an ordinary fuzzy set  $s\text{-fuzzif}(DR^1)$  through s-fuzzification which is given by formula (5.23). By applying this formula  $s\text{-fuzzif}(DR^1)$  becomes:

$$s\text{-fuzzif}(DR^1) = \{0.125/21-23, 0.25/24-26, 0.375/27-29, 0.5/30, 0.375/31, 0.5/32-35, 0.75/36-39, 1/40, 0.75/41-44, 0.5/45-48, 0.375/49, 0.5/50, 0.375/51-55, 0.25/56-60, 0.125/61-65\}.$$

Once demand during replenishment period is represented by an ordinary fuzzy set  $s\text{-fuzzif}(DR^1)$ , the algorithm for determining the optimal order-up-to level  $S_I^1$  continues in the same manner, as demonstrated in Section 5.3.1. This means that the optimal order-up-to level  $S_I^1$  is searched through the set of possible values of demand during the replenishment period,  $\mathcal{DR}^1 = \{21, 29, \dots, 65\}$ . Let us notice that due to the fuzzy lead time and fuzzy length of the replenishment period, demand during the replenishment period have wider range of possible values in comparison with the case where the lead time had a crisp value (see previous example).

The algorithm for determining order-up-to levels is applied for all inventories  $n = 2, \dots, 5$  in *SC 1*, independently. Let us emphasise that lead times to different inventories can be described by different fuzzy sets.

The algorithm for determining replenishment quantities in the presence of uncertainty in lead times is demonstrated using again, as an example, the end-product inventory. Let us suppose that initial stock of end-product inventory is  $I_0^1 = 10$ .

The first step is to calculate uncertain demand  $DL$  during the fuzzy lead time to the end-product inventory. Demand  $DL$  is represented by a level 2 fuzzy set, calculated using formula (5.24). It takes the following fuzzy vales:

- $\{0.25/7, 0.5/8, 0.75/9, 1/10, 0.75/11, 0.5/12, 0.25/13\}$ , with possibility 1, i.e., it is *about* 10 end-products (in the case when the lead time is 1 week),



- $\{0.25/14-15, 0.5/16-17, 0.75/18-19, 1/20, 0.75/21-22, 0.5/23-24, 0.25/25-26\}$ , with possibility 0.5, derived as the fuzzy sum *about* 10 + *about* 10 end-products (in the case when the lead time is 2 weeks).

By applying s-fuzzification of the level 2 fuzzy set  $DL$ , an ordinary fuzzy set  $s\text{-fuzzif}(DL)$  is obtained which represents fuzzy demand during the fuzzy lead time:

$$s\text{-fuzzif}(DL) = \{0.25/7, 0.5/8, 0.75/9, 1/10, 0.75/11, 0.5/12, 0.25/13, 0.125/14-15, 0.25/16-17, 0.375/18-19, 0.5/20, 0.375/21-22, 0.25/23-24, 0.125/25-26\}.$$

The algorithm for determining replenishment quantities now continues in the same manner as it was explained in the previous example.

### 5.3.3. Example 3: Partially coordinated control with fuzzy customer demand and fuzzy supply along a supply chain

This example demonstrates partially coordinated control strategy applied in chain  $SC\ 1$  with characteristics given in Section 5.3.1. The locally optimal order-up-to levels are determined initially in a decentralised manner as described in Section 5.3.1.:  $S_d^1=36, S_d^2=36, S_d^3=36, S_d^4=36, S_d^5=36$ , and then modified in the following way.

At first, the in-process inventory ( $n = 4$ ) is partially coordinated with the raw material inventory ( $n = 5$ ). Since demand  $DR^5$  from inventory 4 on inventory 5 during any replenishment period is fuzzy, replenishment supply from inventory 5 to inventory 4 is also fuzzy. It is approximated based on the order-up-to level  $S_d^5$  of inventory 5 and demand  $DR^5$  on inventory 5. A fuzzy set  $SQ^4$  that represents undelivered quantity of items from inventory 5 to inventory 4 is calculated as follows:

$$SQ^4 = \max(S_d^5 - DR^5, 0) = \{0.75/0-3, 1/4, 0.75/5-8, 0.5/9-12, 0.25/13-16, 0/17\}$$

where  $S_d^5=36$ ,  $DR^5 = \{0/27, 0.25/28-31, 0.5/32-35, 0.75/36-39, 1/40, 0.75/41-44, 0.5/45-48, 0.25/49-52, 0/53\}$  and the rule for fuzzy arithmetic subtraction, given by formula (3.25) is used.

Taking into account the fuzzy undelivered quantity  $SQ^4$ , the optimal order-up-to level  $S_p^4$  is determined using the algorithm in Fuzzy model 2. The possible total cost  $FRS^4(S)$  incurred at inventory 4 during a replenishment period is calculated iteratively, starting from  $S = 36$  and increasing  $S$  by one in each iteration. Table 5.2. shows the following results obtained for  $S = 36$ : (1) the fuzzy undelivered replenishment quantity  $SQ^4$ , including the quantities  $s_{u4}$  that can be undelivered from inventory 5 to inventory 4, and the associated possibilities  $\mu_{SQ^4}(s_{u4})$ , (2) the stock level  $(S-s_{u4})^+$  reached at inventory 4 after the replenishment for each possible  $s_{u4}$ , and (3) the possible costs  $FR^4((S-s_{u4})^+)$  incurred with the inventory levels  $(S-s_{u4})^+$ . The possibility associated with each cost  $FR^4((S-s_{u4})^+)$  is equal to  $\mu_{SQ^4}(s_{u4})$ .

The possible total cost  $FRS^4(S)$ , incurred when the order-up-to level  $S = 36$ , is obtained using the moment rule as the defuzzification method. Then, the order-up-to level  $S$  is increased by one unit, leading to the lower possible total cost. The computation is repeated iteratively, until  $S = 42$  is reached, which generates the higher possible total cost in comparison with the cost incurred when  $S = 41$ . Consequently, the optimal order-up-to level is set to  $S_p^4 = 41$  which minimises  $FRS^4(S)$ . The possible total costs for different  $S$  are illustrated in Figure 5.10.

Once, the order-up-to level  $S_p^4$  is determined, inventory 3 is partially coordinated with inventory 4. The order-up-to level  $S_p^3$  of inventory 3 is calculated, taking into account the new order-up-to level  $S_p^4 = 41$  of inventory 4, and so on, to the end-product inventory. Partial coordination of the inventories along SC 1 gives the following order-up-to levels:  $S_p^5 = 36$ ,  $S_p^4 = 41$ ,  $S_p^3 = 39$ ,  $S_p^2 = 40$ ,  $S_p^1 = 40$ .

Table 5.2. Possible costs incurred at inventory 4,  
with fuzzy replenishment supply

not delivered $s_u^4$	possibility $\mu_{SQ^4}(s_u^4)$	inventory level $(S - s_u^4)^+$	cost $FR^4((S - s_u^4)^+)$
0	0.75	36	38.97
1	0.75	35	38.99
2	0.75	34	39.07
3	0.75	33	39.22
4	1	32	39.43
5	0.75	31	39.69
6	0.75	30	39.99
7	0.75	29	40.32
8	0.75	28	40.68
9	0.5	27	41.07
10	0.5	26	41.46
11	0.5	25	41.85
12	0.5	24	42.24
13	0.25	23	42.63
14	0.25	22	43.02
15	0.25	21	43.41
16	0.25	20	43.80

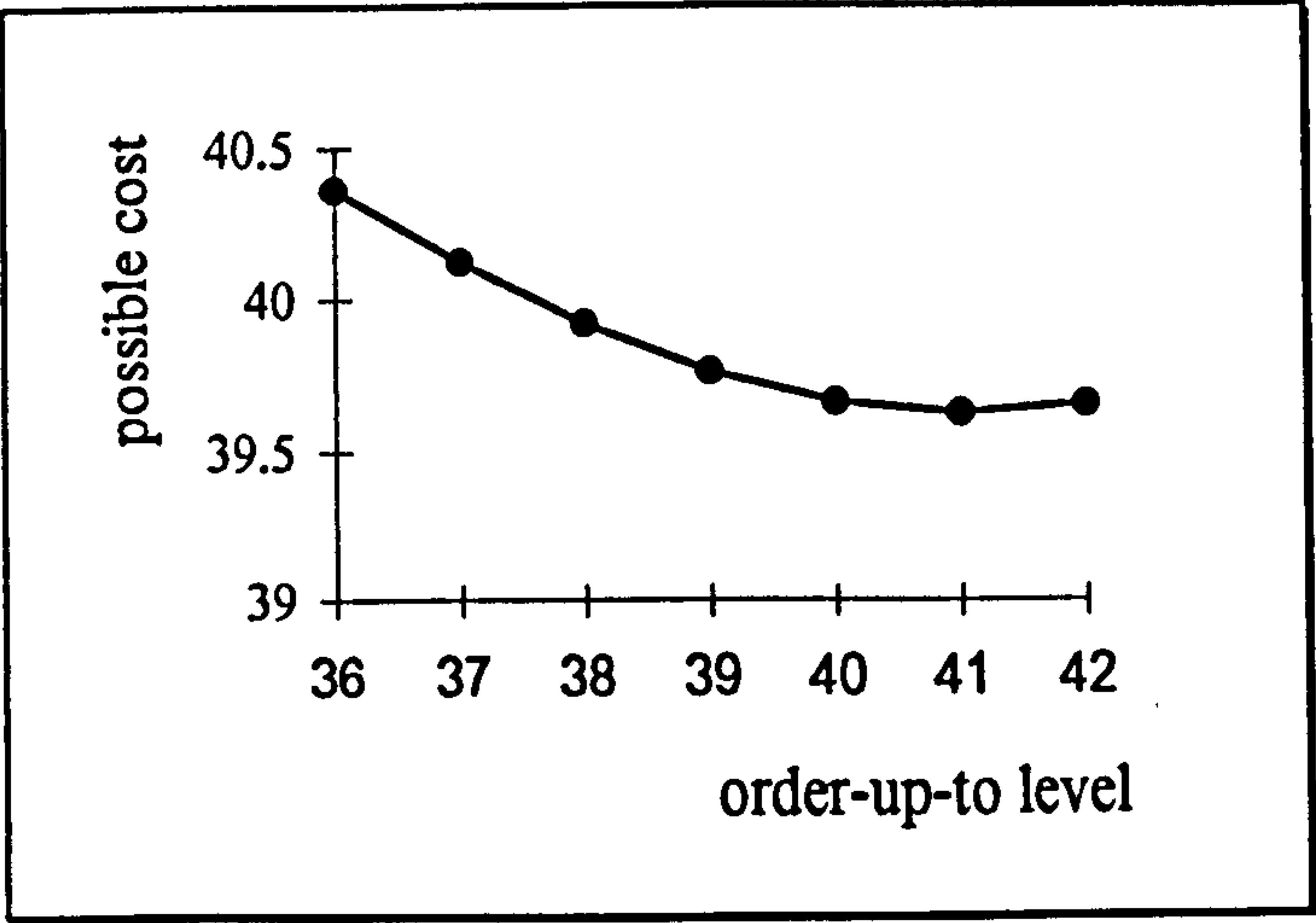


Figure 5.10. The possible total cost for different order-up-to levels



#### **5.3.4. Example 4: Partially coordinated control with fuzzy customer demand, fuzzy supply along a supply chain, and fuzzy lead times to the facilities**

This example demonstrates the calculation of inventory order-up-to levels taking into consideration all three sources of uncertainty: customer demand, supply from one to the succeeding facility and lead times to the facilities. The same chain, denoted by *SC 1*, is considered as in the previous examples. Let us assume that fuzzy customer demand and lead times to the facilities are described in the same way as in Example 2 in Section 5.3.2.

The calculation of order-up-to levels for all the inventories in *SC 1*, in this case when all the three sources of uncertainty are taken into account, involves two steps. In the first step, the order-up-to levels for all the inventories are determined in a decentralised manner, considering fuzzy customer demand and fuzzy lead times, as explained in Section 5.3.2. In the second step, the order-up-to levels are modified by applying partially coordinated control strategy, which takes into account uncertainty in supply along *SC 1*. The coordination is performed in the same way as explained in Section 5.3.3., starting from the raw material inventory ( $n=5$ ) and its successor ( $n=4$ ), towards the end-product inventory ( $n=1$ ) that is coordinated with the preceding inventory ( $n=2$ ).

## **5.4. Summary**

New original SC fuzzy analytical models have been developed, which treat different sources of uncertainty, in particular customer and internal demands, supply from one to the succeeding facility in the chain and lead times to the chain facilities. In contrast to the conventional SC models where uncertainties in SCs are assumed to be random

processes, the SC fuzzy models developed in this research consider different types of uncertainty as imprecision in judgement, lack of evidence or lack of certainty in evidence.

Fuzzy sets have been used to describe subjective judgements in customer demand and lead times. These judgements are usually linguistically expressed by various phrases, such as "demand in a given time unit is *about*  $d$ ", "demand in a given time unit is *more or less* in the interval  $[d_l, d_u]$ ", "lead time is *about*  $l$  weeks". Uncertainty in supply from one to the succeeding facility, caused by insufficient stock levels along the chain, has been modelled by fuzzy sets, too. An important characteristic of the SC fuzzy models developed is that they include all the different sources of uncertainty simultaneously, including uncertainty in customer and internal demands, supply along the SC and lead times to the facilities in the chain.

In these SC fuzzy models, inventory control has been considered as a discrete multistage decision process in a fuzzy environment. Algorithms for determining order-up-to levels which minimise the total possible inventory cost within a given time horizon have been developed.

## CHAPTER 6

### SUPPLY CHAIN SIMULATION MODELLING

In this chapter a new family of special purpose simulators, *SCSIM*, developed and implemented by the author, is described. *SCSIM* is based on dynamic simulation models that emulate operations of a serial SC during a finite time horizon (Petrovic, Petrovic, 1996). The assumptions about SC processes given in Section 5.1. are taken into consideration. *SCSIM* determines SC performance in an uncertain environment over time, including SC fill rate and total SC cost. Simulation is chosen because these SC performance measures cannot be obtained analytically due to the presence of various sources of uncertainty and the complexity of the relations describing SC processes.

A new approach to simulating an SC in a fuzzy environment was built into *SCSIM*. The fuzzy environment is characterised by fuzzy customer demand, fuzzy external supplier reliability, and fuzzy lead times to the SC facilities. The commercially available simulation packages, including both general and special purposed SC simulators, do not operate with fuzzy data, and, therefore, it was necessary to develop and implement a new software tool.

*SCSIM* has three components: (1) *SCBL* which simulates SC operations when customer demand is fuzzy, (2) *SCSUPP* which simulates SC operations when both customer demand and external supplier reliability are fuzzy, and (3) *SCLT* which simulates SC operations when both customer demand and lead times to the facilities in an SC are fuzzy.

The main characteristics and the general structure of *SCSIM*, are presented in Section 6.1. Detailed description of its components is given in Section 6.2., and its



implementation is described in Section 6.3. The potential of applying *SCSIM* as a tool for studying serial SCs is summarised in Section 6.4.

### 6.1. *SCSIM* - a family of *SC SIM*ulators

*SCSIM* is a family of new original SC simulators developed with the following objectives: (1) to obtain SC performance over time, such as SC fill rate and SC total cost, that cannot be obtained analytically, and (2) to examine SC dynamic behaviour in an uncertain environment.

*SCSIM* includes dynamic simulation models that emulate operations of a serial SC during a finite time horizon. It is able to simulate SCs with any number of facilities in series. SC simulation is performed through discrete time instances.

*SCSIM* encompasses the SC fuzzy analytical models, described in Chapter 5, in such a way that order-up-to levels recommended by the fuzzy models are input to the SC simulation models. If the SC performance achieved with the order-up-to levels recommended by the fuzzy models are not satisfactory, the order-up-to levels can be modified in certain ways. This particular aspect of using *SCSIM* is discussed in Chapter 8.

*SCSIM* can simulate uncertainties in customer demand, external supplier reliability and lead times to the facilities. These uncertain data are described by discrete fuzzy sets with any form of membership functions, i.e., possibility distributions. In order to simulate a fuzzy datum, i.e., to generate a prototype of the datum represented by the fuzzy set, the possibility distribution induced by the fuzzy set is transformed into the corresponding probability distribution. This transformation is based on the formula (3.38), given in Section 3.2. This means that simulation of a fuzzy datum, i.e., numerical realisations of the fuzzy datum which is described by a vague linguistic term is performed in *SCSIM* by generating numbers which follow the derived probability distribution.

*SCSIM* is used in this research in the following tasks:

- to determine the order-up-to levels of inventories along a serial SC in an uncertain environment based on the fuzzy SC models, to calculate replenishment quantities to be ordered by the inventories dynamically over time and to assess the SC performance achieved,
- to examine the influence of an uncertain environment on the SC performance and to evaluate the impact of actual or potential changes in the uncertainty in the environment,
- to investigate the impact of different SC control strategies, such as decentralised control and partially coordinated control, on SC performance,
- to enhance making decisions on order-up-to levels of inventories in such a way as to compensate for uncertainty in external supply,
- to analyse sensitivity of SC control decisions to different approximations of uncertainty in SC parameters.

The application of *SCSIM* in various analyses is illustrated in Chapter 7 and Chapter 8.

*SCSIM* measures the following SC performance:

- the total SC cost per end-product demanded;
- the SC holding cost per end-product demanded, and
- the SC fill rate, that is in fact the end-product inventory fill rate.

*SCSIM* also measures performance of each individual inventory in an SC:

- the total inventory cost per item demanded,
- the holding cost per item demanded,
- the inventory fill rate, and
- the total replenishment quantity ordered by the inventory.

The general *SCSIM* structure and the link between the SC simulators and the SC fuzzy models are depicted in Figure 6.1. In *SCSIM*, two types of SC models are combined:

1. fuzzy analytical and generative SC models which are used to determine order-up-to levels of inventories along an SC and to calculate replenishment quantities to be ordered by each inventory, periodically, over a finite time horizon, and
2. evaluative SC simulation models which are used to assess the effects of the selected order-up-to levels on SC performance. If the SC performance determined by simulation are not satisfactory it can be improved in different ways. Iterative procedures built into *SCSIM* change the order-up-to levels of the inventories, determined initially by the fuzzy models, until the satisfactory SC performance is obtained. This particular usage of the SC simulators will be elaborated in Chapter 8, where SC simulation is combined with new original iterative procedures developed for compensation for uncertainty in external supply.

## 6.2. *SCSIM* components

*SCSIM* has three components:

1. *SCBL* which simulates SC operations when customer demand is fuzzy,
2. *SCSUPP* which simulates SC operations when both customer demand and external supplier reliability are fuzzy.
3. *SCLT* which simulates SC operations when both customer demand and lead times to the facilities in the chain are fuzzy.



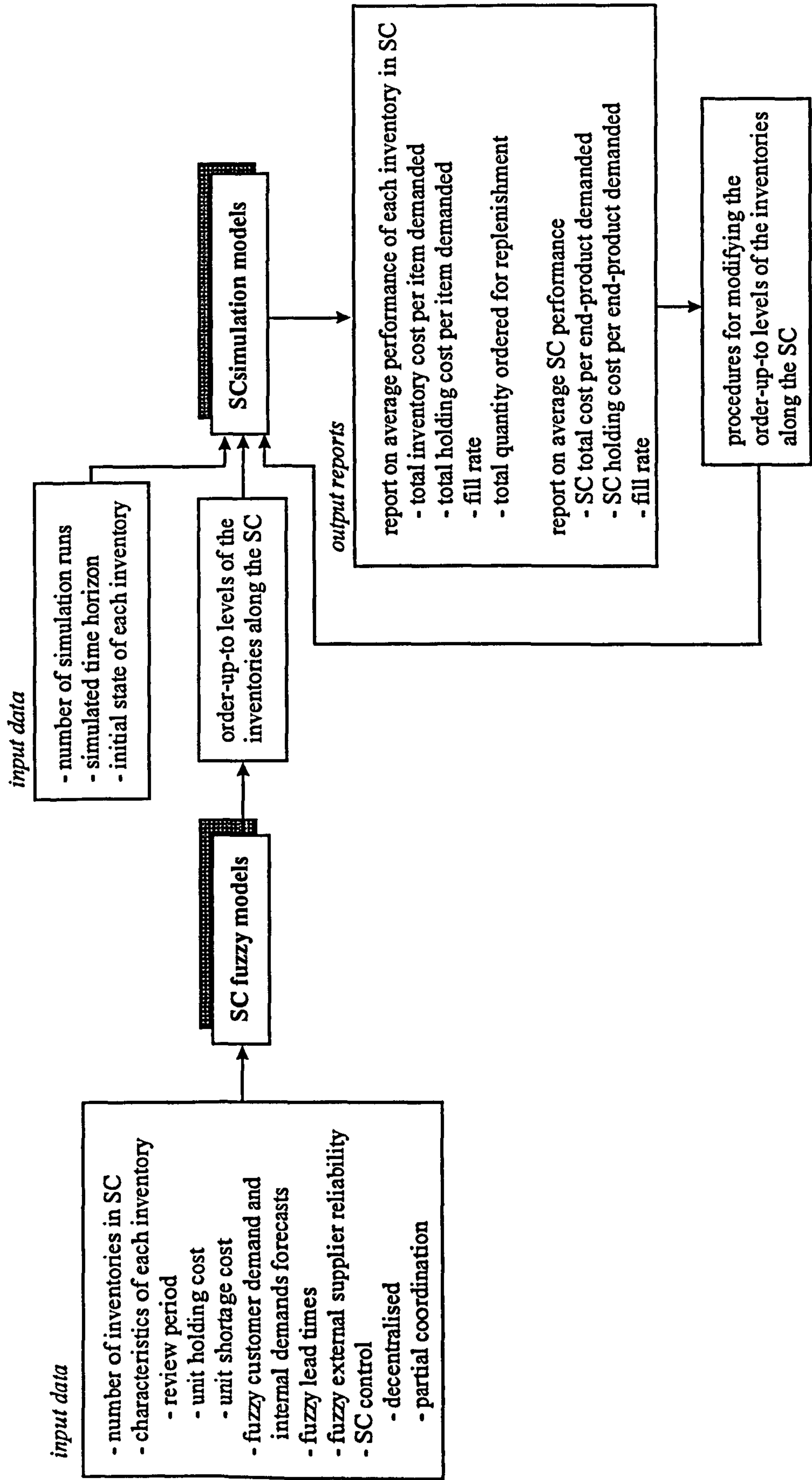


Figure 6.1. The general SCSIM structure

*SCSIM* components are presented in some details in the three sections that follow.

### 6.2.1. *SCBL*

*SCBL* treats only one source of uncertainty inherent in an SC environment, and it is customer demand. It is assumed that an external supplier is absolutely reliable and that lead times along the SC are fixed and precisely known.

Both SC control strategies, described in Section 5.2.2.2. are implemented in *SCBL*: (1) decentralised control and (2) partially coordinated control. Depending on the SC control strategy applied, the order-up-to levels for the inventories along the SC are determined based on one of the fuzzy SC models which are implemented in *SCBL*.

It is assumed that customer demand is time-stationary over review periods of the end-product inventory ( $n = 1$ ); in other words, customer demand has the same pattern during all the end-product inventory review periods. This implies time-stationarity of internal demand on each inventory  $n$ ,  $n = 2, \dots, N$  over the inventory review periods. Therefore, order-up-to levels for the inventories along the SC are calculated for the whole time horizon considered, before the simulation starts.

Simulation can be repeated a number of times in order to get an average SC performance when there is uncertainty in customer demand. In each simulation run a different seed for random number generation is used, and a different sequence of customer demand values is generated. Consequently, SC performance achieved in simulation runs are different. It should be pointed out that the sequence of uncertain customer demand is generated in such a way as to correspond to a fuzzy estimation of customer demand, as discussed in Section 6.1.

**Input data for SCBL are:**

- number of simulation runs,
- $T$  - number of time units to be simulated during each run,
- $N$  - number of inventories in a serial SC,
- name of a file that contains data about each inventory  $n$ ,  $n = 1, \dots, N$ , such as:
  - $R^n$  - number of time units within the inventory review period,
  - $c_h^n$  - unit holding cost,
  - $c_s^n$  - unit shortage cost,
  - $L^n$  - number of time units within a lead time to the inventory,
- name of a file that contains data about the initial state of inventory  $n$ ,  $n = 1, \dots, N$ , such as:
  - $I_0^n$  - initial stock level,
  - $rew_1^n$  - counter of time units within the review period,
  - $repl_1^n$  - indicator of the beginning of a replenishment period,
  - $Q_0^n$  - expected replenishment quantity, ordered before the starting simulated time unit.
- name of a file that contains fuzzy customer demand and fuzzy internal demand forecasts. The file contains fuzzy customer demand forecast for each time unit within one review period of the end-product inventory ( $n = 1$ ), and fuzzy internal demand forecast for each time unit within one review period of inventory  $n$ ,  $n = 2, \dots, N$ , calculated off-line as described in Section 5.2.2.1. In this way, customer demand and internal demands are forecast for the whole finite time horizon in advance, because it is assumed that customer demand and internal demands, consequently, are time-stationary over the review periods of the inventories.
- SC control strategy: (1) fully decentralised control or (2) partially coordinated control.

Each fuzzy datum (e.g., demand forecast) is specified by a possibility distribution, that includes a number of values that data can take and pairs of data



values and associated possibilities. The possibility distribution can have any form. However, in order to transform it into the corresponding probability distribution, it has to be normalised, i.e., at least one of the data values attains the maximum possibility 1.

**Output reports** generated by *SCBL* are presented both on screen and saved into a file.

After each simulated time interval  $t$ ,  $t = 1, \dots, T$ , a report is generated and displayed on request. It includes:

- general data, such as
  - $t$  - number of the current simulated time unit,
  - $d_t^i$  - actual customer demand realised during time unit  $t$ ,
- data about individual inventory  $n$ ,  $n = 1, \dots, N$ , such as
  - $n$  - inventory identification,
  - $rew_t^n$  - counter of the current time interval within the current inventory review period,
  - $Q_t^n$  - quantity received during time unit  $t$  from the preceding facility  $n+1$  or the external supplier, in the case  $n=N$ ,
  - $O_t^n$  - quantity ordered at the beginning of time unit  $t$ ,
  - $I_t^n$  - current stock level,
  - $F_{h,t}^n$  - cumulative holding cost,
  - $F_{s,t}^n$  - cumulative shortage cost,
  - $F_t^n$  - cumulative total cost,
  - $NS_t^n$  - total number of shortages,
  - $BR_t^n$  - part of backordered quantity still owed (i.e., not available for delivery) to succeeding facility  $n-1$ ,
  - $BS_t^n$  - part of backordered quantity ready for the next delivery to succeeding facility  $n-1$ .

A report is printed at the end of each simulation run. It contains:

- total customer demand imposed on the SC, i.e., on the end-product inventory during the simulated time horizon,
- performance of each individual inventory achieved during the simulated time horizon, such as
  - $FD_{T_h}^n$  - total cost per item demanded from inventory  $n$ ; the total inventory cost is calculated as the sum of the holding cost and the shortage cost,
  - $FHD_{T_h}^n$  - holding cost per item demanded from inventory  $n$ ,
  - $FR_{T_h}^n$  - fill rate; an inventory fill rate is calculated as the fraction of demand immediately filled from stock on hand,
  - total replenishment quantity ordered by inventory  $n$ ,
- overall SC performance, such as
  - $FD_{T_h}$  - SC total cost per end-product demanded; the total SC cost is calculated as the sum of holding costs of all inventories and the shortage cost charged on the end-product inventory,
  - $FHD_{T_h}$  - SC holding cost per end-product demanded; the SC holding cost is the sum of the holding costs of all inventories,
  - $FR_{T_h}$  - SC fill rate, that is in fact the end-product inventory fill rate.

Finally, average results of repeated simulation runs are displayed.

### Structure of *SCBL*

*SCBL* encompasses four main modules. Three modules are run iteratively, for each unit time within the time horizon specified:

- a module that simulates processes at each inventory,
- a module that generates uncertain customer demand,
- a module that prints simulation results generated during each time unit.

At the end, a module that calculates and prints the overall SC performance and the performance of each individual inventory achieved during the simulated time horizon is run.

Each of the modules is briefly described below.

#### Module that simulates processes at each inventory along an SC

Processes at each inventory along an SC during each time unit are simulated as follows:

- replenishment phase. Replenishment processes along the SC are simulated, starting from the raw material inventory ( $n = N$ ) towards the end-product inventory ( $n = 1$ ). Inventory  $n$ ,  $n = N, \dots, 1$  is replenished, if the beginning of the current time unit coincides with the beginning of a replenishment period of the inventory. In this case, inventory  $n$ ,  $n = N-1, \dots, 1$ , receives a replenishment quantity from the preceding facility  $n+1$ , including backordered quantity, if it exists. Inventory  $N$  receives a delivery from the external supplier. The current stock level at inventory  $n$  is increased and the backordered quantity of inventory  $n$ , if it exists, is updated in the following way: the part of the backordered quantity ready for delivery from inventory  $n$  to succeeding facility  $n-1$  is increased and the part of the backordered quantity still owed to succeeding facility is decreased.
- ordering phase. Ordering processes along the SC are simulated, starting from the end-product inventory ( $n = 1$ ) towards the raw material inventory ( $n = N$ ). This phase involves inventory  $n$ ,  $n = 1, \dots, N$ , only if the beginning of the current time unit coincides with the beginning of a review period of the inventory. Ordering processes for inventory  $n$ ,  $n = 1, \dots, N-1$  and for inventory  $N$  are different.

A replenishment order for inventory  $n$ ,  $n = 1, \dots, N-1$  is calculated to bring the inventory level after replenishment to the predetermined order-up-to level, taking into account the current inventory level, the backordered quantity still to be



delivered by the preceding facility  $n+1$ , and fuzzy demand on the inventory during the lead time. The quantity that will be received from preceding facility  $n+1$  after the lead time depends on the current stock level at the preceding inventory. The replenishment order placed from inventory  $n$  decreases the stock level at preceding inventory  $n+1$  instantly. If the replenishment order is greater than the stock level, the backordered quantity owed by facility  $n+1$  to facility  $n$  and the number of shortages at inventory  $n+1$  are increased.

The raw material inventory ( $n = N$ ) places an order to the external supplier. The replenishment order is calculated to bring the raw material inventory after replenishment to the predetermined order-up-to level, taking into account the current raw material inventory level and fuzzy demand on the raw material inventory during the supplier lead time.

- receiving customer demand. This phase involves the end-product inventory, only. Customer demand imposed on the end-product inventory decreases the end-product inventory level. If customer demand is greater than end-product stock the backordered quantity owed to the customers and the number of shortages at the end-product inventory are increased.
- cost calculation. In this phase, the cumulative holding cost, the cumulative shortage cost and the cumulative total cost incurred at each inventory  $n$ ,  $n = 1, \dots, N$  are calculated.
- time increment. At the end of a simulated time unit, time increment is performed. The counter of the time unit within the current inventory review period, associated with each inventory is increased by one. The counter is reset to 0 at the end of each review period of the inventory. In addition, indicator of the beginning of a replenishment period, associated with each inventory, is updated.

Module that generates uncertain customer demand

Fuzzy demands are specified by possibility distributions. In order to simulate fuzzy demand, the possibility distribution is transformed into the corresponding probability distribution based on the procedure given in Section 3.2. and formula (3.38). Accordingly, simulation of fuzzy demand, i.e., numerical realisations of demand described by vague linguistic terms are performed by using randomly generated numbers.

Module that prints simulation results obtained during each time unit and during the whole simulated time horizon generates output reports previously described.

The pseudo-code of *SCBL* is given in Appendix 1.

**6.2.2. *SCSUPP***

*SCSUPP* includes two sources of uncertainty that may exist in an SC environment, customer demand and external supplier reliability. It is assumed that the lead times along the SC are fixed and precisely known.

*SCSUPP* is very similar to *SCBL*. The only differences are outlined below.

In addition to all the input data listed for *SCBL*, there is one more input data required by *SCSUPP*:

- name of a file that contains data about fuzzy external supplier reliability *SP*.

*SCSUPP* has one module more than *SCBL*:

- a module that generates uncertain external supply. Fuzzy external supply is simulated in the same manner as fuzzy customer demand, by transforming its possibility distribution into the corresponding probability distribution.

In line with this, there is a change in the module that simulates processes at the raw material inventory ( $n = N$ ), in the ordering phase only. A replenishment order for the raw material inventory is calculated in the same way as in *SCBL*. However, the quantity that will be received from the supplier depends on external supplier reliability and it is determined in the module that generates uncertain external supply.

The pseudo-code of *SCSUPP* is given in Appendix 1.

### 6.2.3. *SCLT*

*SCLT* includes two sources of uncertainty, customer demand and lead times to facilities in an SC. Its structure and modules it contains are very similar to those of *SCBL*. The only differences are outlined below.

In addition to all the input data listed for *SCBL*, *SCLT* uses one more input data:

- name of a file that contains data about fuzzy lead time  $L^n$  to each facility  $n$ ,  $n = 1, \dots, N$  in the SC.

*SCLT* has one additional module:

- a module that generates uncertain lead times. A fuzzy lead time is simulated in the same manner as fuzzy customer demand, by transforming its possibility distribution into the corresponding probability distribution. The fuzzy lead time is simulated over time by generating a sequence of values which follow the probability distribution. This module is run at the beginning of each review period of each inventory in the SC.

All *SCLT* modules are the same as the *SCBL* modules apart from the ordering phase in the module which simulates processes at each facility. In the ordering phase, a replenishment order for each inventory is calculated taking into account fuzzy demand during the fuzzy lead time to the inventory.



The pseudo-code of *SCLT* is given in Appendix 1.

### 6.3. *SCSIM* implementation

The *SCSIM* components, *SCBL* and *SCSUPP* were implemented in C++ object-oriented programming language and runs on PC under the Windows operating system. At the time of writing, the implementation of the third component *SCLT* was still in progress.

Object-oriented programming techniques enabled the *SCSIM* components to be implemented in a modular manner. This enables easy modifications and extensions of the existing SC models. A large amount of data and operations on them, involved in SC simulation, were structured and grouped into classes (i.e., objects). Five classes were implemented which represent: (1) possibility distribution, (2) probability distribution, (3) inventory characteristics, (4) inventory state, and (5) the whole supply chain.

The user-interface was developed in such a manner to enable easy input of SC data and to provide textual and graphical output reports. Data can be input into *SCSIM* either interactively or from files prepared in advance. Output simulation results are presented both on screen and saved into a file.

### 6.4. Summary

New special purpose SC simulators, named *SCSIM*, have been developed by the author. *SCSIM* has been implemented using object-oriented programming language C++.

Two types of SC models developed in this work have been effectively combined into *SCSIM*, including: (1) the SC fuzzy, analytical models and (2) the SC

simulation models. The SC fuzzy models constitute a generative part of *SCSIM* which is used to determine inventory order-up-to levels and replenishment quantities. The SC simulation models, on the other hand, constitute an evaluative part of *SCSIM* which is used to assess the effects of the decision variables, i.e. order-up-to levels and replenishment quantities recommended by the fuzzy models, and to determine SC performance, such as SC fill rate and SC total cost. By combining the two types of models, *SCSIM* provides a dynamic view of SC behaviour and its performance in the presence of uncertainty described by fuzzy sets.

In addition, two new iterative procedures for increasing the order-up-to levels in order to achieve a higher SC fill rate have been built into *SCSIM*. The detailed description of these procedures is given in Chapter 8.

Application of *SCSIM* provides a new methodology for SC modelling and analyses which differs from the existing methodologies, such as system dynamics approach and discrete event simulation, in many respects. The system dynamics methodology was developed in order to study the response of an SC to sudden changes in its external environment, usually manifested as a step change in customer demand. In the SC discrete event simulation models uncertainties in SCs and their environments have been treated as random processes. Uncertainty in SC data has been described by probability distributions with parameters usually assumed to be known. On the other hand, in this research uncertainty in SC data has been described by fuzzy sets and included into the models. *SCSIM* operates with different types of uncertainty in SC data, which are not stochastic in nature, but associated with uncertainty in judgement, lack of evidence or lack of certainty in evidence. In order to bring the fuzzy SC data into a dynamic framework, possibility distributions induced by the fuzzy sets are transformed in *SCSIM* into the corresponding probability distributions. In this way, fuzzy SC data are simulated over time by generating stochastic values which follow the probability distributions. *SCSIM* is the tool in which this approach to simulation in a fuzzy environment has been applied and implemented for the first time.

In summary, *SCSIM* is a powerful tool developed to gain better understanding of SC behaviour and its performance in the presence of uncertainty. It can simulate an SC of any length and can operate with fuzzy customer demand, supplier reliability and lead times, described by discrete fuzzy sets with any form of membership functions. The potential of applying *SCSIM* to study SCs in uncertain environments is demonstrated in the next two chapters.



## CHAPTER 7

### SUPPLY CHAIN ANALYSIS UNDER UNCERTAINTY IN CUSTOMER DEMAND

The aim of this chapter is to demonstrate the application of *SCSIM* tool and *SCBL* simulator, in particular, to analyse a serial SC in the presence of uncertainty. Attention is focused on a serial SC that involves only one source of uncertainty in its operating environment, i.e., it is assumed that uncertainty appears only in customer demand.

First, the effects of two SC control strategies: (1) fully decentralised control and (2) partially coordinated control, formally defined in Section 5.2.2.2., which are applied in the presence of uncertainty in customer demand, are analysed and compared. The results of this work have been presented in a paper accepted for publication in *International Journal of Production Economics* (Petrovic, Roy, Petrovic). Illustrative simulation results are presented in Section 7.1. which show that linking inventories in an SC and applying decentralised control causes deterioration of performance of the inventories. Partially coordinated control is analysed in Section 7.2. The two control strategies are compared using illustrative examples, with a view to testing the following hypothesis: partially coordinated control is a better control strategy with respect to SC performance than decentralised control.

Application of *SCBL* simulator to analyse effects of changing uncertainty in customer demand on optimal policies of SC inventories, inventory performance and overall SC behaviour and performance is described in Section 7.3.

Due to subjectivity inherent in generating a linguistic customer demand estimation and modelling it by a fuzzy set, a question arises on how sensitive the

selection of optimal order-up-to levels is to small changes in the fuzzy customer demand representation. This issue is discussed in Section 7.4.

In all the analyses performed, SC behaviour was examined over time  $T_h$  which was homogeneously discretised and the discrete unit time intervals within  $T_h$  were indexed by index  $t = 1, \dots, T$ . Customer demand during unit time interval  $t$  was represented by a fuzzy set  $D_t$ , defined by a discrete possibility distribution  $\mu_{D_t}(d_t)$ ,  $d_t \in \mathcal{D}_t$ , where  $d_t$  is the possible discrete demand value during time interval  $t$  and  $\mu_{D_t}(d_t)$  is the associated possibility of demand taking the value  $d_t$ .

SC operations during  $T_h$  and performance achieved in the presence of uncertainty in customer demand were simulated using *SCBL* simulator described in Section 6.2.1.

## 7.1. Fully decentralised control

In the case of fully decentralised control each inventory is controlled independently. For each inventory the objective is to minimise the possible total inventory cost per one replenishment period in the presence of uncertainty in demand, under the assumption that the supply from the predecessor is large enough, i.e., that each preceding facility will be able to fill any order imposed. Formally, the order-up-to levels  $S_{d,k^n}^n$ , for each inventory  $n = 1, \dots, N$ , and for each review period  $k^n = 1, \dots, K^n$  are determined in a decentralised manner as described in Section 5.2.2.2., starting from the first review period, and from the end-product inventory ( $n = 1$ ) to the raw material inventory ( $n = N$ ). Once the order-up-to levels are determined, effects of applying decentralised control on performance of each inventory and on overall SC performance during a finite time horizon  $T_h$  are analysed by simulation. Linking inventories in an SC under decentralised control is expected to cause deterioration of inventory fill rates. In particular, the fill rate of each inventory, except that of the raw material, is expected to

decrease in comparison with the fill rate of the inventory considered isolated. The fill rate of each inventory is affected by possible lack of supply from its predecessor. Only the raw material inventory fill rate is unchanged due to an assumption that an absolutely reliable external supplier always delivers all quantities of raw material ordered. An obvious conclusion can be derived: the smaller the fill rate of an inventory in the chain, the larger the decrease in the fill rate of its succeeding inventory.

The performance of an SC depends on the length of the chain. The SC fill rate, commonly defined as the fill rate of the end-product inventory, decreases when a new inventory is added to the chain, unless the inventory added has absolutely reliable delivery, i.e., its fill rate is almost equal to 1.

When inventories are linked in a chain the possible total cost of each inventory achieved with the order-up-to levels determined in the decentralised manner is not locally optimal any more. The stock level  $S_{d,k^n}^n$  would lead to the minimum possible cost of inventory  $n$  during replenishment period  $k^n$  only if all the quantity ordered was available at preceding facility  $n+1$ . However, after replenishment, the target level  $S_{d,k^n}^n$  is not always reached due to the lack of supply from facility  $n+1$  that may happen, and uncertain demand from succeeding facility  $n-1$  during the lead time.

A natural way to safeguard against unreliable supply of predecessors in an SC is to raise the ratios of the unit shortage cost to the unit holding cost of the inventories in the chain. In this way a higher fill rate of the end-product inventory, i.e., higher SC fill rate, can be ensured.

The results of simulation of the SC with the characteristics specified in Example 1 in Section 5.3.1., which is under decentralised control, are presented here. In this chain, all the inventories have the same ratio of the unit shortage cost to the unit holding cost and, consequently, they have the same locally optimal order-up-to levels. If the inventories are considered isolated, not linked in the chain, the same fill rates will be obtained. When the inventories are linked, the adverse effects are clearly observable. A degree of SC fill rate deterioration is measured by comparing the fill



rates of the raw material inventory and the end-product inventory. This is expressed as a percentage by which the end-product inventory fill rate is decreased in comparison with the raw material inventory fill rate. The unit costs ratio of all the inventories is set to different values. The simulation results, graphically presented in Figure 7.1., show fill rate deterioration that occurs when inventories operate in the chain. One can see that higher unit cost ratios lead to smaller decrease in the SC fill rate.

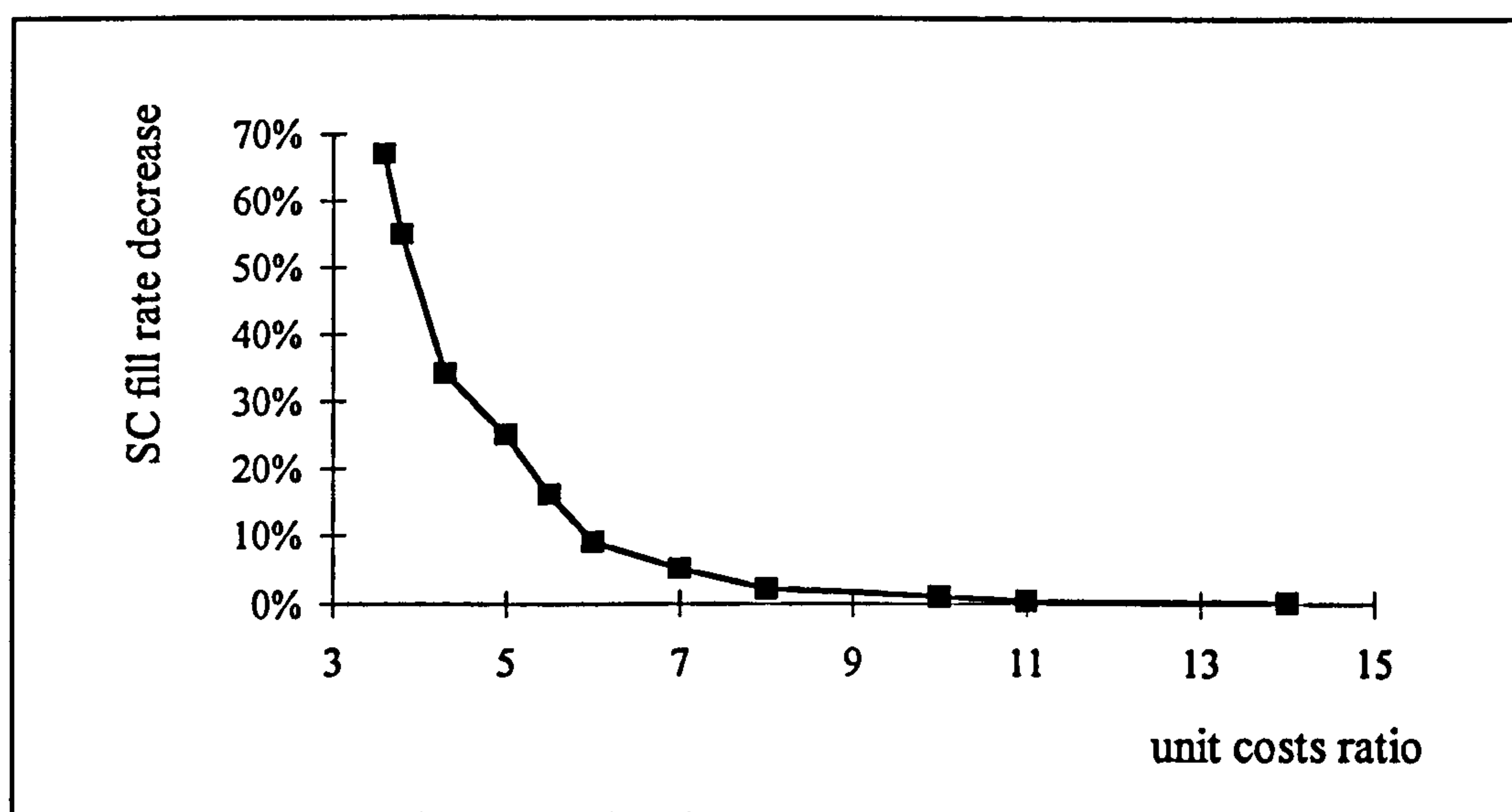


Figure 7.1. SC fill rate deterioration vs unit costs ratio  
of all the inventories along the SC

Some details of the simulation results obtained with two different unit cost ratios are now presented.

### Simulation 7.1.

The purpose of this simulation is to demonstrate that by linking inventories in a series, in the presence of uncertainty in demand, negative effects with respect to fill rates are amplified, starting from the raw material to the end-product inventory.

Consider a chain, denoted here as *SC 1*, with the characteristics specified in Example 1 in Section 5.3.1. The unit shortage costs and the unit holding costs of all the five inventories along the chain, are as follows:

- unit holding costs:  $c_h^1 = 1.5$ ,  $c_h^2 = 1$ ,  $c_h^3 = 0.5$ ,  $c_h^4 = 0.3$ ,  $c_h^5 = 0.1$ ,
- unit shortage costs:  $c_s^1 = 6.5$ ,  $c_s^2 = 4.3$ ,  $c_s^3 = 2.15$ ,  $c_s^4 = 1.29$ ,  $c_s^5 = 0.43$ .

Due to the same unit cost ratio and stationary customer demand, the order-up-to levels for all the inventories, determined in the decentralised manner for all the review periods, are all equal:  $S_d^1=36$ ,  $S_d^2=36$ ,  $S_d^3=36$ ,  $S_d^4=36$ ,  $S_d^5=36$ .

*SC 1* operations are simulated over  $T_h = 52$  weeks. The following initial inventory states are specified:

- initial stock levels:  $I_0^1 = 10$ ,  $I_0^2 = 36$ ,  $I_0^3 = 36$ ,  $I_0^4 = 36$ ,  $I_0^5 = 36$ ,
- the beginning of the initial simulated time interval coincide with the start of the review periods of all the inventories, i.e.,  
 $rew_1^1 = 0$ ,  $rew_1^2 = 0$ ,  $rew_1^3 = 0$ ,  $rew_1^4 = 0$ ,  $rew_1^5 = 0$ ,
- replenishment does not start at any of the inventories at the beginning of the initial simulated time interval, i.e.,  
 $repl_1^1 = 0$ ,  $repl_1^2 = 0$ ,  $repl_1^3 = 0$ ,  $repl_1^4 = 0$ ,  $repl_1^5 = 0$ ,
- there are no replenishment quantities, ordered before the initial time interval, i.e.,  
 $Q_0^1 = 0$ ,  $Q_0^2 = 0$ ,  $Q_0^3 = 0$ ,  $Q_0^4 = 0$ ,  $Q_0^5 = 0$ .

Note that initial end-product stock at the beginning of the first review period,  $I_0^1$ , is selected to be equal to the most possible demand value during the specified lead time of 1 week, i.e., equal to 10. Initial stock levels at other inventories,  $I_n^0$ ,  $n = 2, \dots, 5$ , are chosen to be equal to the target inventory levels 36. By selecting these values the influence of the initial inventory states on the performance of each inventory and the overall SC performance during the simulated time horizon is reduced.

The stock levels at each inventory, simulated in one run, over 52 weeks, are shown as the functions of time in Figure 7.2. It can be noticed that only the raw

material inventory reaches the target order-up-to level 36 after each replenishment. At all other inventories, there are discrepancies between the inventory levels reached after replenishments and the target levels. In addition, these discrepancies are amplified down the chain, starting from the in-process inventory with index 4 towards the end-product inventory with index 1.

Performance of each inventory in the chain and of the overall SC, achieved by setting  $S_d^n = 36$ ,  $n = 1, \dots, 5$ , during time horizon  $T_h = 52$  weeks was obtained by simulation. The selection of one year time horizon was motivated by real-life SC control problems. Simulation was repeated 100 times in order to get average annual SC performance. In each run, *SCBL* simulator used a different seed for generating a sequence of pseudo-random numbers, which was mapped into uncertain customer demand per week within the specified time horizon. Therefore, SC performance achieved in simulation runs were different. The average simulation results are presented in Table 7.1.



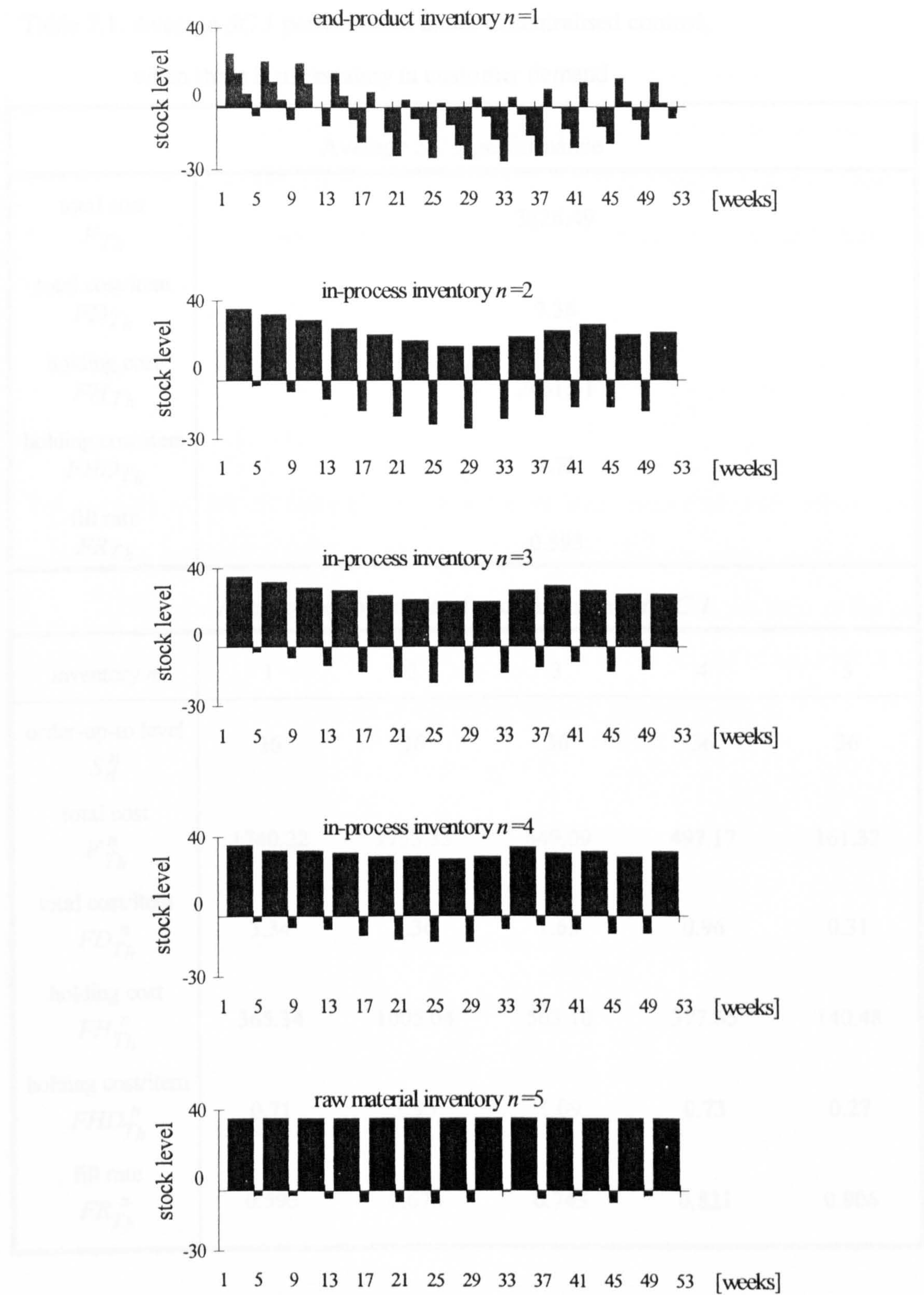


Figure 7.2. Simulation of stock levels along *SC 1* under decentralised control

Table 7.1. Average SC 1 performance under decentralised control,  
when there is uncertainty in customer demand

Average SC 1 performance					
total cost $F_{T_h}$	3826.49				
total cost/item $FD_{T_h}$	7.36				
holding cost $FH_{T_h}$	2451.61				
holding cost/item $FHD_{T_h}$	4.73				
fill rate $FR_{T_h}$	0.595				
Average performance of each inventory in SC 1					
inventory $n$	1	2	3	4	5
order-up-to level $S_d^n$	36	36	36	36	36
total cost $F_{T_h}^n$	1740.22	1733.33	849.09	497.17	161.32
total cost/item $FD_{T_h}^n$	3.34	3.36	1.65	0.96	0.31
holding cost $FH_{T_h}^n$	365.34	1005.04	563.10	377.65	140.48
holding cost/item $FHD_{T_h}^n$	0.71	1.95	1.09	0.73	0.27
fill rate $FR_{T_h}^n$	0.595	0.673	0.743	0.821	0.906



Although the inventories in *SC 1* have the same order-up-to levels, their fill rates are different. The fill rates of the inventories, starting from the in-process inventory with index 4 to the end-product inventory with index 1, are decreased by 9%, 18%, 26% and 34%, respectively, in comparison with the raw material inventory fill rate. The fill rate of the raw material inventory is the same as in the case when it is considered isolated.

### Simulation 7.2.

The purpose of this simulation is to demonstrate that smaller negative effects with regard to fill rates will result if the fill rates of inventories considered isolated are high.

Consider *SC 2*, with all characteristics as those of *SC 1*, but the ratio of the unit shortage cost to the unit holding cost is increased to 11 for all the inventories in the chain. The costs ratio is increased by increasing the unit shortage costs to:

$$- c_s^1 = 16.5, c_s^2 = 11, c_s^3 = 5.5, c_s^4 = 3.3, c_s^5 = 1.1.$$

The order-up-to levels of the inventories calculated in the decentralised manner are now higher:  $S_d^1 = 43, S_d^2 = 43, S_d^3 = 43, S_d^4 = 43, S_d^5 = 43$ .

The initial inventories levels are raised to:

$$- I_0^1 = 10, I_0^2 = 43, I_0^3 = 43, I_0^4 = 43, I_0^5 = 43.$$

The results of simulation of *SC 2* operations over 52 weeks are shown in Figure 7.3. One can see that discrepancies between the inventory levels reached after replenishments and the target levels are substantially reduced in contrast with *SC 1*.

The average simulation results are given in Table 7.2. In this case, linking the inventories in series has a less negative impact on the fill rates of the inventories than in *SC 1*. The fill rates of the in-process inventories and the end-product inventory are decreased insignificantly in comparison with the raw material inventory fill rate. This better *SC 2* fill rate is achieved at the expense of a higher inventory cost per end-



product demanded. The high order-up-to levels along *SC* 2 caused a greater holding cost and a smaller shortage cost at each inventory.

## 7.2 Partially coordinated control

Order-up-to levels of inventories in an SC determined in the decentralised manner do not provide satisfactory control of the chain as a whole, with respect to the SC fill rate and total cost. Therefore, a partial coordination control (as defined in Section 5.2.2.2.) is introduced. Each inventory is controlled taking into consideration not just uncertainty in demand but also uncertainty in supply, requiring coordination with its predecessor.

The partial coordination between each of the two neighbouring inventories leads to the modification of the order-up-to levels determined in the decentralised manner. It starts with the coordination of the pair of inventories with indices  $N$  and  $N-1$ , followed by inventories indexed by  $N-1$  and  $N-2$ , and so on until the pair indexed by 2 and 1 is reached. The new order-up-to levels are derived;  $S_{p,k^n}^n$ ,  $n = 1, \dots, N$ ,  $k^n = 1, \dots, K^n$ . Generally, the coordinated order-up-to levels  $S_{p,k^n}^n$  are equal to, or increased, in comparison with the corresponding  $S_{d,k^n}^n$  determined in the decentralised manner. Note that the order-up-to levels for the raw material inventory are not changed, i.e.,  $S_{d,k^N}^N = S_{p,k^N}^N$ ,  $k^N = 1, \dots, K^N$ .

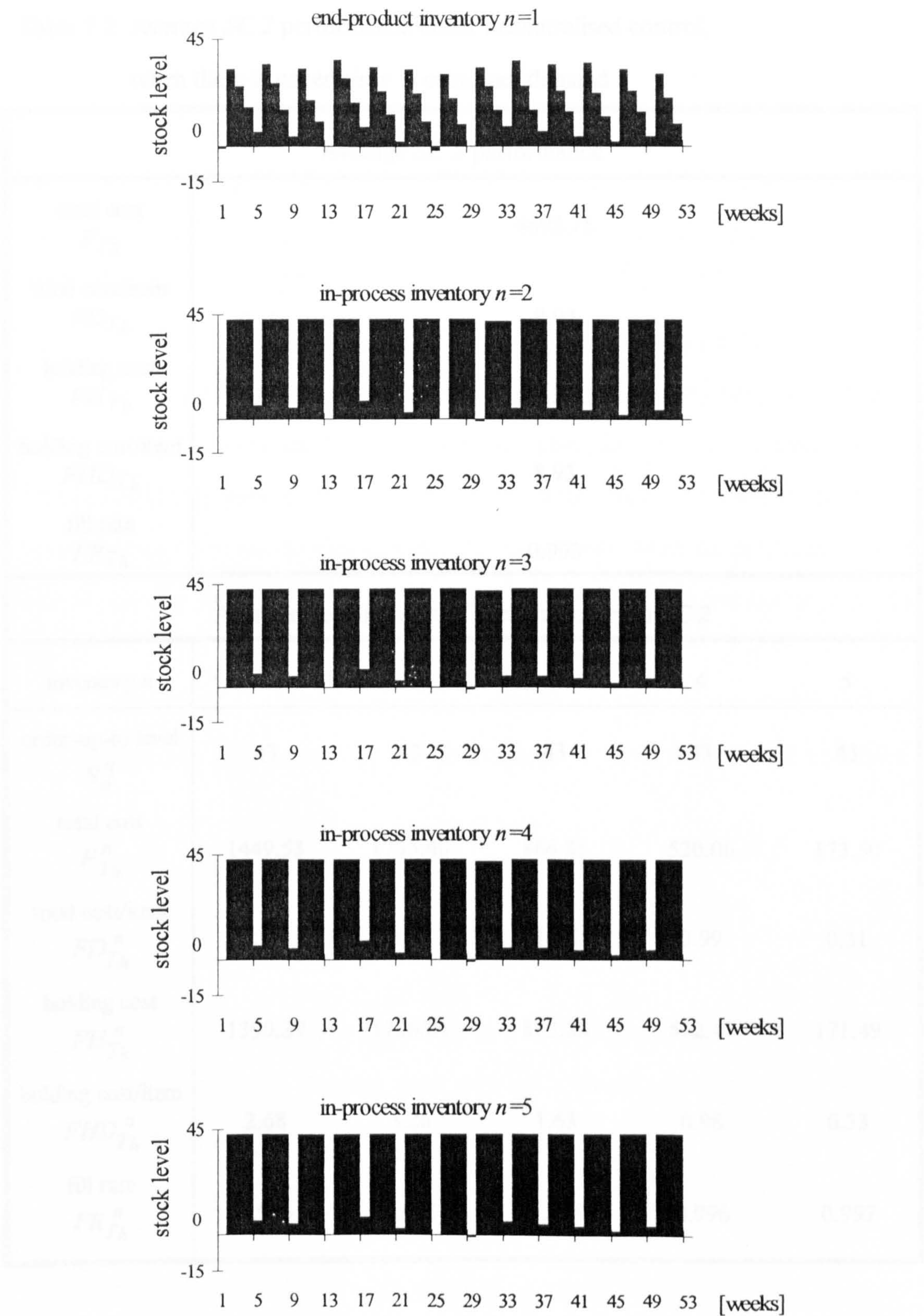


Figure 7.3. Simulation of stock levels along SC 2 under decentralised control



Table 7.2. Average SC 2 performance under decentralised control,  
when there is uncertainty in customer demand

Average SC 2 performance					
total cost $F_{T_h}$	4692.78				
total cost/item $FD_{T_h}$	9.04				
holding cost $FH_{T_h}$	4633.55				
holding cost/item $FHD_{T_h}$	8.93				
fill rate $FR_{T_h}$	0.993				
Average performance of each inventory in SC 2					
inventory $n$	1	2	3	4	5
order-up-to level $S_d^n$	43	43	43	43	43
total cost $F_{T_h}^n$	1449.53	1733.40	866.81	520.06	173.50
total cost/item $FD_{T_h}^n$	2.79	3.32	1.66	0.99	0.31
holding cost $FH_{T_h}^n$	1390.29	1706.01	853.28	512.47	171.49
holding cost/item $FHD_{T_h}^n$	2.68	3.26	1.63	0.98	0.33
fill rate $FR_{T_h}^n$	0.993	0.995	0.995	0.996	0.997



Simulating SC behaviour and its performance under partially coordinated control and comparing it with decentralised control reveal some interesting phenomena. Due to the higher order-up-to levels  $S_{p,k^n}^n$ ,  $n = 1, \dots, N$ ,  $k^n = 1, \dots, K^n$ , the fill rates of all the inventories, except the raw material inventory, are increased. The inventories approximately reach the fill rates which would have been reached if they were not linked in the chain, with order-up-to levels  $S_{d,k^n}^n$ . Therefore, the effects of decrease in the SC fill rate as a result of linking the inventories in series are reduced.

The possible total cost of each inventory in the SC under partially coordinated control is smaller in comparison with the case when decentralised control is applied. The reason is that each of the order-up-to levels is determined to minimise the possible total inventory cost taking into account uncertain supply from the predecessor. Under partial coordination greater holding costs and smaller shortage costs are incurred along the SC.

The effects of applying partially coordinated control on two chains, SC 1 and SC 2, defined in Simulation 7.1. and Simulation 7.2., respectively, are presented as follows.

### Simulation 7.3.

The purpose of this simulation is to show that by applying partial coordination higher SC fill rate with lower holding cost can be achieved, in comparison with fully decentralised control.

Starting from the order-up-to levels of the inventories in SC 1 determined independently:

$$S_d^1 = 36, S_d^2 = 36, S_d^3 = 36, S_d^4 = 36, S_d^5 = 36,$$

the modified partially coordinated order-up-to levels obtained are:

$$S_p^1 = 40, S_p^2 = 40, S_p^3 = 39, S_p^4 = 41, S_p^5 = 36.$$

The initial inventory levels are changed accordingly to:

$$I_0^1 = 10, I_0^2 = 40, I_0^3 = 39, I_0^4 = 41, I_0^5 = 36.$$

The stock level changes at each inventory along *SC 1* during the simulated time horizon  $T_h = 52$  weeks are shown in Figure 7.4. Higher order-up-to levels along *SC 1* cause more stocks held at each inventory in comparison with the case when *SC 1* is controlled decentrally. As was expected, discrepancies between the inventory levels after replenishments and the target levels prevailed at each location, except the raw material inventory. However, due to partially coordinated control their amplification along the chain is nearly eliminated.

SC simulation over  $T_h$ , repeated 100 times, generates the average results given in Table 7.3.



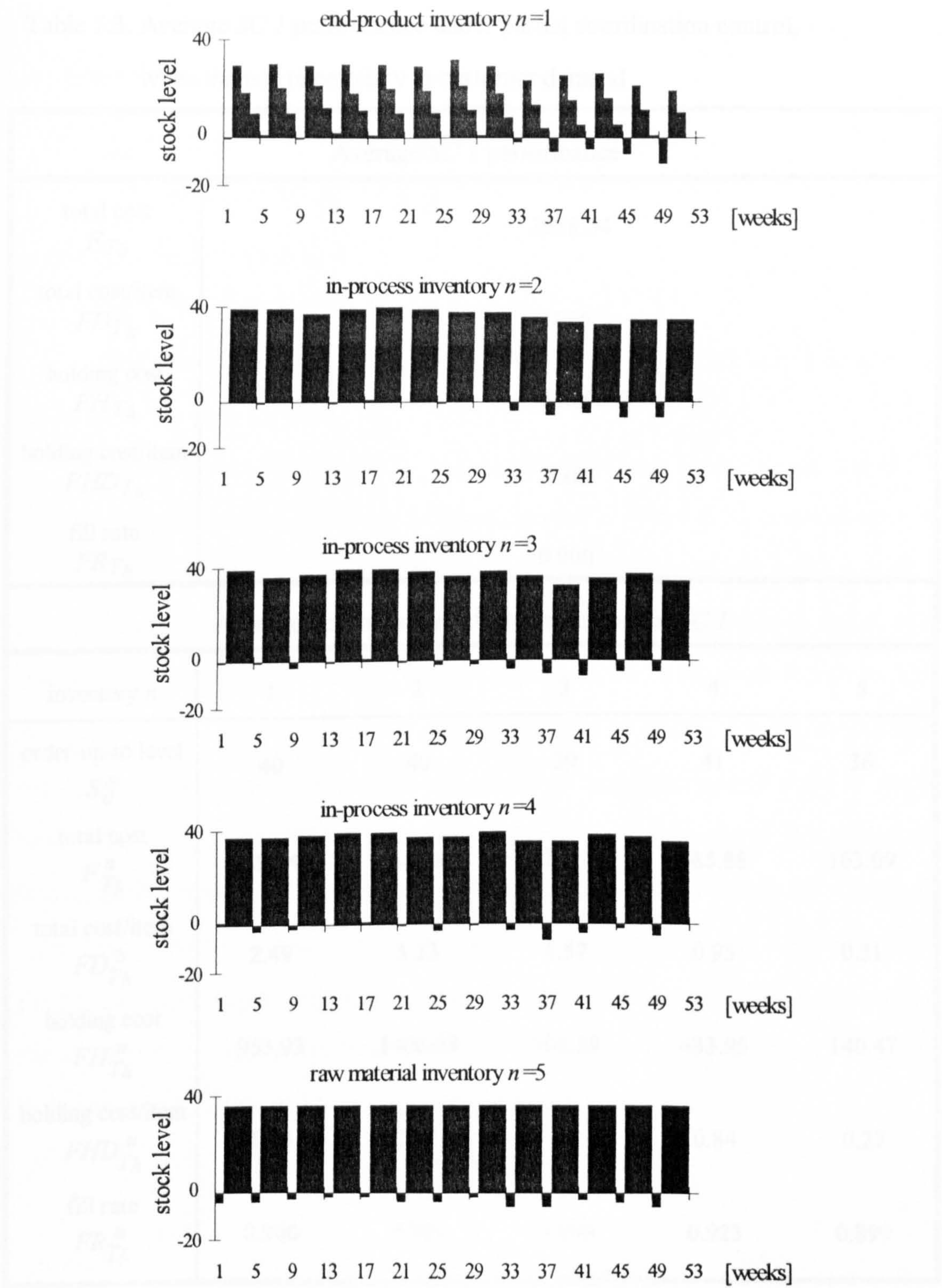


Figure 7.4. Simulation of stock levels along SC 1 under partial coordination control



Table 7.3. Average SC 1 performance under partial coordination control,  
when there is uncertainty in customer demand

Average SC 1 performance					
total cost $F_{T_h}$	3976.54				
total cost/item $FD_{T_h}$	7.65				
holding cost $FH_{T_h}$	3636.00				
holding cost/item $FHD_{T_h}$	7.00				
fill rate $FR_{T_h}$	0.900				
Average performances of each inventory in SC 1					
inventory $n$	1	2	3	4	5
order-up-to level $S_d^n$	40	40	39	41	36
total cost $F_{T_h}^n$	1294.46	1627.85	814.54	485.88	163.09
total cost/item $FD_{T_h}^n$	2.49	3.13	1.57	0.93	0.31
holding cost $FH_{T_h}^n$	953.93	1406.08	701.58	433.95	140.47
holding cost/item $FHD_{T_h}^n$	1.84	2.71	1.35	0.84	0.27
fill rate $FR_{T_h}^n$	0.900	0.901	0.899	0.923	0.899

Partial coordination applied to SC 2 causes minor order-up-to level modifications. The order-up-to levels are changed from

$$S_d^1 = 43, S_d^2 = 43, S_d^3 = 43, S_d^4 = 43, S_d^5 = 43 \text{ to}$$

$$S_p^1 = 45, S_p^2 = 45, S_p^3 = 45, S_p^4 = 46, S_p^5 = 43$$

and initial inventory levels are:

$$I_0^1 = 10, I_0^2 = 45, I_0^3 = 45, I_0^4 = 46, I_0^5 = 43.$$

The average simulation results achieved with these partially coordinated order-up-to levels are given in Table 7.4.

In both chains, SC 1 and SC 2, as it can be seen from Table 7.3. and Table 7.4., all the fill rates along the chains are balanced, around 0.90 and 0.99, respectively. This is achieved by setting the ratios of the unit shortage costs to the unit holding costs of all the inventories to the same value. Consequently, the initial order-up-to levels of all the inventories considered isolated, before the partial coordination, are the same, 36 in SC 1 and 43 in SC 2. By modifying the order-up-to levels, partial coordination provides the fill rate of each inventory in an SC to be approximately equal to the fill rate of the inventory considered isolated before linking it in the chain. This means that linking the inventories in series under partially coordinated control does not change the individual inventory fill rate. The fill rates of approximately 0.90 and 0.99, achieved at the partially coordinated inventories along SC 1 and SC 2, respectively, correspond to the fill rates that would be obtained if the inventories were operated isolated, by setting their order-up-to levels to 36 and 43, respectively.

Table 7.4. Average SC 2 performance under partial coordination control,  
when there is uncertainty in customer demand

Average SC 2 performance					
total cost $F_{T_h}$	5029.16				
total cost/item $FD_{T_h}$	9.70				
holding cost $FH_{T_h}$	5007.54				
holding cost/item $FHD_{T_h}$	9.66				
fill rate $FR_{T_h}$	0.998				
Average performances of each inventory in SC 2					
inventory $n$	1	2	3	4	5
order-up-to level $S_d^n$	45	45	45	46	43
total cost $F_{T_h}^n$	1578.99	1820.27	910.37	557.40	175.88
total cost/item $FD_{T_h}^n$	3.03	3.47	1.73	1.06	0.33
holding cost $FH_{T_h}^n$	1557.38	1814.88	907.62	556.05	171.62
holding cost/item $FHD_{T_h}^n$	3.01	3.47	1.73	1.06	0.33
fill rate $FR_{T_h}^n$	0.998	0.999	0.999	0.999	0.993



Comparing the *SC 1* and *SC 2* fill rates achieved using the two control strategies, it can be clearly established that partial coordination control approach is superior to decentralised control in the sense that a higher SC fill rate can be achieved with a lower SC holding cost. This was further tested by additional simulation. *SC 1* was again used as the reference chain, with a specific feature that the cost ratios of all the inventories in the chain were equal. The cost ratio was increased in each simulation to enhance the SC fill rate. The results of the simulations are illustrated in Figure 7.5. When the chain was partially coordinated, a better chain performance was achieved. Partial coordination was especially superior in the case when the ratios of the unit shortage costs to the unit holding costs were not high enough to provide high inventory fill rates and reliable delivery along the SC under decentralised control.

The hypothesis that partial coordination is a better control strategy than decentralised control can be fully tested using *SCBL* simulator. This would, however, require extensive simulation experiments, in which various SC data have to be varied, including the number of facilities in the chain, the unit holding costs, the unit shortage costs, the lead times to the facilities, inventory review periods and others, using an experimental design approach (Pidd, 1984). This is suggested for the further research.

### 7.3. Effects of changing uncertainty in customer demand

Various uncertainties which are a natural part of market characteristics have a substantial effect on the estimation of customer demand. This estimation depends to a great extent on a manager's personal attitude, and the estimated demand values can be different from the actual values. One can talk about overestimation and underestimation or optimistic and pessimistic estimations, however, this section deals only with consequences which are likely to appear due to uncertainty in the external environment. It is assumed that demand estimation can match actual demand.

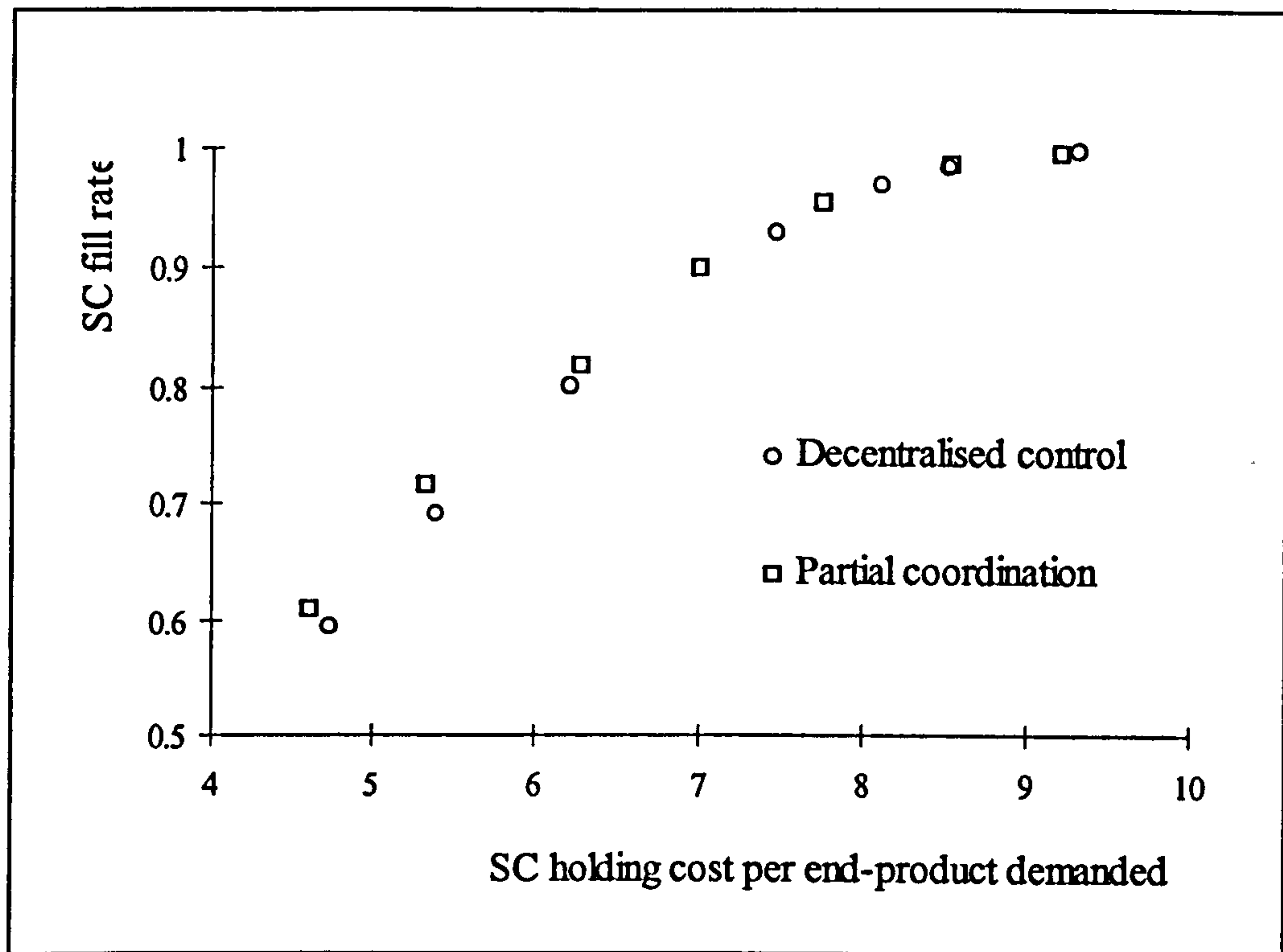


Figure 7.5. SC fill rate vs SC holding cost per end-product demanded, under two different SC control strategies

Naturally, the questions arise as to how to model various uncertainties in the external environment which affect customer demand, and how to measure uncertainty in customer demand. In this research, generating a linguistic demand estimation (forecast) and representing it by a fuzzy set is proposed. Different approaches to measuring uncertainty represented by fuzzy sets are described in Section 3.3. In this work, the measure of uncertainty associated with fuzzy customer demand has to be assessed taking into account two issues: (1) the number of demand values that can happen and the range to which the values belong (or using the terminology of fuzzy set theory, it is the support of the fuzzy set *customer demand* and its size) and (2) the possibilities of demand taking these values. As the range of values that demand can take increases, the characterisation of demand becomes less specific. Also, a larger number of demand values with high possibility grades indicates that information about demand is spread over many highly possible alternative values. Accordingly, a measure

of customer demand uncertainty has to consider both the size of the support of the corresponding fuzzy set and its membership (i.e., possibility) grades. In this research, a measure of non-specificity, defined by formula (3.53), is used to assess uncertainty in fuzzy customer demand.

Changes in the external environment can reflect on fuzzy customer demand estimation in different ways. It is therefore important to investigate the influence of changing uncertainty in customer demand on SC behavior and its performance. In this section it is illustrated how two such changes can be examined using *SCBL* simulator:

1. changes in the range of values that demand can take; for example, demand is estimated to be "*about* certain value", but the range of the possible demand values can have different boundaries,
2. qualitative changes in customer demand that require different linguistic estimations of customer demand; for example, there are changes in the possibility of an event such that demand takes a value isolated from the interval of usual demand values; for example, an event of very high demand or very low demand can change the demand description from "demand is *about* certain value" to "demand is *about* certain value, but there is a possibility of demand taking a new value".

The relationship between changes in uncertainty in customer demand and dynamic changes in inventory levels and orders placed along an SC, and SC fill rate are next investigated.

### 7.3.1. Effects of changing the range of possible demand values

The range of possible demand values, which corresponds to the range of the support of the fuzzy set which represents customer demand, indicates the amount of uncertainty associated with customer demand. A larger range of possible values reflects higher uncertainty in determining the values that demand can take. Similarly, more certainty



about possible demand values narrows its range. A general analysis of the effects of changing the range of possible demand values are given first, and the conclusions drawn are then illustrated using the SC example described in Example 1 in Section 5.3.1.

The effects of changing uncertainty in customer demand on optimal order-up-to levels of inventories along an SC are examined first. An optimal order-up-to level for an inventory in the SC under decentralised control is determined using the fuzzy model for an isolated inventory control, described in Section 5.2.1.1. The first step in finding the optimal order-up-to level  $S$  of the inventory for one review period is the calculation of customer demand  $DR$  on the inventory during the corresponding replenishment period. The standard fuzzy arithmetic rule, given in formula (5.1), is used for adding fuzzy customer demand estimations  $D_t$  for each time interval  $t$ ,  $t = 1, \dots, R$  within the replenishment period. Obviously, widening the range of possible values of demand  $D_t$ ,  $t = 1, \dots, R$  widens the range of possible values of demand  $DR$ . It should be also mentioned that in the case when the replenishment period includes more than one unit time interval, i.e.,  $R > 1$ , there are more possible values of demand  $DR$  than that of each individual demand  $D_t$ ,  $t = 1, \dots, R$ ; this means that uncertainty in demand estimation for a replenishment period accumulates with an increase in the length of the replenishment period. This is in accordance with common sense that the larger the time period for which demand is estimated, the more uncertain the estimation.

More uncertainty in demand  $DR$  also causes more uncertainty in the fuzzy total cost,  $FR(S)$ , incurred during the replenishment period. Widening the range of possible values of demand  $DR$  causes more uncertainty in  $FR(S)$ , i.e., an increase of both the number of possible cost values and their range. Of course, changing fuzzy total cost  $FR(S)$  leads to a different defuzzified value, and consequently, to a different optimal order-up-to level  $S$ .

Next, effects of changing the range of fuzzy customer demand on SC behavior and its performance are analysed. Larger range of possible customer demand values causes higher dynamic variability in stock levels of the inventories along the chain, and in orders placed from one to the preceding facility. These variations create undesirable situations: a high stock level will often be kept at an inventory in the SC when the order from the succeeding facility is small, and demand for a large quantity of items is received when the inventory stock level is low.

Using fully decentralised control, each inventory in the SC except the raw material inventory, is affected by variations in ordering from the succeeding facility and variations in delivery from the preceding facility, caused by changing the range of possible demand values. However, using partially coordinated control, uncertainty in delivery from one to the succeeding facility and its variations are taken into account. In this way, the effects of changing the range of possible demand values are diminished.

#### Simulation 7.4.

The purpose of this simulation is to show that increased uncertainty in customer demand causes a larger deterioration of SC performance.

SC 1 with characteristics specified in Example 1 in Section 5.3.1. under decentralised control is considered. Customer demand is estimated to be *about* 10 end-products per week. Different degrees of uncertainty are associated with this estimation, reflecting on the range of the possible demand values clustered around value 10. Three test cases are defined:

Case 1. It is estimated that customer demand in time interval  $t$ ,  $t = 1, \dots, 52$  can have 13 possible integer values in the range from 4 to 16 and it is modelled by a discretised time stationary fuzzy set

$$D_t = \{0/3, 0.25/4, 0.25/5, 0.25/6, 0.25/7, 0.5/8, 0.75/9, 1/10, 0.75/11, 0.5/12, 0.25/13, 0.25/14, 0.25/15, 0.25/16, 0/17\}, t = 1, \dots, 52.$$

Case 2. Customer demand in time interval  $t$ ,  $t = 1, \dots, 52$  has a smaller number of possible values than in Case 1; customer demand can take 7 values in the range from 7 to 13 and it is modelled by a discretised time stationary fuzzy set

$$D_t = \{0/6, 0.25/7, 0.5/8, 0.75/9, 1/10, 0.75/11, 0.5/12, 0.25/13, 0/14\},$$

$$t = 1, \dots, 52, \text{ and}$$

Case 3. Customer demand can have only 3 values (9, 10 and 11), with the following possibilities:

$$D_t = \{0/8, 0.75/9, 1/10, 0.75/11, 0/12\}, t = 1, \dots, 52.$$

The possibility distributions of the fuzzy sets which represent customer demands defined in the three cases are illustrated in Figure 7.6. (a).

Customer demands in the three cases have different degrees of uncertainty measured by their non-specificities. Using the formula (3.53), the non-specificities  $U(D_t)$ ,  $t = 1, \dots, 52$  in customer demands are:

$$\begin{aligned} \text{Case 1. } U(D_t) &= \int_0^1 \log_2 |D_t| d\alpha \\ &= \int_0^{0.25} \log_2 13 d\alpha + \int_{0.25}^{0.5} \log_2 5 d\alpha + \int_{0.5}^{0.75} \log_2 3 d\alpha + \int_{0.75}^1 \log_2 1 d\alpha = 1.90 \end{aligned}$$

$$\begin{aligned} \text{Case 2. } U(D_t) &= \int_0^1 \log_2 |D_t| d\alpha \\ &= \int_0^{0.25} \log_2 7 d\alpha + \int_{0.25}^{0.5} \log_2 5 d\alpha + \int_{0.5}^{0.75} \log_2 3 d\alpha + \int_{0.75}^1 \log_2 1 d\alpha = 1.68 \end{aligned}$$

$$\begin{aligned} \text{Case 3. } U(D_t) &= \int_0^1 \log_2 |D_t| d\alpha \\ &= \int_0^{0.75} \log_2 3 d\alpha + \int_{0.75}^1 \log_2 1 d\alpha = 1.19 \end{aligned}$$

The fuzzy set in Case 1 have the highest degree of non-specificity due to the widest range of the possible demand values.

Next, how non-specificity of customer demand during unit time interval reflects on non-specificity of demand calculated for a replenishment period is examined.



Demands  $DR_k^n$  imposed on inventory  $n$ ,  $n = 1, \dots, 5$ , during replenishment periods  $k = 1, \dots, 13$ , within the time horizon of 52 weeks are time stationary and, therefore, the index  $k$  for the replenishment periods can be omitted. Demands  $DR^n$ ,  $n = 1, \dots, 5$ , are the same, i.e.,  $DR^1 = DR^2 = \dots = DR^5$ . Using formula (5.1),  $DR^n$ ,  $n = 1, \dots, 5$  for each of the three cases are calculated as follows:

Case 1.  $DR^n = \{0/15, 0.25/16-31, 0.5/32-35, 0.75/36-39, 1/40, 0.75/41-44, 0.5/45-48, 0.25/49-64, 0/65\}$ ,  $n = 1, \dots, 5$ ,

Case 2.  $DR^n = \{0/27, 0.25/28-31, 0.5/32-35, 0.75/36-39, 1/40, 0.75/41-44, 0.5/45-48, 0.25/49-52, 0/53\}$ ,  $n = 1, \dots, 5$ ,

Case 3.  $DR^n = \{0/35, 0.75/36-39, 1/40, 0.75/41-44, 0/45\}$ ,  $n = 1, \dots, 5$ .

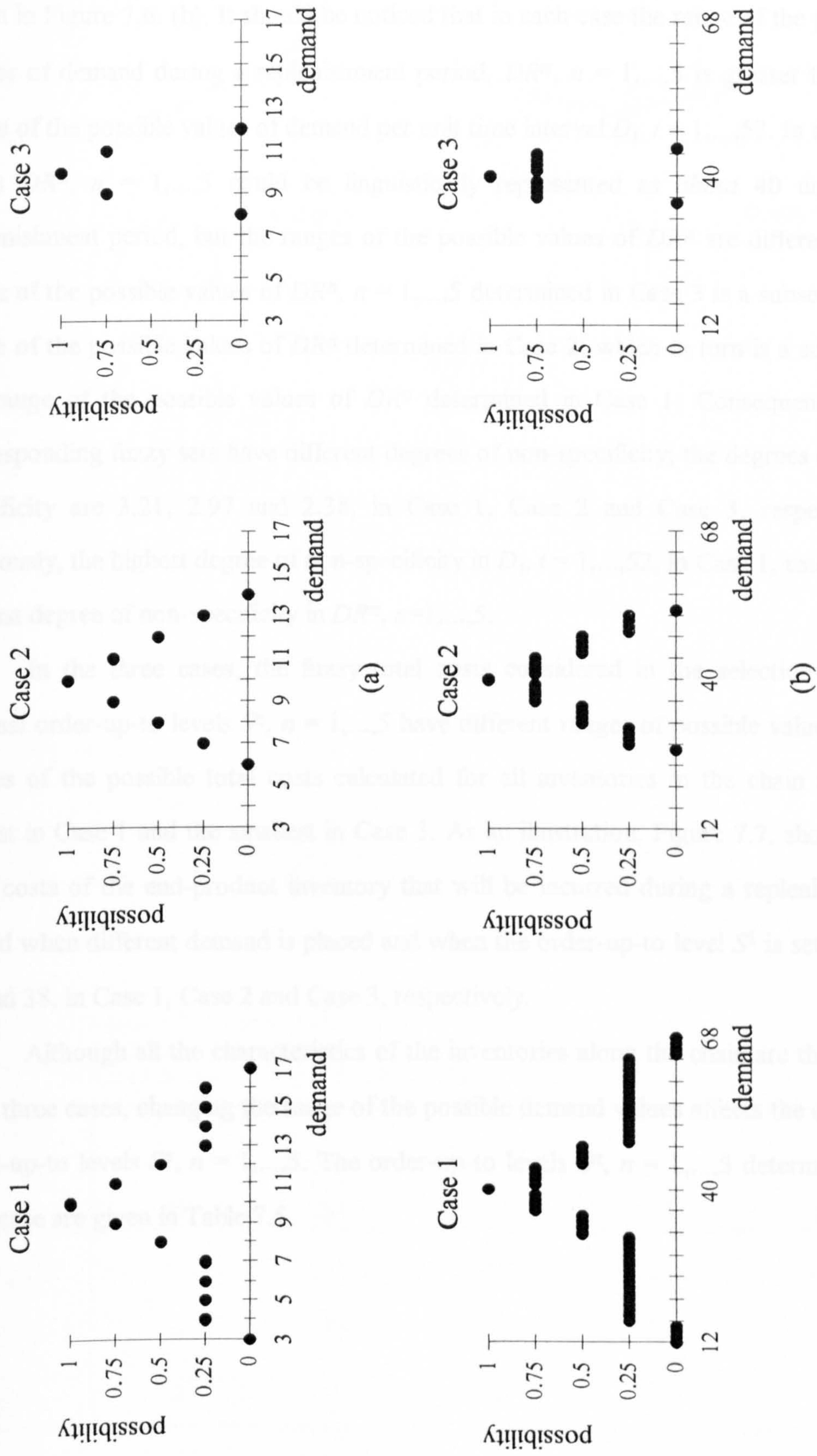


Figure 7.6. (a) Fuzzy sets which represent customer demands that have different ranges of the possible values  
(b) Fuzzy sets which represent demands during a replenishment period that have different ranges of the possible values

The possibility distributions of  $DR^n$ ,  $n = 1, \dots, 5$  in the three cases examined are given in Figure 7.6. (b). It should be noticed that in each case the range of the possible values of demand during a replenishment period,  $DR^n$ ,  $n = 1, \dots, 5$  is greater than the range of the possible values of demand per unit time interval  $D_t$ ,  $t = 1, \dots, 52$ . In all three cases  $DR^n$ ,  $n = 1, \dots, 5$  could be linguistically represented as *about* 40 units per replenishment period, but the ranges of the possible values of  $DR^n$  are different. The range of the possible values of  $DR^n$ ,  $n = 1, \dots, 5$  determined in Case 3 is a subset of the range of the possible values of  $DR^n$  determined in Case 2, which in turn is a subset of the range of the possible values of  $DR^n$  determined in Case 1. Consequently, the corresponding fuzzy sets have different degrees of non-specificity; the degrees of non-specificity are 3.21, 2.97 and 2.38, in Case 1, Case 2 and Case 3, respectively. Obviously, the highest degree of non-specificity in  $D_t$ ,  $t = 1, \dots, 52$ , in Case 1, causes the highest degree of non-specificity in  $DR^n$ ,  $n=1, \dots, 5$ .

In the three cases, the fuzzy total costs considered in the selection of the optimal order-up-to levels  $S^n$ ,  $n = 1, \dots, 5$  have different ranges of possible values. The ranges of the possible total costs calculated for all inventories in the chain are the largest in Case 1 and the smallest in Case 3. As an illustration, Figure 7.7. shows the total costs of the end-product inventory that will be incurred during a replenishment period when different demand is placed and when the order-up-to level  $S^1$  is set to 35, 36 and 38, in Case 1, Case 2 and Case 3, respectively.

Although all the characteristics of the inventories along the chain are the same in all three cases, changing the range of the possible demand values affects the optimal order-up-to levels  $S^n$ ,  $n = 1, \dots, 5$ . The order-up-to levels  $S^n$ ,  $n = 1, \dots, 5$  determined in each case are given in Table 7.5.



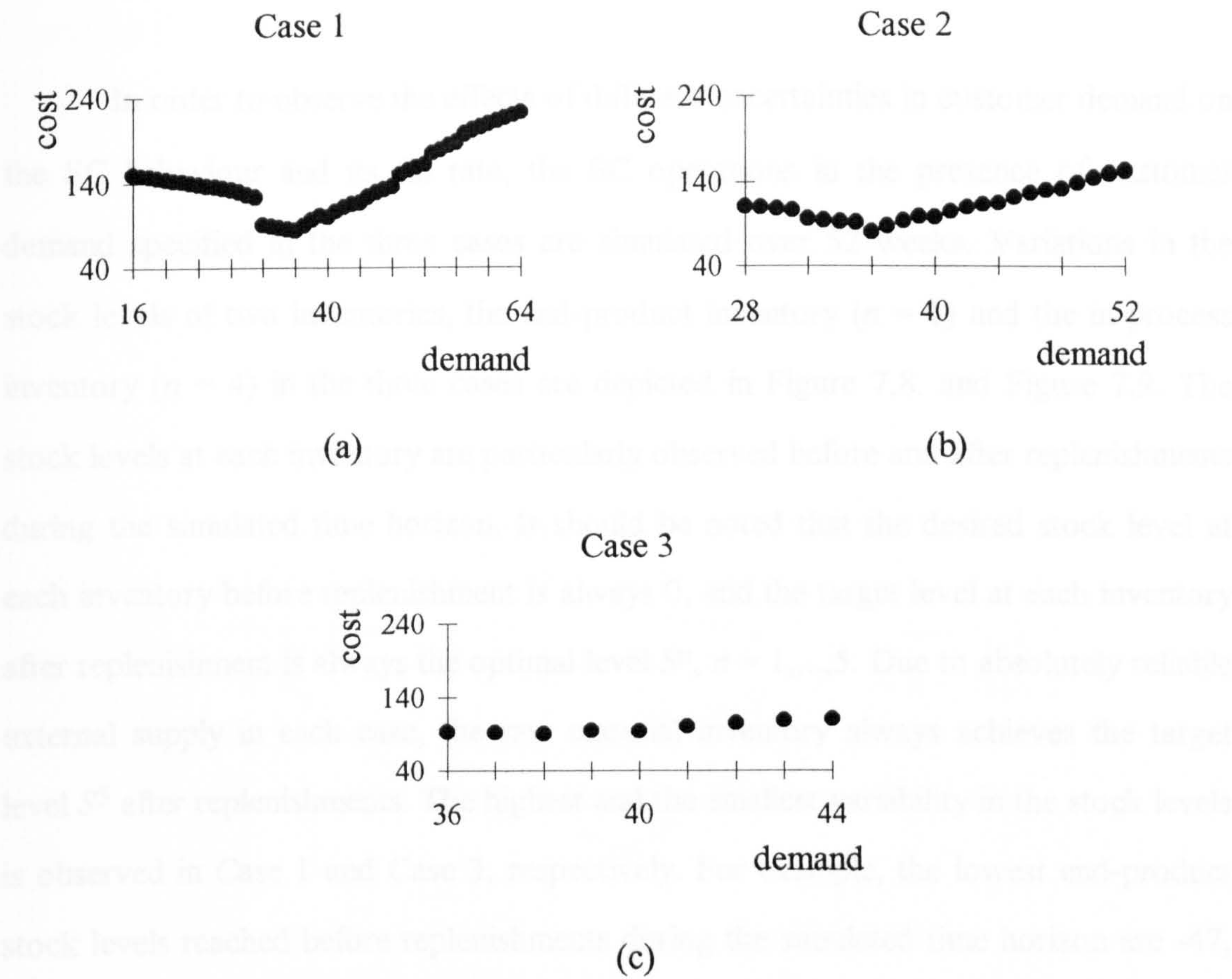


Figure 7.7. Possible total cost of the end-product inventory when the order-up-to level is (a)  $S^1 = 35$ , in Case 1, (b)  $S^1 = 36$ , in Case 2, and (c)  $S^1 = 38$ , in Case 3

Table 7.5. Optimal order-up-to levels for different ranges of the possible customer demand values

	order-up-to levels				
	$S^1$	$S^2$	$S^3$	$S^4$	$S^5$
Case 1	35	33	33	33	33
Case 2	36	36	36	36	36
Case 3	38	38	38	38	38

In order to observe the effects of different uncertainties in customer demand on the SC behaviour and its fill rate, the SC operations in the presence of customer demand specified in the three cases are simulated over 52 weeks. Variations in the stock levels of two inventories, the end-product inventory ( $n = 1$ ) and the in-process inventory ( $n = 4$ ) in the three cases are depicted in Figure 7.8. and Figure 7.9. The stock levels at each inventory are particularly observed before and after replenishments during the simulated time horizon. It should be noted that the desired stock level at each inventory before replenishment is always 0, and the target level at each inventory after replenishment is always the optimal level  $S^n$ ,  $n = 1, \dots, 5$ . Due to absolutely reliable external supply in each case, the raw material inventory always achieves the target level  $S^5$  after replenishments. The highest and the smallest variability in the stock levels is observed in Case 1 and Case 3, respectively. For example, the lowest end-product stock levels reached before replenishments during the simulated time horizon are -47, -27 and -11 in Case 1, Case 2 and Case 3, respectively. (The negative stock levels denote the existence of shortages). On the other hand, the stock levels at the end-product inventory after replenishments ranged from -2 to 33, from 14 to 37 and from 29 to 38 in the three cases, respectively. In each case it is observed that the variability in the stock levels amplifies along the chain, starting from the raw material inventory towards the end-product inventory.

Similar variations are observed in the orders placed from one to the preceding facility. First, let us explain the ordering processes along SC 1. The beginning of the review periods of all the inventories coincide. Due to the backorder stock-out policy applied at each inventory and absolutely reliable external supply that fills any order imposed from the raw material inventory, the quantities ordered from each facility  $n$ ,  $n = 1, \dots, 5$  at the beginning of each review period  $k$ ,  $k = 1, \dots, 13$  are the same. This means that the quantity ordered from facility 1 to facility 2 at the beginning of review period  $k$  is equal to the quantity ordered from facility 2 to facility 3 and so on.



Changing the uncertainty in customer demand in the three cases affects the orders placed along the chain. Figure 7.10. illustrates variations in the orders, simulated along the chain during 52 weeks. One can see that increasing uncertainty in customer demand by extending the range of its possible values amplifies variations in the orders.

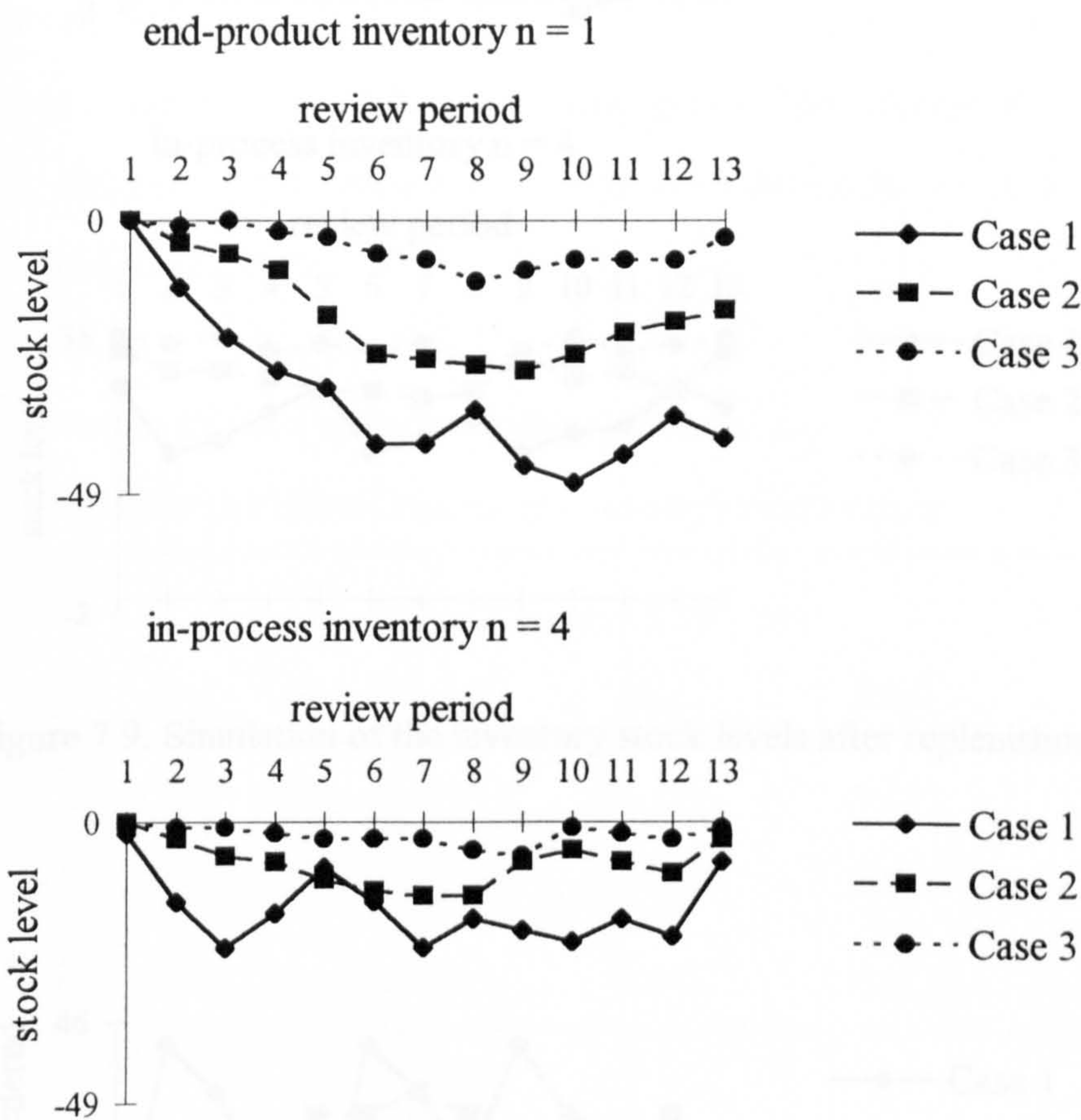


Figure 7.8. Simulation of the inventory stock levels before replenishments



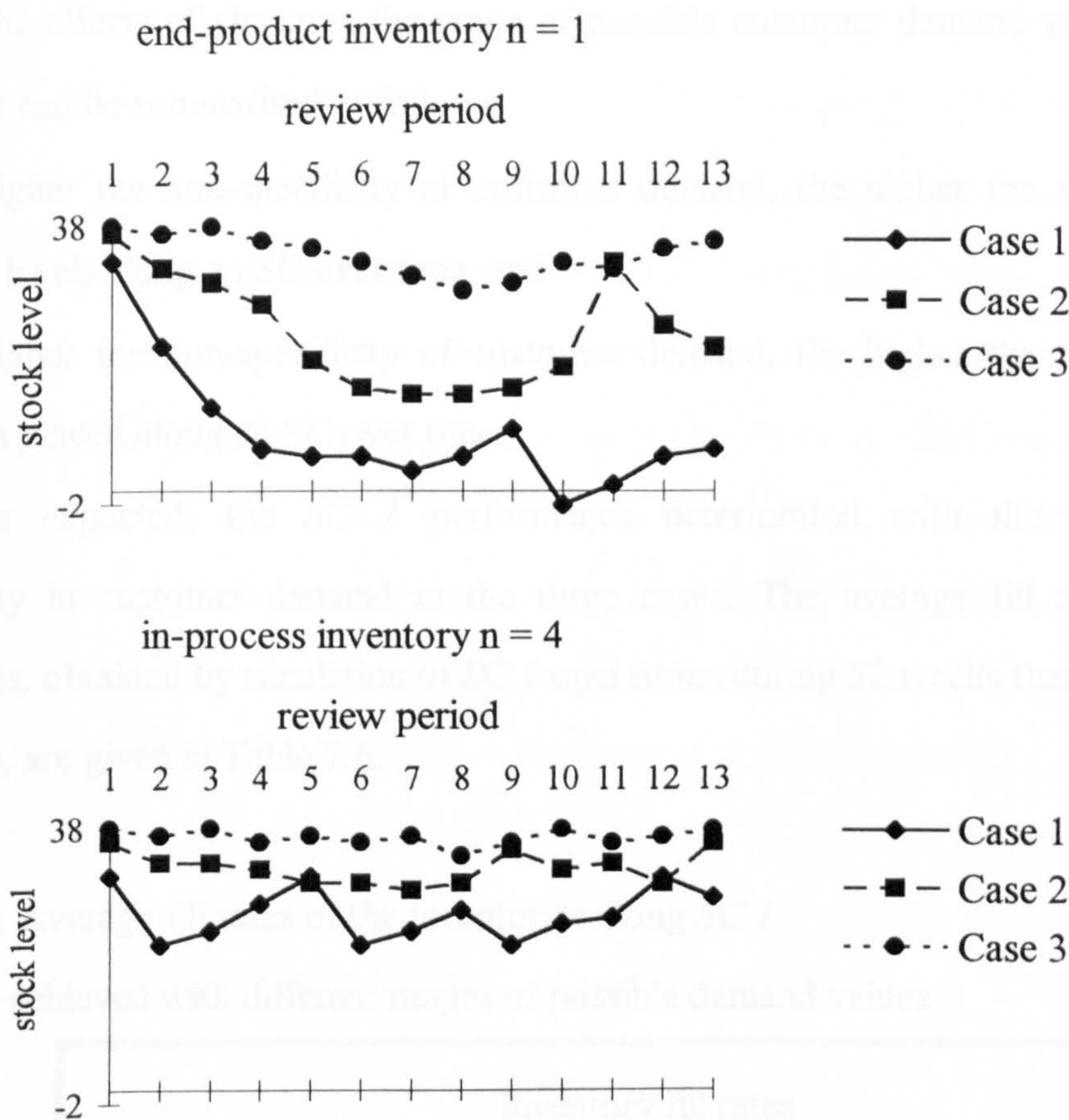


Figure 7.9. Simulation of the inventory stock levels after replenishments

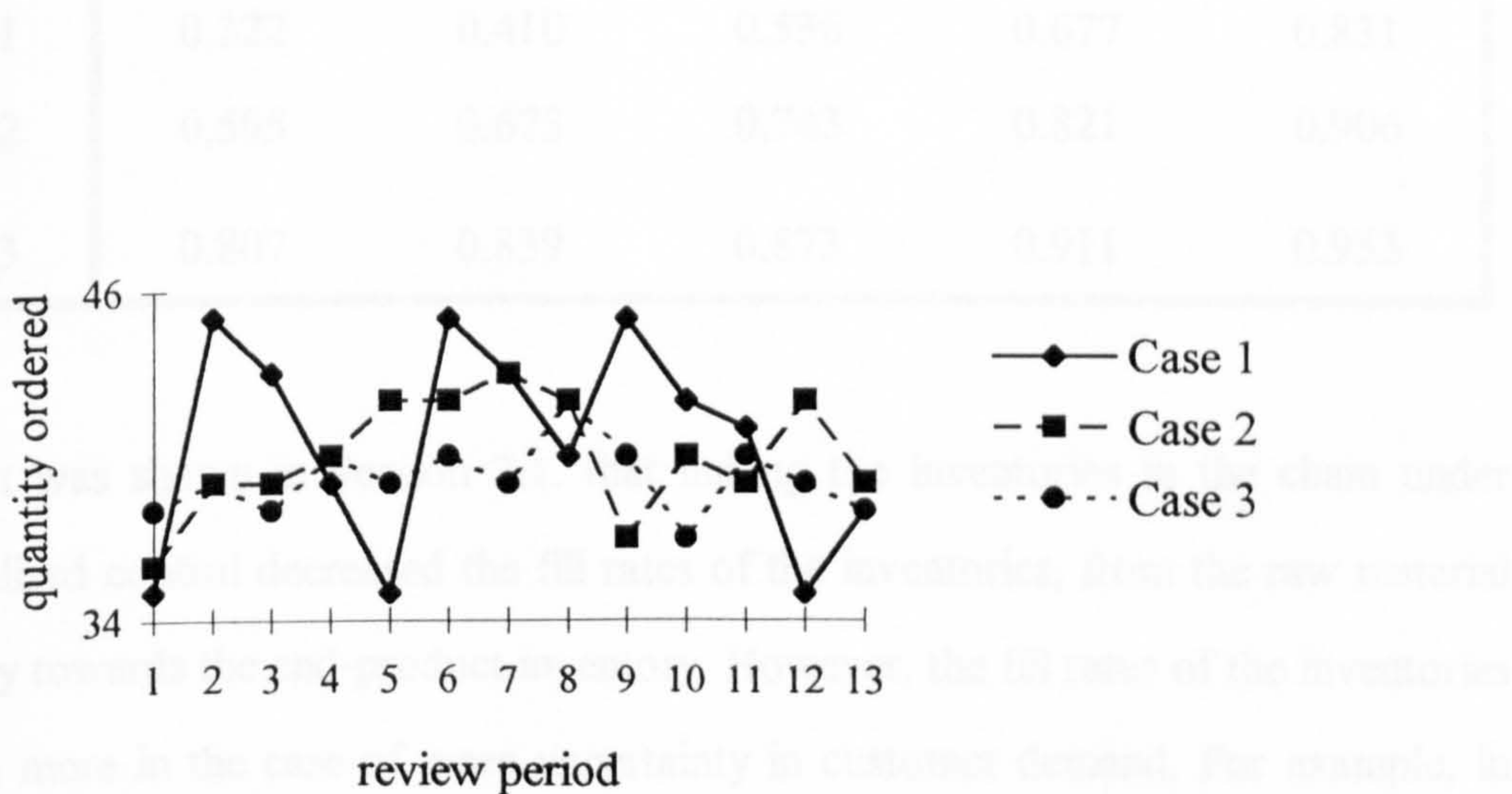


Figure 7.10. The influence of the range of the possible demand values on orders placed along SC 1

The effects of changing the range of possible customer demand values on SC behaviour can be summarised as follows:

- a) the higher the non-specificity of customer demand, the higher the variations in stock levels along an SC over time, and
- b) the higher the non-specificity of customer demand, the higher the variations in orders placed along an SC over time.

As expected, the SC 1 performance deteriorated with the increase in uncertainty in customer demand in the three cases. The average fill rates of the inventories, obtained by simulation of SC 1 operations during 52 weeks that is repeated 100 times, are given in Table 7.6.

Table 7.6. Average fill rates of the inventories along SC 1, achieved with different ranges of possible demand values

	inventory fill rates				
	$FR_{T_h}^1$	$FR_{T_h}^2$	$FR_{T_h}^3$	$FR_{T_h}^4$	$FR_{T_h}^5$
Case 1	0.322	0.410	0.536	0.677	0.831
Case 2	0.595	0.673	0.743	0.821	0.906
Case 3	0.807	0.839	0.873	0.911	0.953

It was shown in Section 7.1. that linking the inventories in the chain under decentralised control decreased the fill rates of the inventories, from the raw material inventory towards the end-product inventory. However, the fill rates of the inventories decrease more in the case of more uncertainty in customer demand. For example, in Case 1, the fill rate of the end-product inventory, (i.e.,the SC 1 fill rate) dropped by 61% in comparison with the fill rate of the raw material inventory, while in Case 2 and Case 3 the percentages are smaller, 34% and 15%, respectively.

In order to establish the relationship between non-specificity of customer demand and SC fill rate extensive simulation can be done using *SCBL* simulator. To illustrate this let us use chain *SC 1* with characteristics given in Example 1 in Section 5.3.1. *SC 1* operations were simulated when fuzzy customer demand estimation was *about* 10 end-products per each week, but non-specificity associated with the estimation in each simulation experiment was different. The non-specificities in customer demand were varied from 0, in the case when customer demand was precisely specified to be 10 end-products per week and modelled by fuzzy set  $D_t = \{1/10\}$ ,  $t = 1, \dots, 52$ , to 2.07, in the case when customer demand was modelled by fuzzy set  $D_t = \{0.25/0-7, 0.5/8, 0.75/9, 1/10, 0.75/11, 0.5/12, 0.25/13-20\}$ ,  $t = 1, \dots, 52$ . Based on the simulation results, which are illustrated in Figure 7.11., the following conclusion can be made: as the non-specificity in customer demand increases, the SC fill rate decreases.

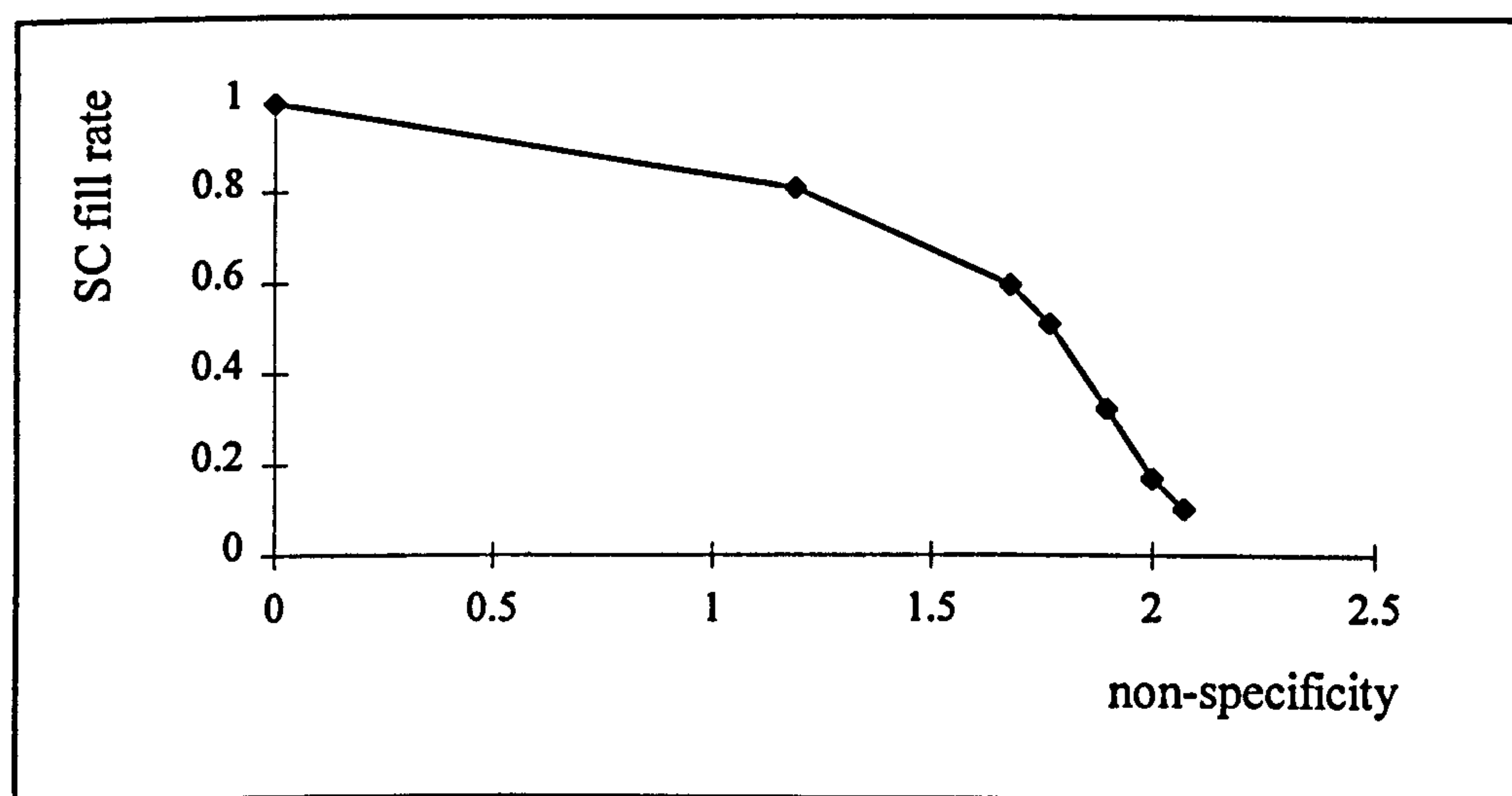


Figure 7.11. Non-specificity of customer demand vs SC fill rate



### 7.3.2. Effects of qualitative changes in a customer demand estimation

Fuzzy customer demands examined so far have been described by linguistic expressions, in the form "demand is *about*  $d$  products per unit time interval", and consequently, represented by triangular possibility distributions. However, customer demand may be more vaguely expressed by different linguistic terms and modelled by possibility distributions of various shapes. In this section, attention is focused on cases when customer demand is described by complex terms, such as "demand is *about*  $d$  products, but there is a high possibility of demand taking value  $d_i$ ". The corresponding fuzzy set contains values around value  $d$  and the particular value  $d_i$ .

As one would expect, the influence of a particular demand value  $d_i$  on optimal order-up-to levels of inventories along an SC depends on the possibility associated with the corresponding demand value. The higher the possibility of the particular demand value the stronger the impact on the selection of the optimal-order-up-to levels. In addition, the ratios of the unit shortage cost to the unit holding cost of the inventories determine the influence of the particular demand value. For example, if a particular demand value is much greater than other possible values and the unit costs ratio of an inventory in the SC is high, then such a judgment of customer demand causes greater increase in the optimal order-up-to level than in the cases when this ratio is small.

A general conclusion can be drawn that increasing the possibility of a particular demand value deteriorates performance of an SC. The degree of deterioration, however, depends on the unit cost ratios of the inventories along the SC. This is demonstrated by the following simulation results.

**Simulation 7.5.**

The purpose of this simulation is to show that if customer demand is very vaguely expressed, the degree of deterioration of SC performance can be very high.

In the first part of simulation *SC 1* is considered, when customer demand is *about* 10 products per week, but there is also a possibility of demand being 20 products per week. Five cases are examined:

Case 1. The possibility distribution of demand in week  $t$  had a simple triangular form;

$$D_t = \{0/6, 0.25/7, 0.5/8, 0.75/9, 1/10, 0.75/11, 0.5/12, 0.25/13, 0/14\},$$

$$t = 1, \dots, 52.$$

Case 2. The possibility distribution of demand is the same as in Case 1, except for a possibility 0.25 that 20 products per week may be demanded;

$$D_t = \{0/6, 0.25/7, 0.5/8, 0.75/9, 1/10, 0.75/11, 0.5/12, 0.25/13, 0/14,$$

$$0.25/20\}, t = 1, \dots, 52.$$

Case 3. Similar to Case 2, but the possibility of demand being 20 products per week is 0.5;

$$D_t = \{0/6, 0.25/7, 0.5/8, 0.75/9, 1/10, 0.75/11, 0.5/12, 0.25/13, 0/14, 0.5/20\},$$

$$t = 1, \dots, 52.$$

Case 4. Similar to Case 2, but the possibility of demand being 20 products per week is 0.75;

$$D_t = \{0/6, 0.25/7, 0.5/8, 0.75/9, 1/10, 0.75/11, 0.5/12, 0.25/13, 0/14, 0.75/20\},$$

$$t = 1, \dots, 52,$$

Case 5. Similar to Case 2, but the possibility of demand being 20 products per week is 1;

$$D_t = \{0/6, 0.25/7, 0.5/8, 0.75/9, 1/10, 0.75/11, 0.5/12, 0.25/13, 0/14, 1/20\},$$

$$t = 1, \dots, 52.$$

Table 7.7. gives the order-up-to levels of the inventories, calculated for the five cases. The order-up-to levels of the inventories along *SC 1* become higher with increase in possibility of the higher demand value per week.

Table 7.7. The order-up-to levels of the inventories along *SC 1*,  
for different possibilities of customer demand taking a higher value

	order-up-to levels				
	$S^1$	$S^2$	$S^3$	$S^4$	$S^5$
Case 1	36	36	36	36	36
Case 2	39	38	38	38	38
Case 3	41	39	39	39	39
Case 4	42	40	40	40	40
Case 5	43	41	41	41	41

In the second part of simulation, the optimal order-up-to levels of all the inventories along *SC 2*, with higher unit cost ratios of the inventories described in Simulation 7.2., are calculated for the five cases of fuzzy customer demands (see Table 7.8.). They are compared with the corresponding order-up-to levels of the inventories along *SC 1*. It is demonstrated that the high particular demand value has greater influence on the order-up-to levels of the inventories along *SC 2*, due to higher unit costs ratio of each inventory than in *SC 1*. By increasing the possibility of customer demand taking value 20, from 0 to 0.25, 0.5, 0.75 and 1, the optimal order-up-to levels of the inventories along *SC 2* are increased by 16%, 30%, 35% and 37%. The corresponding increases in the optimal order-up-to levels of the inventories along *SC 1* are smaller (6%, 8%, 11% and 14%, respectively). By simulating *SC 1* and *SC 2* operations, effects similar to those discussed in the previous section are observed, namely, increased possibility of customer demand for 20 products causes higher



dynamic variations in stock levels and orders placed along the chains. However, the *SC 2* fill rate is less affected by changing the possibility of demand for 20 products than the *SC 1* fill rate, but at the expense of higher inventory cost per end-product demanded.

Table 7.8. The order-up-to levels of the inventories along *SC 2*,  
for different possibilities of customer demand taking a high value

	order-up-to levels				
	$S^1$	$S^2$	$S^3$	$S^4$	$S^5$
Case 1	43	43	43	43	43
Case 2	50	50	50	50	50
Case 3	57	56	56	56	56
Case 4	58	58	58	58	58
Case 5	59	59	59	59	59

#### 7.4. Sensitivity analysis

Modelling of uncertain parameters using fuzzy sets generally relies on one's subjective opinion, human belief or accrued experience. Similarly, a fuzzy set that represents uncertain customer demand can be derived from a manager's subjective judgement. Since specifying a possibility distribution of fuzzy customer demand is subjective in nature, it is, therefore, appropriate to examine sensitivity of optimal order-up-to levels of inventories along an SC and sensitivity of SC performance to small changes of input data, such as estimation of possibilities of customer demand values.

Generally, possibilities associated with customer demand values are important in the selection of the optimal order-up-to level of an inventory. Let us refer to the

fuzzy model for finding the optimal order-up-to level of an inventory for one review period in the presence of fuzzy customer demand, described in Section 5.2.1.1. In the fuzzy model proposed, the possibility distributions of customer demand in unit time intervals determine the possibility distributions of the shortage cost and the holding cost incurred at the inventory during a replenishment period, and affect the selection of the optimal order-up-to level of the inventory. It is supposed that small changes in the possibilities of customer demand values cause small changes in the possibilities of the shortage cost and the holding cost. The defuzzification method that transforms the fuzzy shortage cost and the fuzzy holding cost into scalar values reacts to small changes in the possibilities of the costs. However, searching for the optimal order-up-to level of the inventory is performed over a discretised finite set of levels, and consequently, the selection of the optimal order-up-to level is not sensitive to small changes in possibilities of fuzzy customer demand.

The fact that the optimal inventory order-up-to level is not sensitive to small changes in possibilities of customer demand values gives more credibility to the fuzzy model used.

In addition, it can be shown using *SCBL* simulator that small changes in the possibilities of fuzzy customer demand cause small changes in SC behaviour and its performance. This is illustrated in Simulation 7.6.

### **Simulation 7.6.**

The purpose of this simulation is to demonstrate that small changes in customer demand estimation does not generate substantial changes in the SC performance, namely, SC fill rate and SC holding cost.

Consider *SC 1* with characteristics given in Section 5.3.1. Three cases with different fuzzy estimations of customer demand placed on *SC 1* are considered.

Case 1. Customer demand is *about* 10 products per week  $t$ ,  $t = 1, \dots, 52$ ; it is modelled by a triangular possibility distribution

$$D_t = \{0/6, 0.25/7, 0.5/8, 0.75/9, 1/10, 0.75/11, 0.5/12, 0.25/13, 0/14\},$$

$$t = 1, \dots, 52.$$

Case 2. Customer demand is similar to that in Case 1, but with 5% higher possibilities of demand taking values greater than 10 products per week; the possibility distribution of customer demand is

$$D_t = \{0/6, 0.25/7, 0.5/8, 0.75/9, 1/10, 0.79/11, 0.52/12, 0.26/13, 0/14\},$$

$$t = 1, \dots, 52.$$

Case 3. The possibilities of customer demand taking the values less than or equal to 10 products per week are the same as in Case 1, and the possibilities of the values greater than 10 products per week are 10% higher than in Case 1; the possibility distribution of customer demand is

$$D_t = \{0/6, 0.25/7, 0.5/8, 0.75/9, 1/10, 0.82/11, 0.55/12, 0.27/13, 0/14\},$$

$$t = 1, \dots, 52.$$

The possibility distributions of customer demand specified in the three cases are graphically represented in Figure 7.12.

The small differences in specifying the possibilities of the customer demand values in the three cases cause only small differences between their degrees of non-specificity. The non-specificities of customer demands are 1.68, 1.73 and 1.77 for Case 1, Case 2 and Case 3, respectively.

The changes in the possibility distributions of fuzzy customer demand does not alter the optimal order-up-to levels of the inventories. They are  $S^1 = 36$ ,  $S^2 = 36$ ,  $S^3 = 36$ ,  $S^4 = 36$ ,  $S^5 = 36$ , in all three cases.



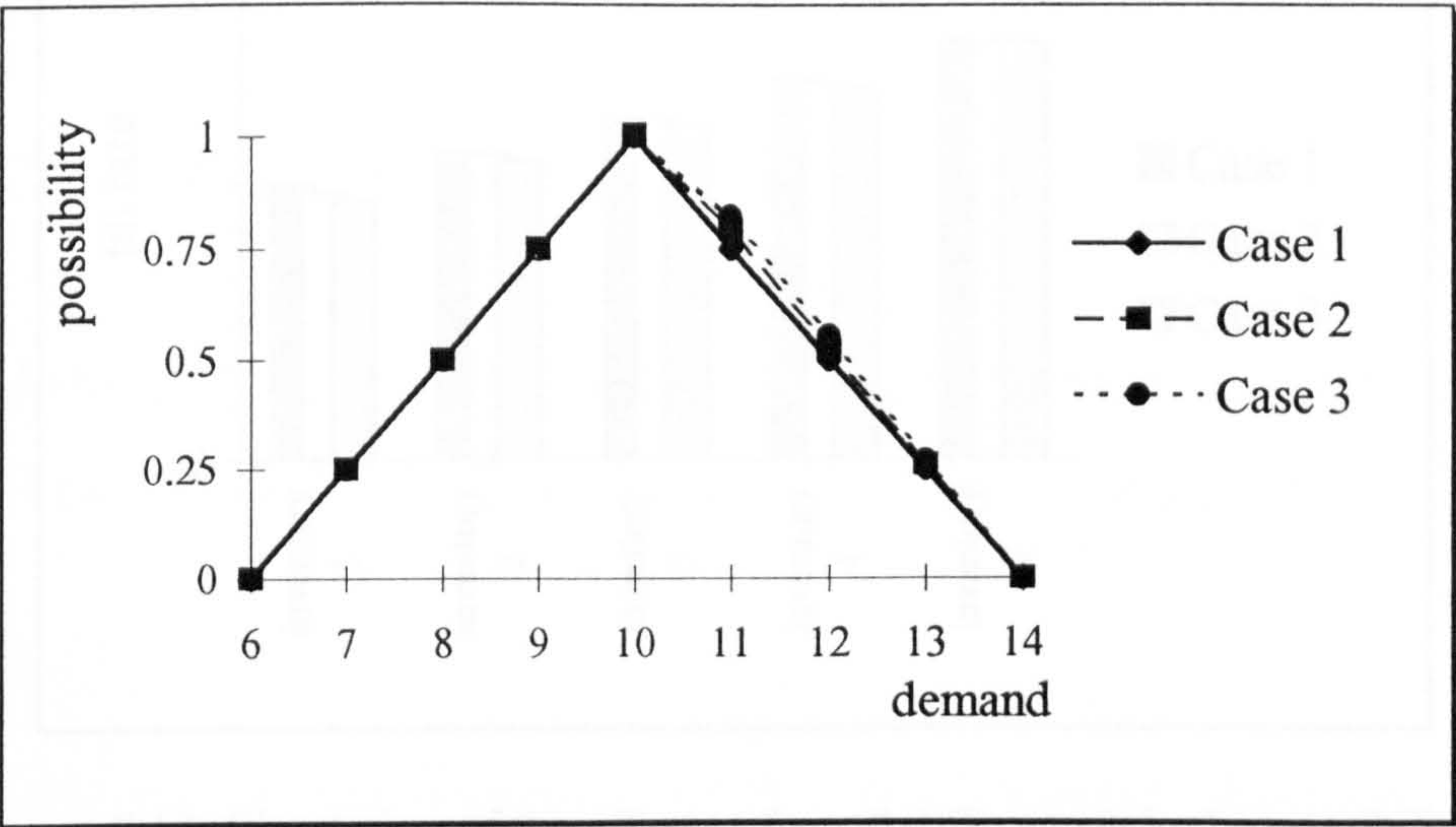


Figure 7.12. Three possibility distributions associated with customer demand

*SC I* performance when customer demand is modelled by the three possibility distributions is simulated over 52 weeks and simulation is repeated 100 times. The average fill rates achieved at each of the inventories are graphically compared in Figure 7.13. Figure 7.14. shows the average holding cost for each inventory per item demanded. The *SC I* performance achieved in the three cases support the important conclusion that *SC I* is not sensitive to small changes in the possibilities of the customer demand values.

Figure 7.14. Simulation of the effects of small differences in the possibilities of the customer demand values on the average inventory holding cost per item demanded



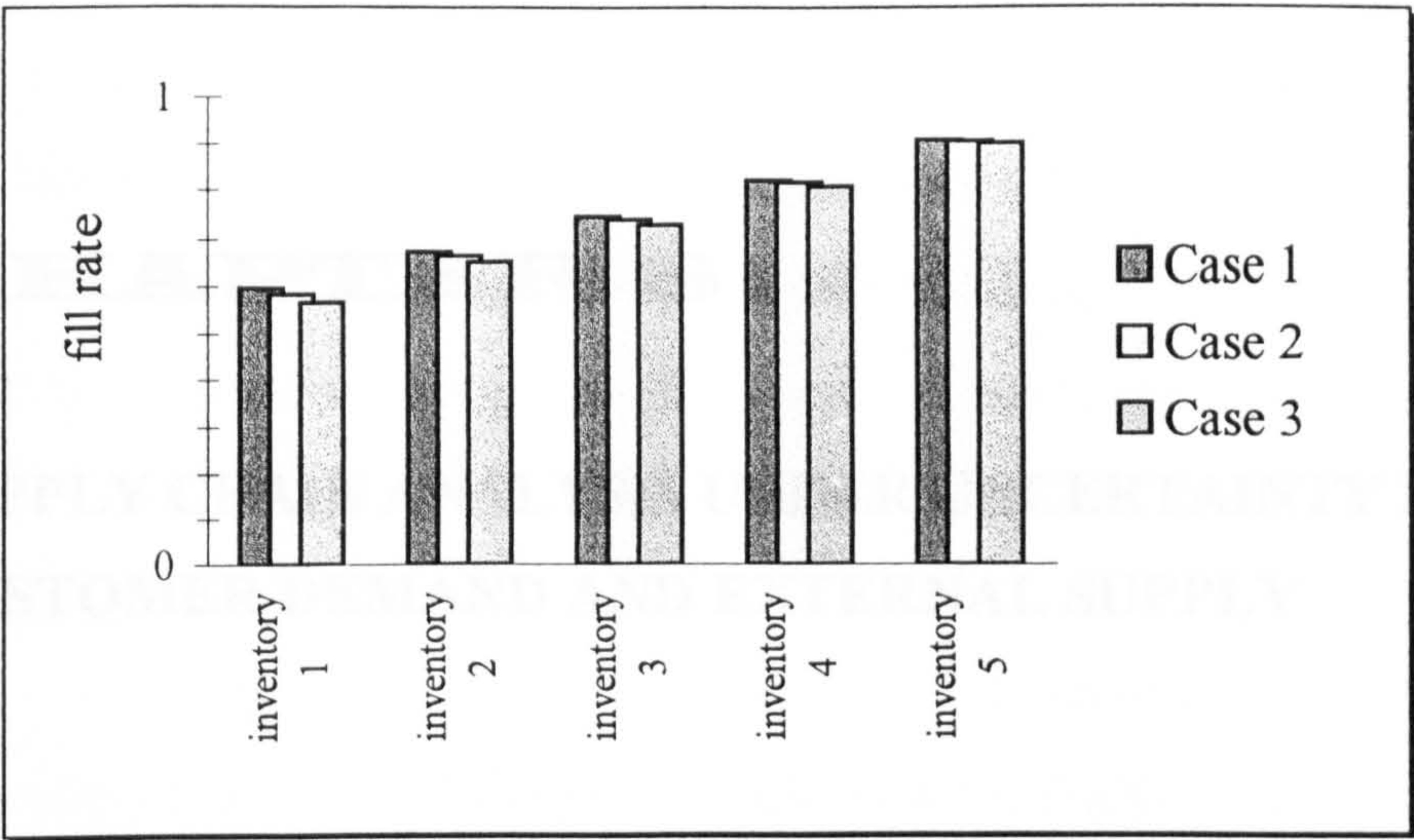


Figure 7.13. Simulation of the effects of small differences in the possibilities of the customer demand values on the average inventory fill rates

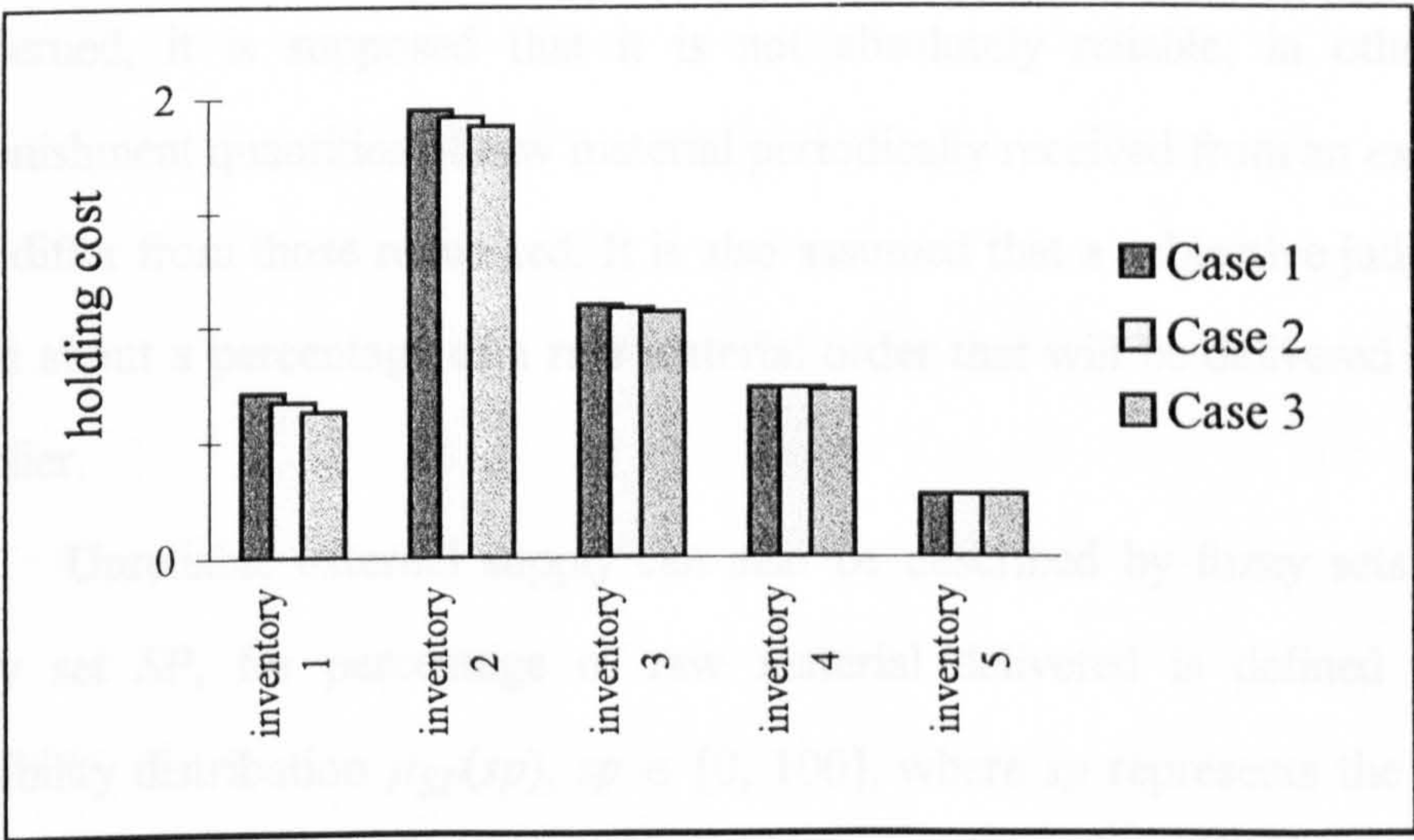


Figure 7.14. Simulation of the effects of small differences in the possibilities of the customer demand values on the average inventory holding cost per item demanded

## CHAPTER 8

### SUPPLY CHAIN ANALYSIS UNDER UNCERTAINTY IN CUSTOMER DEMAND AND EXTERNAL SUPPLY

In this chapter the application of *SCSUPP* simulator, described previously in Section 6.2.2., to analyse behaviour of a serial SC in the presence of uncertainties in both customer demand and external supply is demonstrated. It is assumed that the SC operates in the same fashion and that uncertainty in customer demand is described in the same manner as it is done in Chapter 7. As far as external supply of raw material is concerned, it is supposed that it is not absolutely reliable; in other words, the replenishment quantities of raw material periodically received from an external supplier may differ from those requested. It is also assumed that a subjective judgement can be given about a percentage of a raw material order that will be delivered by an external supplier.

Unreliable external supply can also be described by fuzzy sets. The discrete fuzzy set  $SP$ , for percentage of raw material delivered is defined by a discrete possibility distribution  $\mu_{SP}(sp)$ ,  $sp \in [0, 100]$ , where  $sp$  represents the percentage of raw material order delivered by the external supplier and  $\mu_{SP}(sp)$  is the associated possibility.

In Section 8.1, results of the analysis of the effects of unreliable external supply on SC behaviour and its performance using *SCSUPP* simulator are presented. The analysis has been performed to investigate SC characteristics that make the SC less vulnerable to unreliable external supply.



The author has proposed and analysed two new procedures for compensating the negative effects of unreliable external supply. These procedures, developed for the two SC control strategies: (1) fully decentralised control and (2) partially coordinated control, are presented in Sections 8.2. and 8.3. respectively. The results of this work has been accepted for publication in *European Journal of Operational Research* (Petrovic, Roy, Petrovic).

Examples of the effects of changing the estimation of external supplier reliability using *SCSUPP* simulator are shown in Section 8.4.

### 8.1. Effects of uncertainty in external supply

As one can expect, external supplier uncertainty is a serious threat to successful SC operation. Due to the unreliable external supplier, the raw material inventory does not reach the target stock level after replenishments. As a consequence, lower stocks are held downstream, i.e., from the raw material inventory towards the end-product inventory. The effects of the uncertain external supplier on the fill rates of the inventories are spread along the whole SC. The fill rates of the inventories are decreased in comparison with the case when the external supplier is absolutely reliable.

The intensity of influence of an uncertain external supplier on the end-product inventory fill rate (i.e., on the SC fill rate) depends on the fill rates of inventories along an SC achieved with absolutely reliable supply. A rather obvious conclusion can be made that the higher the fill rates of inventories in an SC are, the lower is the vulnerability of the SC to external supply uncertainty.

A series of simulations was performed to support this conclusion. The simulated SC had the characteristics given in Example 1, in Section 5.3.1. The decentralised control strategy was applied. Increasing the ratio of the unit shortage cost to the unit holding cost was used as a mechanism for increasing the fill rate of

each inventory in the chain. The cost ratios of all the inventories in the chain under consideration were equal and they were increased from one to another simulation, providing higher fill rates of the inventories. In all the simulations, SC performance was observed when an external supplier was estimated to be *moderately* reliable. It was assumed that the external supplier could deliver 80% of the raw material ordered in each review period with possibility 1, 90% with possibility 0.5 and the whole order, i.e., 100% with possibility 0.25. In each simulation, the SC fill rate achieved with the *moderately* reliable supplier was compared with the SC fill rate obtained when the supplier was absolutely reliable. Figure 8.1. shows how deterioration of the *SC 1* fill rate, caused by the *moderately* reliable external supplier, can be reduced by increasing the cost ratio of the inventories. The SC fill rate achieved with the *moderately* reliable external supplier was compared to the SC fill rate with the absolutely reliable supplier and the corresponding percentage decrease is presented in the diagram. Obviously, the SC under consideration became less vulnerable to unreliable external supply by increasing the cost ratio of the inventories, i.e., by providing higher fill rates of the inventories.

As far as the inventory cost is concerned, generally, unreliable supply produces a lower holding cost of an inventory in an SC, but shortage cost is increased.

Some interesting effects of uncertain external supply on SC performance are demonstrated in Simulation 8.1. and Simulation 8.2.

### **Simulation 8.1.**

The purpose of Simulation 8.1. is to show that uncertainty in external supply deteriorates the performance of an SC, in particular SC fill rate and the total SC cost per end-product demanded.

Consider *SC 1* specified in Example 1, Section 5.3.1., but with a *moderately* reliable external supplier. Let the *moderately* reliable external supplier mean that, in



this specific context, the supplier can deliver 80%, 90% or 100% of the raw material ordered in each review period with possibilities: 1, 0.5 and 0.25, respectively.

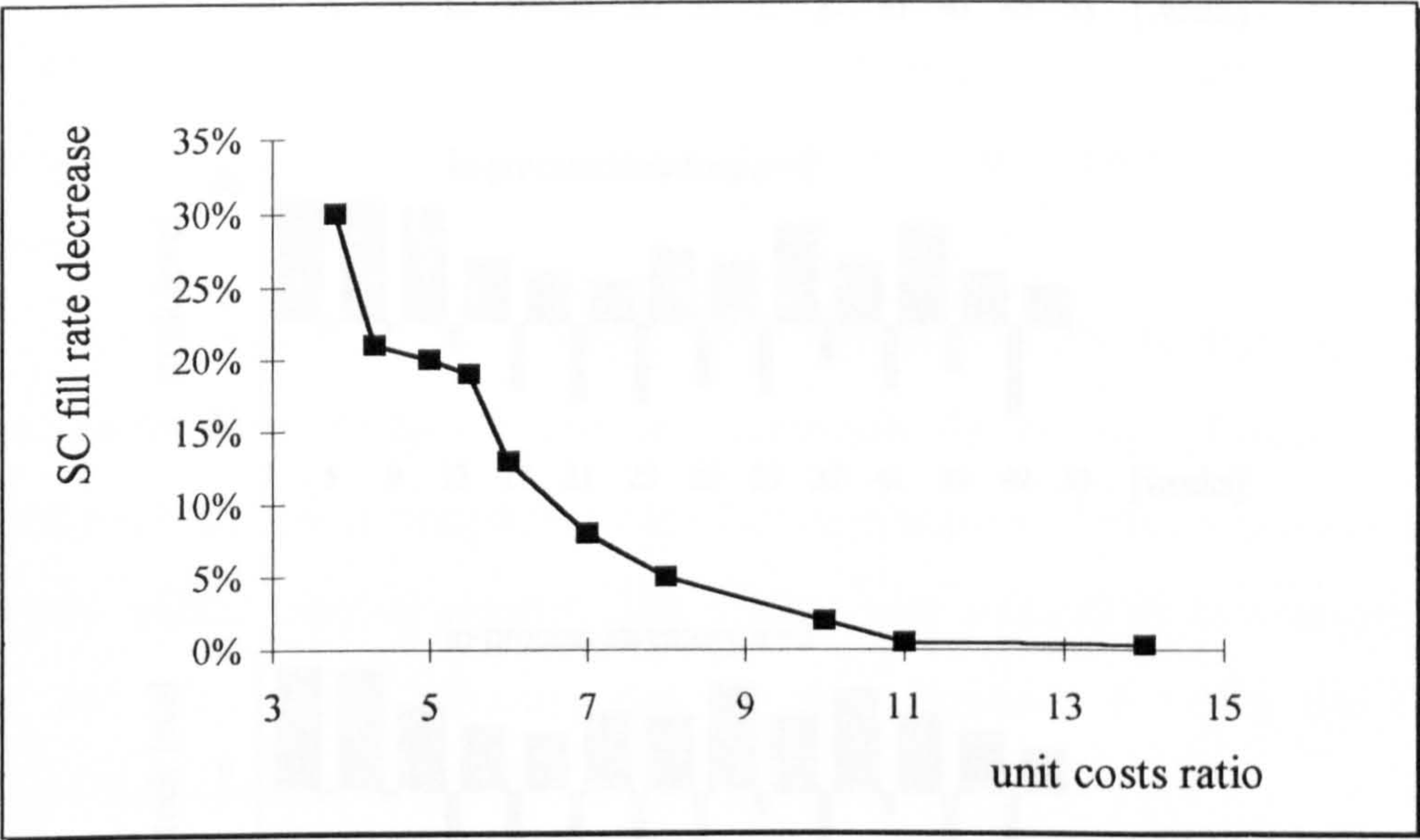


Figure 8.1. Deterioration of the SC fill rate vs cost ratio of the inventories when there is uncertainty in external supply

The order-up-to levels determined in the decentralised manner are the same:  $S_d^1=36, S_d^2=36, S_d^3=36, S_d^4=36, S_d^5=36$ .

The stock levels during the finite time horizon of 52 weeks are depicted in Figure 8.2. One can see that due to the *moderately* reliable external supplier, the raw material inventory does not reach the target stock level after replenishments. As a consequence, lower stocks are held in the downstream part of SC 1, in comparison with the case when external supplier is absolutely reliable (see Figure 7.2.).



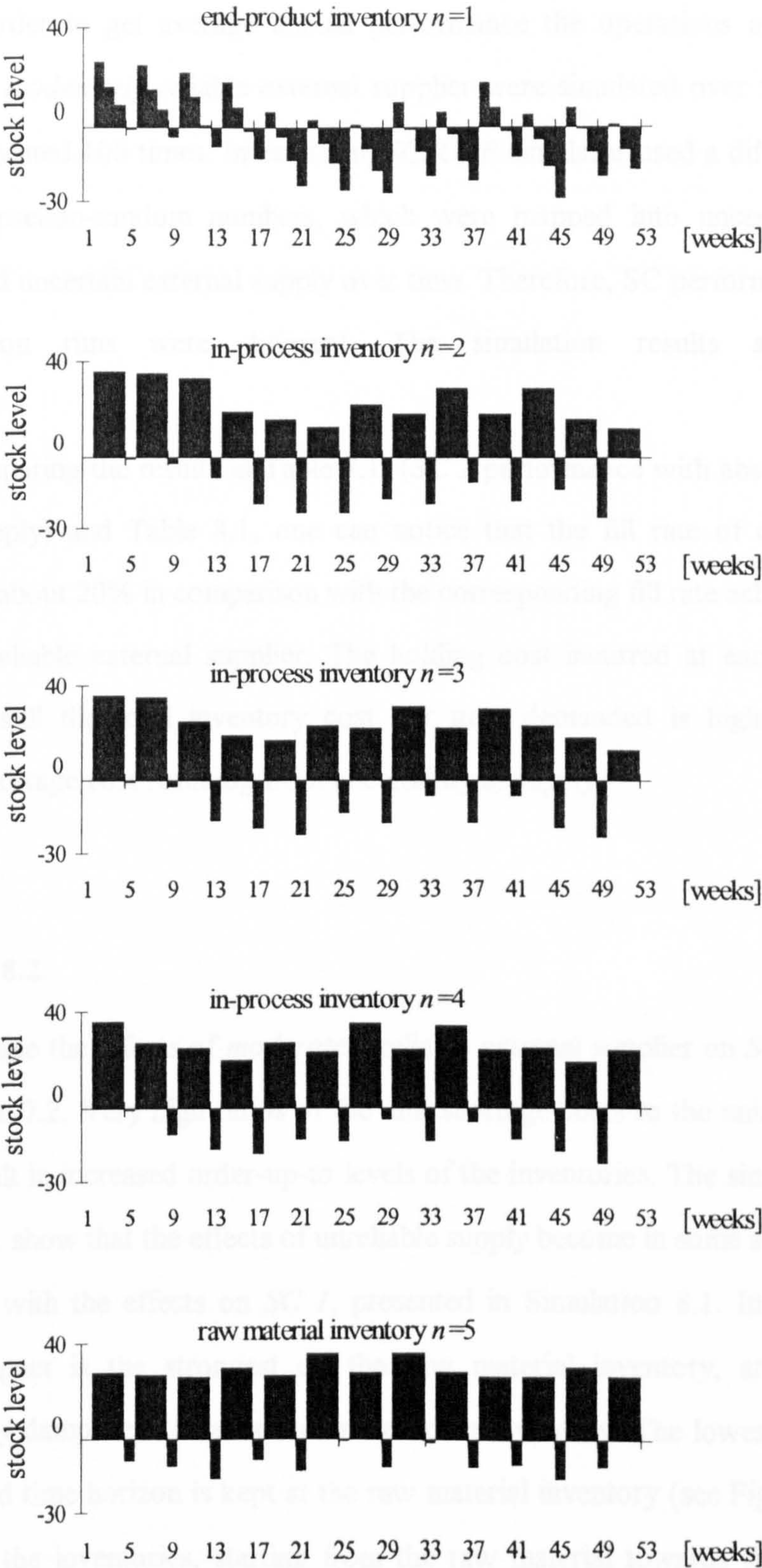


Figure 8.2. Simulation of stock levels along SC 1 under decentralised control when customer demand and external supply are uncertain

In order to get average annual performance the operations of *SC 1* in the presence of *moderately* reliable external supplier were simulated over 52 weeks with each run repeated 100 times. In each run, *SCSUPP* simulator used a different seed for generating pseudo-random numbers, which were mapped into uncertain customer demands and uncertain external supply over time. Therefore, SC performance achieved in simulation runs were different. The simulation results are given in Table 8.1.

Comparing the results in Table 7.1. (*SC 1* performance with absolutely reliable external supply) and Table 8.1, one can notice that the fill rate of each inventory dropped by about 20% in comparison with the corresponding fill rate achieved with the absolutely reliable external supplier. The holding cost incurred at each inventory is lower, but still the total inventory cost per item demanded is higher due to the increased shortage cost resulting from uncertainty in supply.

### Simulation 8.2.

Let us examine the effects of *moderately* reliable external supplier on *SC 2*, described in Simulation 7.2. Very high ratios of the unit shortage costs to the unit holding costs in *SC 2* result in increased order-up-to levels of the inventories. The simulation results in Table 8.2. show that the effects of unreliable supply become in some sense weaker in comparison with the effects on *SC 1*, presented in Simulation 8.1. In this case, the negative impact is the strongest on the raw material inventory, and it becomes progressively dampened towards the end-product inventory. The lowest stock during the simulated time horizon is kept at the raw material inventory (see Figure 8.3.). The fill rates of the inventories, starting from the raw material towards the end-product inventory were decreased by 11%, 6%, 3%, 1% and 1%, in comparison with the fill rates obtained in the presence of the absolutely reliable external supplier. It means that *moderately* reliable supply of raw material changes the *SC 2* fill rate by 1% only.



Table 8.1. Average SC 1 performance under decentralised control,  
when customer demand and external supply are uncertain

Average SC 1 performance					
total cost $F_{T_h}$	3663.72				
total cost/item $FD_{T_h}$	7.06				
holding cost $FH_{T_h}$	1872.71				
holding cost/item $FHD_{T_h}$	3.62				
fill rate $FR_{T_h}$	0.470				
Average performance of each inventory in SC 1					
inventory $n$	1	2	3	4	5
order-up-to level $S_d^n$	36	36	36	36	36
total cost $F_{T_h}^n$	2051.13	1788.93	884.18	521.93	170.58
total cost/item $FD_{T_h}^n$	3.95	3.48	1.72	1.01	0.33
holding cost $FH_{T_h}^n$	260.12	776.92	434.74	292.07	108.87
holding cost/item $FHD_{T_h}^n$	0.50	1.51	0.85	0.57	0.21
fill rate $FR_{T_h}^n$	0.470	0.544	0.595	0.654	0.721



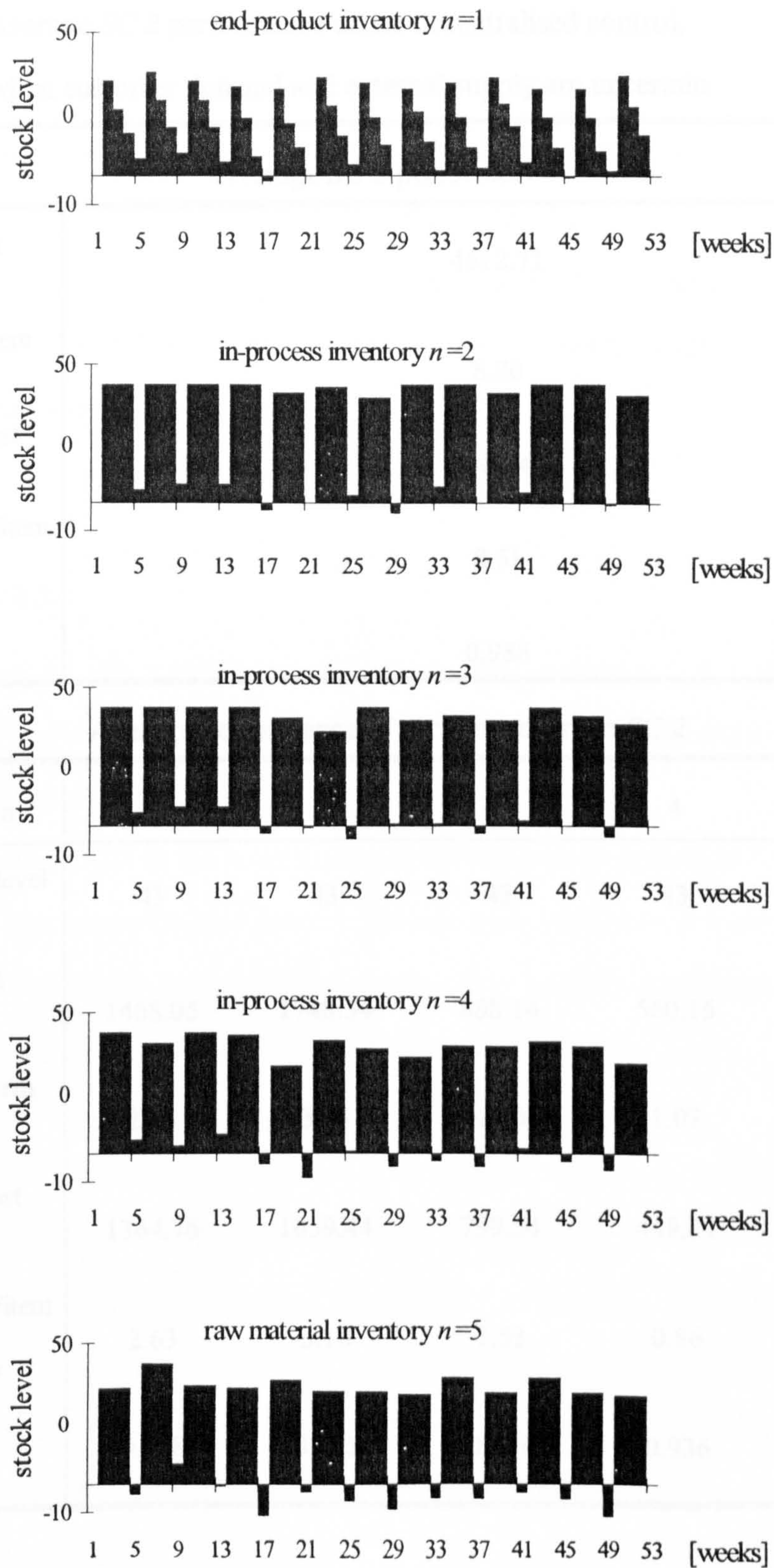


Figure 8.3. Simulation of stock levels along SC 2 under decentralised control when customer demand and external supply are uncertain

Table 8.2. Average SC 2 performance under decentralised control,  
when customer demand and external supply are uncertain

Average SC 2 performance					
total cost $F_{T_h}$	4512.71				
total cost/item $FD_{T_h}$	8.70				
holding cost $FH_{T_h}$	4409.42				
holding cost/item $FHD_{T_h}$	8.51				
fill rate $FR_{T_h}$	0.988				
Average performance of each inventory in SC 2					
inventory $n$	1	2	3	4	5
order-up-to level $S_d^n$	43	43	43	43	43
total cost $F_{T_h}^n$	1468.05	1748.54	888.14	560.15	206.88
total cost/item $FD_{T_h}^n$	2.83	3.35	1.70	1.07	0.40
holding cost $FH_{T_h}^n$	1364.76	1659.44	799.54	449.24	136.44
holding cost/item $FHD_{T_h}^n$	2.63	3.18	1.53	0.86	0.26
fill rate $FR_{T_h}^n$	0.988	0.985	0.970	0.936	0.878

Let us now analyse effects of uncertainty in external supply on a partially coordinated SC. Order-up-to levels of inventories along the partially coordinated SC are higher or at least equal to the corresponding order-up-to levels determined in the



decentralised way. Consequently, the fill rates of all the inventories achieved along the partially coordinated SC with absolutely reliable external supply are higher than in the case of the corresponding decentrally controlled SC. Therefore, it would be reasonable to suppose that although an uncertain external supplier adversely affects performance of a partially coordinated SC, its negative influence on the fill rates of the inventories and the overall SC fill rate becomes smaller. This is clearly illustrated by Simulation 8.3. and Simulation 8.4.

### Simulation 8.3.

The purpose of this simulation is to show that uncertainty in external supplier deteriorates performance of a partially coordinated SC, but the negative effects are smaller than in the case when decentralised control is applied.

Consider *SC 1* in the same uncertain environment described in Simulation 8.1., but under partially coordinated control. The uncertain environment includes both fuzzy customer demand and fuzzy external supply. The partially coordinated order-up-to levels of the inventories along the *SC 1* are:

$$S_p^1 = 40, S_p^2 = 40, S_p^3 = 39, S_p^4 = 41, S_p^5 = 36.$$

The results of the simulation of *SC 1* operations given in Table 8.3. show that the fill rates of the inventories along the chain, starting from the raw material inventory towards the end-product inventory, dropped by 21%, 18%, 16%, 14% and 13%, respectively, in comparison with the fill rates achieved with absolutely reliable external supply. The uncertain external supplier decreased the *SC 1* fill rate by 13%. Note that the *SC 1* fill rate dropped by 21% in the same uncertain environment under decentralised control, (see Table 8.1.).



Table 8.3. Average SC 1 performance under partial coordination control, when customer demand and external supply are uncertain

Average SC 1 performance					
total cost $F_{T_h}$	3664.17				
total cost/item $FD_{T_h}$	7.00				
holding cost $FH_{T_h}$	2900.18				
holding cost/item $FHD_{T_h}$	5.58				
fill rate $FR_{T_h}$	0.781				
Average performance of each inventory in SC 1					
inventory $n$	1	2	3	4	5
order-up-to level $S_d^n$	40	40	39	41	36
total cost $F_{T_h}^n$	1443.44	1682.66	849.35	508.80	173.14
total cost/item $FD_{T_h}^n$	2.77	3.23	1.63	0.98	0.33
holding cost $FH_{T_h}^n$	699.45	1174.27	571.61	345.89	108.95
holding cost/item $FHD_{T_h}^n$	1.35	2.26	1.10	0.66	0.21
fill rate $FR_{T_h}^n$	0.781	0.774	0.753	0.758	0.714

#### Simulation 8.4.

The purpose of this simulation is to show that in the case when SC operates under partially coordinated control and has high inventory fill rates, unreliability of external supplier does not produce significant negative effects.

Consider now SC 2 described in Simulation 8.2., but under partially coordinated control. Note that SC 2 operates in the same uncertain environment as SC 1, examined in the previous simulation experiments.

Due to the high cost ratio of all inventories in SC 2 which produced the high order-up-to levels, ( $S_p^1 = 45$ ,  $S_p^2 = 45$ ,  $S_p^3 = 45$ ,  $S_p^4 = 46$ ,  $S_p^5 = 43$ ), the negative effects of the *moderately reliable* external supplier are gradually reduced, from the raw material inventory to all the downstream inventories. The simulation results in Table 8.4. indicate that this influence of uncertainty in external supply is the strongest on the raw material inventory, reducing its fill rate by 11%. However, this effect completely disappears at the end-product inventory; the fill rate of the end-product inventory, i.e., the SC 2 fill rate is the same as the SC 2 fill rate recorded in the presence of absolutely reliable supply of raw material (Table 7.4.).

From the cost point of view, an interesting result is worth noticing. Comparing the results given in Table 8.4. and Table 7.4., one can see that the total SC 2 cost per end-product demanded is smaller when external supply was uncertain than in the case when external supply is absolutely reliable. This can be explained in the following way. Uncertain raw material supply reduces the stocks held at the raw material inventory and decreases the SC 2 holding cost. On the other hand, the stocks held along SC 2 are still sufficient to prevent deterioration of the fill rate of the end-product inventory due to the high ratios of the unit shortage to the unit holding costs. Hence, the shortage cost charged on the end-product inventory is not increased. The lower holding cost of the inventories along SC 2 and unchanged shortage cost of the end-product inventory result in lower SC 2 cost per end-product demanded.

Table 8.4. Average SC 2 performance under partial coordination control,  
when customer demand and external supply are uncertain

Average SC 2 performance					
total cost $F_{T_h}$	4860.30				
total cost/item $FD_{T_h}$	9.35				
holding cost $FH_{T_h}$	4840.17				
holding cost/item $FHD_{T_h}$	9.31				
fill rate $FR_{T_h}$	0.988				
Average performance of each inventory in SC 2					
inventory $n$	1	2	3	4	5
order-up-to level $S_d^n$	45	45	45	46	43
total cost $F_{T_h}^n$	1567.10	1809.01	899.63	542.30	211.57
total cost/item $FD_{T_h}^n$	3.01	3.44	1.71	1.03	0.40
holding cost $FH_{T_h}^n$	1546.97	1797.90	875.05	484.32	135.94
holding cost/item $FHD_{T_h}^n$	2.98	3.42	1.67	0.92	0.26
fill rate $FR_{T_h}^n$	0.998	0.998	0.992	0.967	0.870



## 8.2. Compensation of uncertainty in external supply under decentralised SC control strategy

The negative effects of an unreliable external supplier on fill rates of inventories and on SC fill rate can be compensated by increasing the stock levels of the inventories in the SC. Suppose that management objective is to achieve the same SC fill rate as in the case of an absolutely reliable external supplier. Then, the order-up-to levels of some stock points in the SC have to be increased.

Two original approaches are examined:

- one-site compensation, where the stock at only one inventory in the SC is increased,
- multi-site compensation, where stock levels at more than one inventory along the SC are increased; it is done iteratively in such a way as to ensure that the increment of the SC fill rate per cost incurred is the largest.

Both approaches require a system view of the SC as a whole. They are based on the combined use of an SC generative model and an SC evaluative model. First, the order-up-to levels of the inventories through the SC are computed using one of the fuzzy SC models and the effects of selected order-up-to levels on the SC performance are evaluated using *SCSUPP* simulator. Then, the order-up-to levels of the inventories along the SC are increased using the generative model for one-site or multi-site compensation and the SC performance are evaluated using *SCSUPP*, and so on. The generative and the evaluative models are used in an iterative way, until the SC fill rate obtained by the evaluative model reaches the SC fill rate achieved under absolutely reliable supply.

Our hypothesis is that generally, the multi-site approach has certain advantages over one-site compensation. However, it is computationally more extensive and the number of simulation runs to be performed in each iteration increase (linearly) with an increasing number of inventories in an SC.

### 8.2.1. One-site compensation

One-site compensation for uncertain external supply allows increasing the order-up-to levels of only one inventory in an SC. The algorithm for compensation at the selected inventory denoted by  $\bar{n}$  consists of two steps:

Step 1: The order-up-to levels  $S_{d,k\bar{n}}^{\bar{n}}$  for all the review periods indexed by  $k^{\bar{n}} = 1, \dots, K^{\bar{n}}$  during the time horizon  $T_h$  are increased by 1 unit. In other words, all the order-up-to levels of inventory  $\bar{n}$  during  $T_h$  are changed.

Step 2: The SC fill rate during  $T_h$  is evaluated with the new set of order-up-to levels, that includes  $S_{d,k\bar{n}}^{\bar{n}} + 1$ ,  $k^{\bar{n}} = 1, \dots, K^{\bar{n}}$  and  $S_{d,k^n}^n$ , which are unchanged for  $n = 1, \dots, N$ ,  $n \neq \bar{n}$ ,  $k^n = 1, \dots, K^n$ .

The two-step algorithm is repeated until the SC fill rate becomes approximately equal to the SC fill rate recorded under absolutely reliable supply. The new order-up-to levels are denoted by  $S_{do,k^n}^n$ ,  $n = 1, \dots, N$ ,  $k^n = 1, \dots, K^n$ .

The pseudo-code of the algorithm is given in Appendix 2.

Increasing order-up-to levels at one point only seems to be interesting if it is performed at: (a) a raw material inventory or (b) an end-product inventory. The consequences are the following:

(a) Increasing the raw material order-up-to levels has an impact on all downstream inventories. The raw material inventory fill rate becomes higher causing increased fill rates of all the inventories along the SC. The costs of the inventories are also changed, starting from the raw material inventory through the whole chain to the end-product inventory. The holding cost of each inventory is increased, while the shortage cost of each inventory is decreased.

(b) Increasing the end-product order-up-to levels changes only the end-product inventory performance. Its fill rate becomes higher, the holding cost is increased, but its shortage cost is decreased.

The question could be raised: is it reasonable to look for the most appropriate location in an SC to increase order-up-to levels so as to minimise the costs incurred? The most cost effective location in the SC to increase its order-up-to levels depends on the ratios of the unit shortage cost to the unit holding cost of the inventories. In the case of higher ratios along the whole SC and higher order-up-to levels, consequently, the effects of uncertain external supply are weaker. In this case a smaller increment in order-up-to levels of the most appropriate inventory is sufficient for compensation.

A number of simulations is performed to show that it is possible to compensate for the unreliable external supplier by increasing either the raw material order-up-to levels or the end-product order-up-to levels, with an ultimate objective to achieve the SC fill rate approximately equal to the one obtained when external supply was absolutely reliable. Some simulation results are given in Simulation 8.5.

### **Simulation 8.5.**

The purpose of Simulation 8.5. is to demonstrate that unreliable external supply can be compensated by substantially increasing either case (a), the raw material order-up-to level, or case (b), the end-product order-up-to level.

Consider *SC 1* with the *moderately* reliable external supplier. The objective of compensation is to achieve the SC fill rate approximately equal to 0.6, the same value obtained when external supply was absolutely reliable. Simulation results in Tables 8.5. and 8.6. show that it is necessary to increase either order-up-to level of raw material by 28% , in case (a), or order-up-to level of end-product by 25%, in case (b).

Comparing the results in Tables 8.5. and 8.6., one can see that compensation applied at the end-product inventory is more cost effective. It incurs lower SC total



cost and SC holding cost per end-product demanded, and, also, lower holding cost of each inventory per item demanded.

In addition it is demonstrated that moderate reliability of the external supplier, in the case when an SC operates with higher fill rates, can be compensated by smaller increase either in the order-up-to levels of raw material or the order-up-to levels of the end-product.

In *SC 2* where high ratios of the unit shortage costs to the unit holding costs of all the inventories ensure high fill rates along the chain, it is enough to increase the order-up-to level of raw material stock level by 5% only, or the order-up-to level of the end-product by 2% only, to achieve the *SC 2* fill rate recorded in the case of absolutely reliable external supply (Table 7.2.). The *SC 2* performance and performance of each individual inventory in two cases: case (a), increased order-up-to level of raw material and case (b), increased order-up-to level of the end-product, are given in Tables 8.7. and 8.8, respectively.

Table 8.5. Case (a), average *SC 1* performance under decentralised control,  
when compensation for uncertainty in external supply is applied  
at the raw material inventory

Average SC 1 performance					
total cost $F_{T_h}$	3877.78				
total cost/item $FD_{T_h}$	7.43				
holding cost $FH_{T_h}$	2518.57				
holding cost/item $FHD_{T_h}$	4.84				
fill rate $FR_{T_h}$	0.600				
Average performance of each inventory in SC 1					
inventory $n$	1	2	3	4	5
order-up-to level $S_d^n$	36	36	36	36	46
total cost $F_{T_h}^n$	1731.97	1731.89	847.54	495.64	166.44
total cost/item $FD_{T_h}^n$	3.31	3.34	1.64	0.96	0.32
holding cost $FH_{T_h}^n$	372.75	1027.98	577.97	388.90	150.96
holding cost/item $FHD_{T_h}^n$	0.72	1.99	1.12	0.75	0.29
fill rate $FR_{T_h}^n$	0.600	0.685	0.759	0.841	0.931

Table 8.6. Case (b), average SC 1 performance under decentralised control,  
when compensation for uncertainty in external supply is applied  
at the end-product inventory

Average SC 1 performance					
total cost $F_{T_h}$	3190.90				
total cost/item $FD_{T_h}$	6.12				
holding cost $FH_{T_h}$	1824.08				
holding cost/item $FHD_{T_h}$	3.51				
fill rate $FR_{T_h}$	0.598				
Average performance of each individual inventory in SC 1					
inventory $n$	1	2	3	4	5
order-up-to level $S_d^n$	45	36	36	36	36
total cost $F_{T_h}^n$	1735.67	1887.57	924.49	542.02	176.14
total cost/item $FD_{T_h}^n$	3.32	3.58	1.76	1.03	0.33
holding cost $FH_{T_h}^n$	368.85	668.73	397.55	280.19	108.77
holding cost/item $FHD_{T_h}^n$	0.71	1.27	0.76	0.53	0.21
fill rate $FR_{T_h}^n$	0.598	0.463	0.535	0.615	0.703



Table 8.7. Case (a), average SC 2 performance under decentralised control,  
when compensation for uncertainty in external supply is applied  
at the raw material inventory

Average SC 2 performance					
total cost $F_{T_h}$	4598.94				
total cost/item $FD_{T_h}$	8.86				
holding cost $FH_{T_h}$	4534.26				
holding cost/item $FHD_{T_h}$	8.74				
fill rate $FR_{T_h}$	0.993				
Average performance of each inventory in SC 2					
inventory $n$	1	2	3	4	5
order-up-to level $S_d^n$	43	43	43	43	46
total cost $F_{T_h}^n$	1450.26	1732.11	859.48	529.37	187.08
total cost/item $FD_{T_h}^n$	2.79	3.32	1.66	1.01	0.36
holding cost $FH_{T_h}^n$	1385.58	1689.76	830.43	478.78	149.71
holding cost/item $FHD_{T_h}^n$	2.67	3.24	1.59	0.92	0.29
fill rate $FR_{T_h}^n$	0.993	0.993	0.987	0.971	0.935

Table 8.8. Case (b), average SC 2 performance under decentralised control,  
when compensation for uncertainty in external supply is applied  
at the end-product inventory

Average SC 2 performance					
total cost $F_{T_h}$	4565.17				
total cost/item $FD_{T_h}$	8.78				
holding cost $FH_{T_h}$	4506.76				
holding cost/item $FHD_{T_h}$	8.67				
fill rate $FR_{T_h}$	0.993				
Average performance of each inventory in SC 2					
inventory $n$	1	2	3	4	5
order-up-to level $S_d^n$	45	43	43	43	43
total cost $F_{T_h}^n$	1543.56	1773.69	905.22	569.28	208.73
total cost/item $FD_{T_h}^n$	2.97	3.38	1.72	1.08	0.40
holding cost $FH_{T_h}^n$	1485.15	1643.34	793.46	447.77	137.04
holding cost/item $FHD_{T_h}^n$	2.86	3.13	1.51	0.85	0.26
fill rate $FR_{T_h}^n$	0.993	0.978	0.962	0.930	0.876

### 8.2.2. Multi-site compensation

Multi-site compensation for an uncertain external supplier involves all the inventories along an SC. It might be preferred to one-site compensation because it may offer better balance between the fill rates of the inventories along an SC. In addition, it can provide better SC performance with respect to the total SC cost than one-site compensation.

The problem arises as to selecting a particular inventory  $\bar{n}$  in each iteration for which it is the most reasonable to increase the order-up-to levels. Generally, determination of the best location in an SC for increasing the order-up-to levels at each iteration depends on many factors, among others on the ratio of the unit shortage cost to the unit holding cost of each inventory, the values of the unit holding costs along the SC and their mutual ratios, the position of each inventory in the SC, and of course, on the degree of uncertainty in external supply, etc. A variety of algorithms can be defined to select inventory  $\bar{n}$ , leading to different overall SC performance. One new original algorithm is proposed and analysed below.

The algorithm for multi-site compensation for uncertain external supply developed for this research has a similar structure as the algorithm for one-site compensation described in the previous section. It consists of three steps that are iteratively repeated until the desired SC fill rate is achieved. The only difference is that in this algorithm each inventory in an SC is considered in each iteration as a potential location for increasing the order-up-to levels by one unit for all the review periods. This means that in each iteration order-up-to levels of different inventories can be increased by one unit.

A potential location for stock increase is a particular inventory  $\bar{n}$  for which the highest increase in its fill rate will result from a unit increase in its order-up-to levels. Increasing the order-up-to levels of inventory  $\bar{n}$  changes its fill rate, but also the fill rates of all downstream inventories, with indices  $\bar{n}-1, \bar{n}-2, \dots, 1$ . However, the level of changes in the fill rates depends on two factors: (1) the fill rate increase achieved at



inventory  $\bar{n}$  by increasing its order-up-to levels by one unit; that is to say the higher the increase in fill rate of inventory  $\bar{n}$ , the stronger its effect on the fill rates of the downstream inventories and the SC fill rate, and (2) the location of inventory  $\bar{n}$  in the SC; the further the inventory  $\bar{n}$  from the end-product inventory, the weaker its influence on the SC fill rate. From the cost point of view, stock at different inventory locations along the SC has different holding costs; due to value-added processes along the SC, holding costs are getting higher, starting from the raw material towards the end-product inventory.

The algorithm proposed for selecting inventory  $\bar{n}$  in each iteration takes into account the SC fill rate obtained by increasing the order-up-to levels of inventory  $\bar{n}$ , the fill rate increase achieved at inventory  $\bar{n}$ , and the total holding cost of the whole SC, and makes a trade-off between these variables.

In each iteration, Step 1 and Step 2 are repeated for each inventory  $n$ ,  $n = 1, \dots, N$  and Step 3 is applied afterwards.

Step 1: The order-up-to levels  $S_{d,k^n}^n$  for all review periods  $k^n = 1, \dots, K^n$  of inventory  $n$  are increased by 1 unit.

Step 2: The following data are recorded using *SCSUPP* simulator:

- $FR_{Th}$  - the SC fill rate obtained with the new set of order-up-to levels of inventory  $n$ ,
- $\Delta FR_{Th}^n$  - the difference between the fill rates of inventory  $n$ , achieved before and after the unit order-up-to levels increase and
- $FHD_{Th}$  - the sum of the holding costs incurred at all inventories along the SC per end-product demanded.

Step 3: The inventory  $\bar{n}$  that has the maximum associated value  $MF^{\bar{n}} = FR_{Th} \cdot \Delta FR_{Th}^{\bar{n}} / FHD_{Th}$  achieved is selected as the best location for the unit order-up-to levels increase. This algorithm favours that inventory for which the unit

increase in its order-up-to levels provides high SC fill rate and high increment in its own fill rate, but taking into account the holding cost incurred in the SC.

In the next iteration the new set of order-up-to levels that includes  $S_{d,k^{\bar{n}}}^{\bar{n}} + 1$ ,  $k^{\bar{n}} = 1, \dots, K^{\bar{n}}$  is considered. The final order-up-to levels that provide the target  $FR_{Th}$  are denoted by  $S_{dm,k^n}^n$ ,  $n = 1, \dots, N$ ,  $k^n = 1, \dots, K^n$ .

The pseudo-code of the algorithm is given in Appendix 2.

The application of the proposed algorithm for multi-site compensation and simulation of SC performance with multi-site compensation applied are given in Simulation 8.6.

### Simulation 8.6.

The purpose of this simulation is to show that in the case of unreliable external supply, multi-site compensation, i.e., increasing order-up-to levels along an SC, provides better SC performance than in the case when one-site compensation is applied.

Let us consider SC 1 in the uncertain environment, and let us perform multi-site compensation for its *moderately reliable* external supplier. In the first iteration, initial  $S_d^n = 36$  is increased by 1 unit, sequentially, for  $n = 1, \dots, 5$ , generating five possible new sets of the order-up-to levels. Simulation of SC 1 operations over 52 weeks is repeated 100 times for each set of the order-up-to levels. The simulation results obtained in the first iteration are given in Table 8.9.

Table 8.9. Results of the first iteration of the algorithm for multi-site compensation

$n$	$S_{dm}^1$	$S_{dm}^2$	$S_{dm}^3$	$S_{dm}^4$	$S_{dm}^5$	$FR_{Th}$	$\Delta FR_{Th}^n$	$FHD_{Th}$	$MF^n$
1	37	36	36	36	36	0.480	0.010	3.58	0.00134
2	36	37	36	36	36	0.480	0.013	3.67	0.00170
3	36	36	37	36	36	0.476	0.014	3.69	0.00181
4	36	36	36	37	36	0.474	0.016	3.68	0.00206
5	36	36	36	36	37	0.471	0.023	3.68	0.00294

In the first iteration, inventory 5 is selected as the best location to increase the order-up-to level by 1 unit. The new set of the order-up-to levels includes  $S_{dm}^1=36, S_{dm}^2=36, S_{dm}^3=36, S_{dm}^4=36, S_{dm}^5=37$ .

Following iterations recommended different locations for unit increase of the order-up-to levels. The simulation results are presented in Table 8.10.

Table 8.10. The results after applying six iterations  
of the algorithm for multi-site compensation

iteration	selected	order-up-to levels					fill rate
	inventory						
	$\bar{n}$	$S_{dm}^1$	$S_{dm}^2$	$S_{dm}^3$	$S_{dm}^4$	$S_{dm}^5$	$FR_{Th}$
1	5	36	36	36	36	37	0.471
2	5	36	36	36	36	38	0.500
3	4	36	36	36	37	38	0.512
4	1	37	36	36	37	38	0.534
5	2	37	37	36	37	38	0.576
6	3	37	37	37	37	38	0.595



After six iterations, setting the order-up-to levels to  $S_{dm}^1=37$ ,  $S_{dm}^2=37$ ,  $S_{dm}^3=37$ ,  $S_{dm}^4=37$ ,  $S_{dm}^5=38$  ensured the SC 1 fill rate approximately equaled the fill rate achieved in the case of absolutely reliable external supply ( $FR_{Th}^n \approx 0.6$ ). The SC 1 average performance and average performance of each individual inventory obtained using multi-site compensation for uncertain external supply are presented in Table 8.11.

Multi-site compensation for the *moderately* reliable external supplier involved all the inventories along SC 1. It provided better SC 1 performance with respect to the total SC 1 cost and the holding SC 1 cost than one-site compensation applied at the raw material inventory, but it was less cost effective than one-site compensation applied at the end-product inventory. However, if the SC fill rate ( $FR_{Th}$ ) and the SC holding cost per end-product demanded ( $FHD_{Th}$ ) were the only two factors taken into consideration for selecting the best location to increase the order-up-to levels, the end-product inventory would be selected in each iteration. On the other hand, by taking into account the increment in an inventory fill rate achieved by increasing the inventory order-up-to levels ( $\Delta FR_{Th}^n$ ), a different inventory was selected in each iteration, making the stock distribution along the chain more even. Consequently, the additional costs incurred by compensating for uncertainty in external supply were more evenly spread among the members of the SC. Multi-site compensation offered better balance between the fill rates of the inventories along the chain; the differences between the inventory fill rates were smaller, as illustrated in Figure 8.4.

Table 8.11. Average SC I performance under decentralised control,  
when multi-site compensation for uncertainty in external supply is applied

Average SC 1 performance					
total cost $F_{T_h}$	3658.83				
total cost/item $FD_{T_h}$	7.05				
holding cost $FH_{T_h}$	2288.70				
holding cost/item $FHD_{T_h}$	4.42				
fill rate $FR_{T_h}$	0.595				
Average performance of each inventory in SC 1					
inventory $n$	1	2	3	4	5
order-up-to level $S_d^n$	37	37	37	37	38
total cost $F_{T_h}^n$	1752.40	1737.00	860.28	510.43	167.85
total cost/item $FD_{T_h}^n$	3.37	3.37	1.67	0.99	0.33
holding cost $FH_{T_h}^n$	382.26	952.72	508.82	327.46	117.45
holding cost/item $FHD_{T_h}^n$	0.74	1.85	0.99	0.64	0.23
fill rate $FR_{T_h}^n$	0.595	0.647	0.684	0.725	0.773



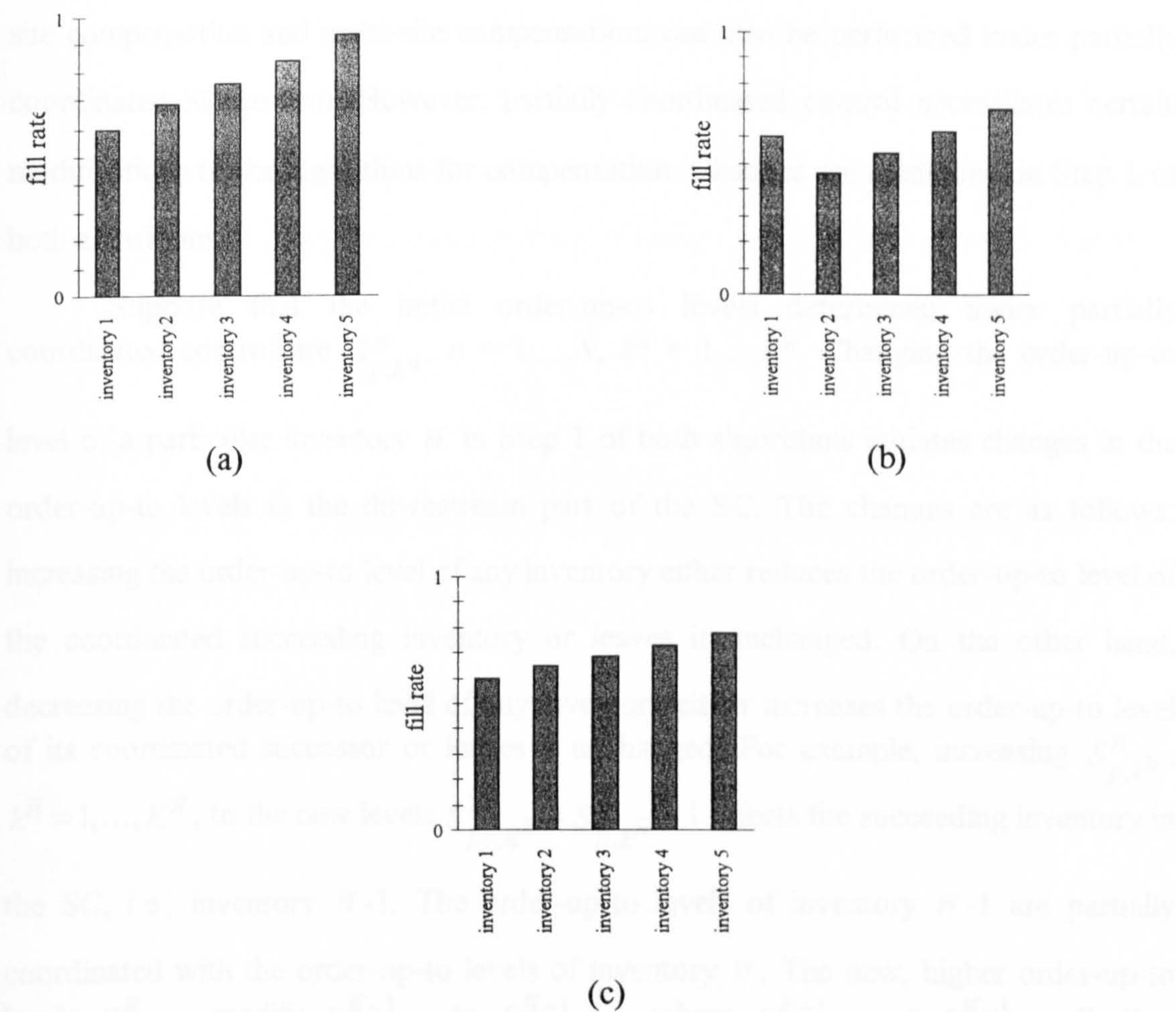


Figure 8.4. Inventory fill rates achieved by applying the algorithms for

- (a) one-site compensation at the raw material,
- (b) one-site compensation at the end-product inventory
- (c) multi-site compensation

8.3. Compensation of uncertainty in external supply under partially coordinated SC control strategy

The negative effects of uncertainty in external supply on a partially coordinated SC can be compensated in the same manner as in the case of the decentrally controlled SC. Both approaches to compensation developed for fully decentralised SC control, one-



site compensation and multi-site compensation, can also be performed under partially coordinated SC control. However, partially coordinated control necessitates certain modifications to the algorithms for compensation. Changes are made only in Step 1 of both algorithms.

Suppose that the initial order-up-to levels determined under partially coordinated control are  $S_{p,k^n}^n$ ,  $n = 1, \dots, N$ ,  $k^n = 1, \dots, K^n$ . Changing the order-up-to level of a particular inventory  $\bar{n}$  in Step 1 of both algorithms initiates changes in the order-up-to levels in the downstream part of the SC. The changes are as follows: increasing the order-up-to level of any inventory either reduces the order-up-to level of the coordinated succeeding inventory or leaves it unchanged. On the other hand, decreasing the order-up-to level of any inventory either increases the order-up-to level of its coordinated successor or leaves it unchanged. For example, increasing  $S_{p,k^{\bar{n}}}^{\bar{n}}$ ,  $k^{\bar{n}} = 1, \dots, K^{\bar{n}}$ , to the new levels  $S_{po,k^{\bar{n}}}^{\bar{n}} = S_{p,k^{\bar{n}}}^{\bar{n}} + 1$  affects the succeeding inventory in the SC, i.e., inventory  $\bar{n}-1$ . The order-up-to levels of inventory  $\bar{n}-1$  are partially coordinated with the order-up-to levels of inventory  $\bar{n}$ . The new, higher order-up-to levels  $S_{po,k^{\bar{n}}}^{\bar{n}}$  modify  $S_{p,k^{\bar{n}-1}}^{\bar{n}-1}$  to  $S_{po,k^{\bar{n}-1}}^{\bar{n}-1}$ , where  $S_{po,k^{\bar{n}-1}}^{\bar{n}-1} \leq S_{p,k^{\bar{n}-1}}^{\bar{n}-1}$ . Further partial coordination involves inventories  $\bar{n}-1$  and  $\bar{n}-2$ , new order-up-to levels of inventory  $\bar{n}-2$  are generated, where  $S_{po,k^{\bar{n}-2}}^{\bar{n}-2} \geq S_{p,k^{\bar{n}-2}}^{\bar{n}-2}$ , and so on, until the end-product inventory is reached. Obviously, if the selected inventory for increasing the order-up-to levels is the end-product inventory, i.e.,  $\bar{n} = 1$ , the order-up-to levels of the rest of the inventories in the SC remain the same, i.e.,  $S_{po,k^n}^n = S_{p,k^n}^n$ ,  $n = 2, \dots, N$ ,  $k^n = 1, \dots, K^n$ .

The pseudo code of the algorithm for one-site compensation under partially coordinated control is given in Appendix 2.

An interesting phenomenon can be observed here: increasing the order-up-to level of one inventory in an SC in one iteration might decrease some of the downstream order-up-to levels, due to the partially coordinated control. Consequently, the new set of order-up-to levels might lead to a SC fill rate that is lower than SC fill

rate achieved in the previous iteration. In other words, increasing the order-up-to level of one inventory under partial coordination does not always guarantee higher SC fill rate. This is clearly illustrated in Simulation 8.7. which also shows that a target SC fill rate can still be achieved by increasing the order-up-to levels in subsequent iterations.

### Simulation 8.7.

The purpose of this simulation is to demonstrate that unreliable external supplier of an SC under partial coordination can be compensated by increasing the stock at one site of the SC.

Let us focus on *SC 1* under partial coordination, specified in Simulation 8.3. Suppose that objective is to compensate for the *moderately reliable* external supplier, increasing the stock at the raw material inventory only.

The order-up-to level of the raw material inventory is increased iteratively by 1 unit, causing changes in all the order-up-to levels along *SC 1*. Table 8.12. contains the results of the iterations of the algorithm for one-site compensation applied to the raw material inventory. The results of each iteration include the new set of the order-up-to levels of the inventories along *SC 1* partially coordinated, and the corresponding average fill rates of the inventories, generated by simulation that is repeated 100 times over 52 weeks.

The order-up-to levels generated after 11 iterations provided the target *SC 1* fill rate, i.e., the same fill rate (equal to 0.900) as was achieved with absolutely reliable external supply.

Examining the results in Table 8.12. one can notice that some increments of the raw material stock do not increase the SC fill rate. For example, the *SC 1* fill rate obtained in iteration 7, when  $S_{po}^5$  is increased to 43, is lower than the fill rate achieved in the previous iteration, when  $S_{po}^5$  is set to 42. This unexpected SC behavior can be explained in the following way. The partial coordination of  $S_{po}^4$  with  $S_{po}^5$  reduced

$S_{po}^4 = 38$  in iteration 7 in comparison with  $S_{po}^4 = 39$  in iteration 6, and the order-up-to levels of the downstream inventories are the same in both iterations. The lower fill rate of inventory 4,  $FR_{T_h}^4$ , recorded in iteration 7, reduced the fill rates of the downstream inventories and the SC *I* fill rate.

Table 8.12. Iterations of the algorithm for one-site compensation applied to SC *I*, under partially coordinated control

iteration	order-up-to levels					inventory fill rates				
	$S_{po}^1$	$S_{po}^2$	$S_{po}^3$	$S_{po}^4$	$S_{po}^5$	$FR_{T_h}^1$	$FR_{T_h}^2$	$FR_{T_h}^3$	$FR_{T_h}^4$	$FR_{T_h}^5$
1	40	40	39	41	37	0.791	0.785	0.768	0.777	0.735
2	40	40	40	40	38	0.811	0.805	0.792	0.776	0.758
3	40	40	40	40	39	0.825	0.821	0.811	0.799	0.785
4	40	40	40	40	40	0.844	0.840	0.831	0.820	0.809
5	40	40	40	39	41	0.854	0.848	0.836	0.823	0.835
6	40	40	40	39	42	0.872	0.868	0.858	0.846	0.858
7	40	40	40	38	43	0.858	0.856	0.848	0.837	0.878
8	40	40	40	38	44	0.873	0.870	0.861	0.851	0.893
9	40	40	40	38	45	0.881	0.881	0.877	0.869	0.914
10	41	39	41	37	46	0.897	0.876	0.895	0.864	0.934
11	41	39	41	37	47	0.900	0.880	0.902	0.874	0.948



#### 8.4. Effects of changing estimation of external supplier reliability

It has been argued in the previous sections that an unreliable external supplier of raw material affects SC behavior and SC performance. Uncertainty in reliability of the external supplier can be expressed by various, imprecise linguistic terms. Generally, the more reliable external supplier means that higher percentages of raw material order will be delivered by the supplier and degrees of belief in delivering high percentages of the ordered quantities are higher. Similar to the case of uncertainty in customer demand, subjectivity is present in the estimation of both the range of percentages of raw material orders delivered and the associated possibilities. Therefore, the sensitivity of SC behavior and its performance to changes in estimation of external supplier reliability has to be examined. Assuming that a possibility distribution used is a good estimation of actual external supply, the same analysis can be used to examine the influence of changing actual external supply of raw material on SC performance. This type of analysis may be useful in selecting an external supplier.

As discussed previously in this chapter, an SC is less vulnerable to uncertainty in external supply in the case of high ratios of the unit costs of inventories or under partially coordinated control. Therefore, the following statement may be made and tested using *SCSUPP*: high ratios of the unit shortage costs to the unit holding costs of inventories along an SC or partially coordinated control of an SC make the SC less sensitive to subjectivity in estimation of external supplier reliability.

*SCSUPP* has been used to examine whether small differences in estimating uncertainty in external supply reliability change SC operations and performance significantly or, in other words, whether SC performance can still be well evaluated, regardless of the small differences that may exist in estimating external supplier reliability. Differences in the estimations of external supplier reliability reflect on the range of percentages of orders delivered by the supplier or on the associated possibilities. The results of Simulation 8.8. and Simulation 8.9. have shown that small

changes in estimation of external supplier reliability cause only small changes in SC performance.

### Simulation 8.8.

The purpose of this simulation is to show that small changes in the description of estimation of external supplier reliability produce only small changes in SC performance: SC fill rate and SC holding cost per end-product demanded.

Let us consider the performance of *SC 1*, but let us suppose that the external supplier is *very reliable*. Different perceptions of the *very reliable* external supplier can be modelled by different fuzzy sets. Let us define four fuzzy sets that can represent the *very reliable* external supplier. The possibility distributions of the four fuzzy sets, depicted in Figure 8.5., are:

Case 1.  $SP = \{0/60, 0.25/70, 0.5/80, 0.75/90, 1/100\}$ ,

Case 2.  $SP = \{0/68, 0.25/76, 0.5/84, 0.75/92, 1/100\}$ ,

Case 3.  $SP = \{0/78, 0.25/82, 0.5/88, 0.75/94, 1/100\}$ ,

Case 4.  $SP = \{0/84, 0.25/88, 0.5/92, 0.75/96, 1/100\}$ .

The fuzzy sets proposed, differ in their ranges only. In all four cases, the possibility of the *very reliable* supplier filling the order fully is equal to 1. Among these cases, Case 4 represents the strictest definition of the *very reliable* supplier, because it is assumed that the *very reliable* supplier delivers more than 84% of raw material ordered.

The average *SC 1* performance obtained by simulating *SC 1* operations over 52 weeks, 100 times, using the four different representations of the *very reliable* external supplier, are similar to each other. The average fill rates of the inventories along *SC 1* and the average holding costs per item demanded, that are incurred at the inventories

during the finite time horizon, are given in Figure 8.6. and Figure 8.7., respectively. Changing the estimation of the *very reliable* external supplier, from the estimate in Case 1 to the estimate in Case 4, increases the average SC 1 fill rate by 6%, and increases the average total SC 1 cost per end-product demanded by 0.9% only.

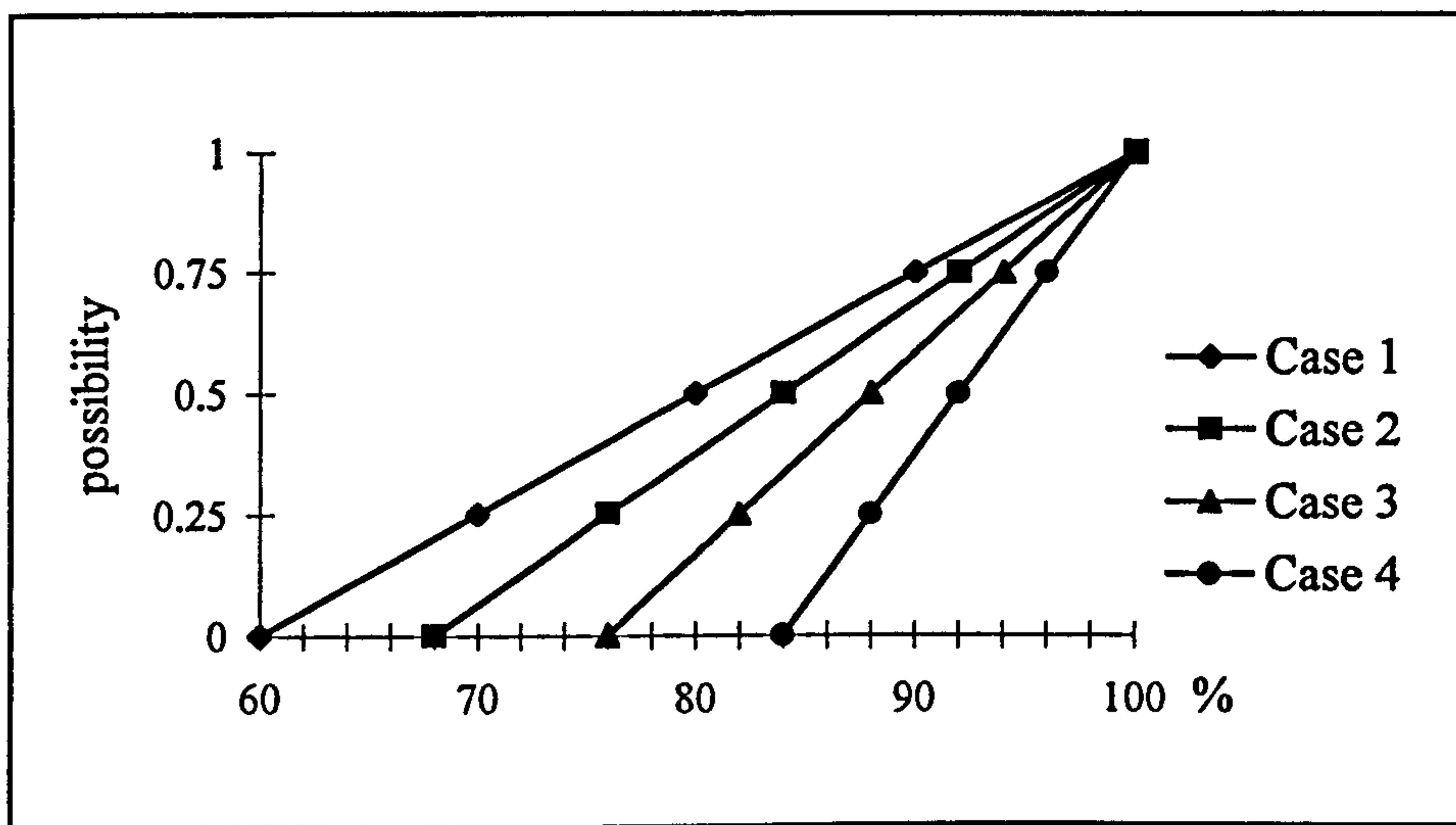


Figure 8.5. Four fuzzy sets to represent the *very reliable* external supplier

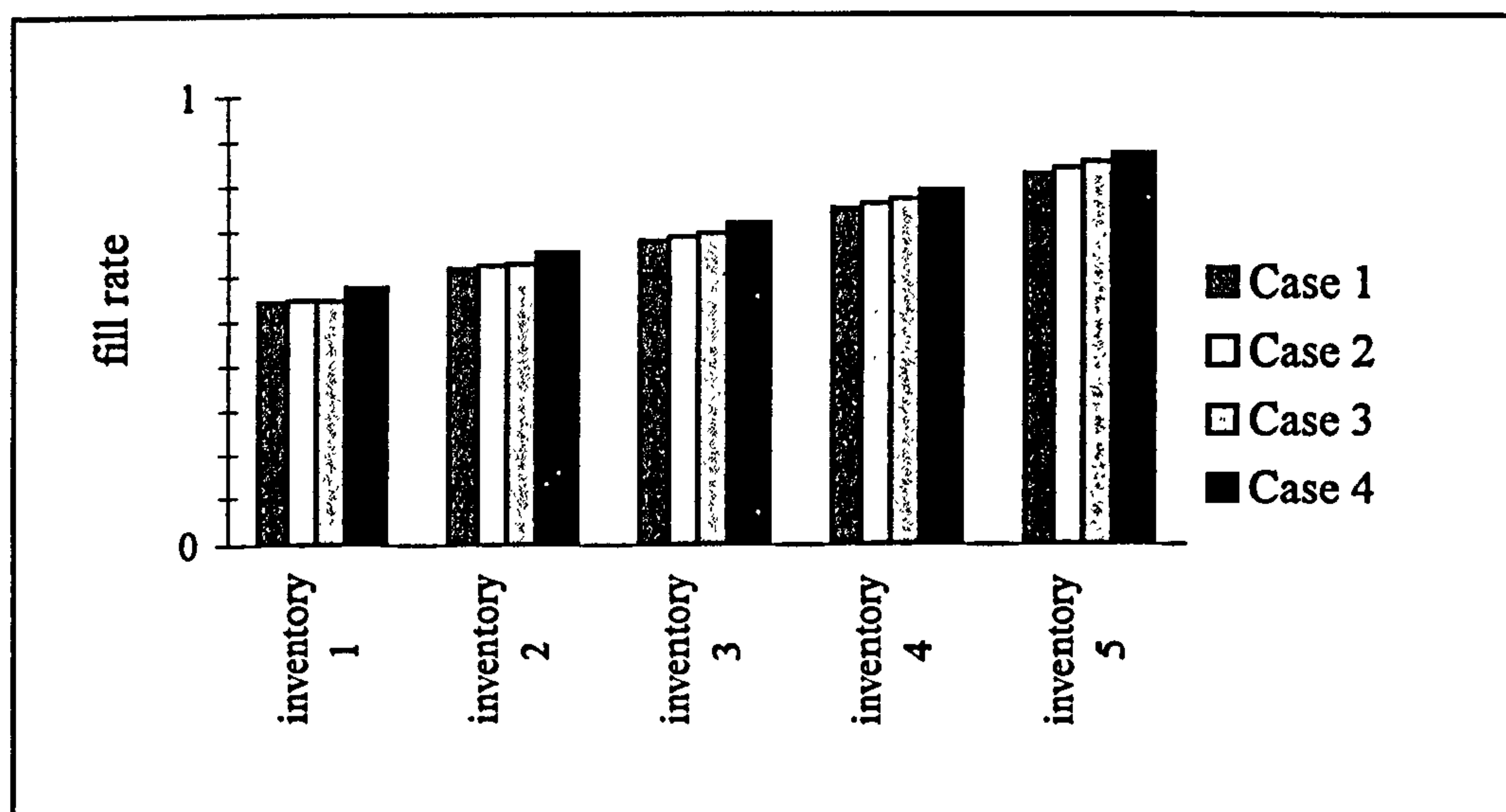




Figure 8.6. Average fill rates of the inventories,  
achieved with the *very reliable* external supplier

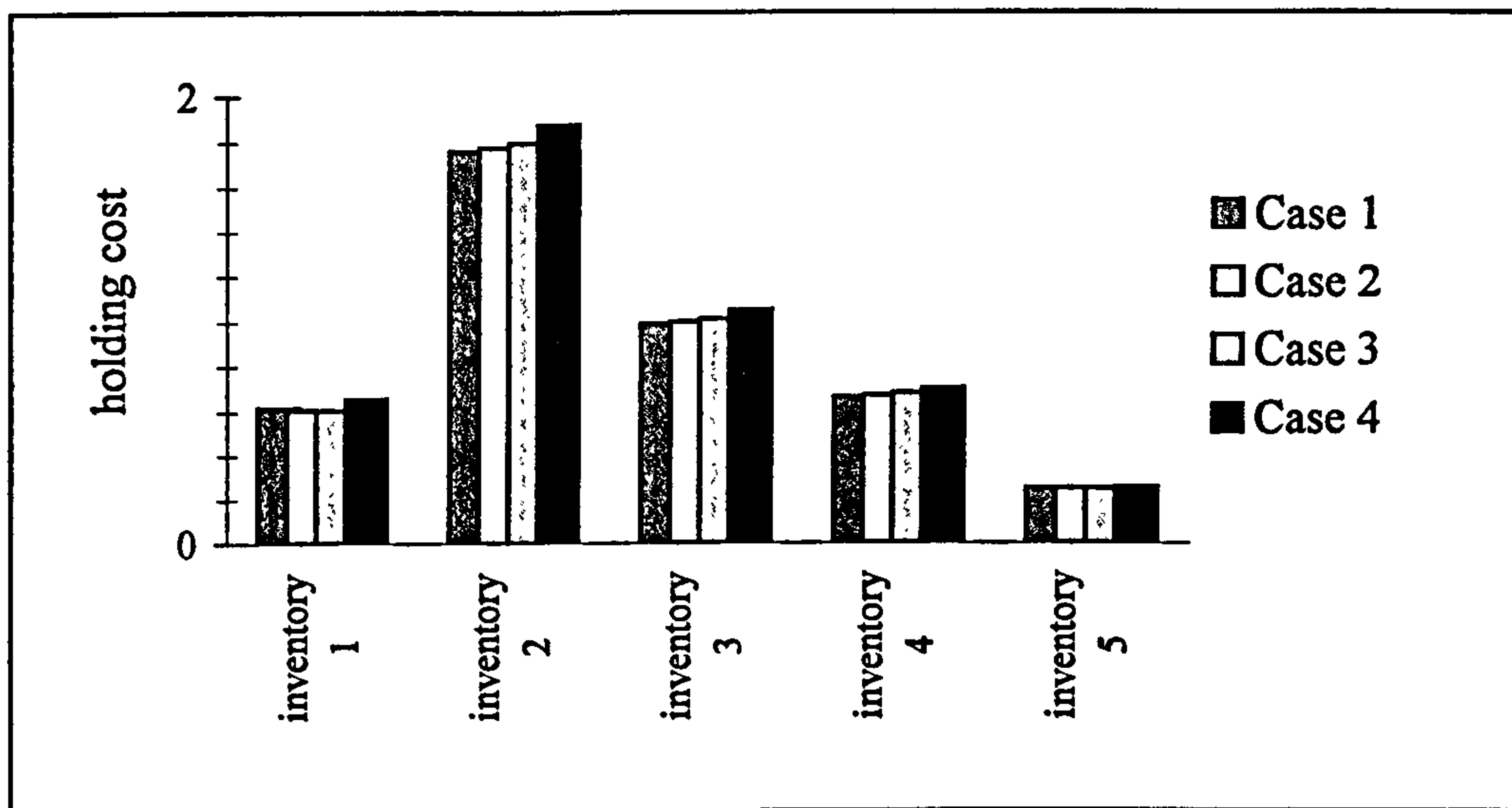


Figure 8.7. Average holding costs of the inventories per item demanded,  
when the external supplier is *very reliable*

### Simulation 8.9.

The purpose of this simulation is to show that small changes around an initial estimation of supplier reliability cause small changes in SC performance.

Consider *SC 1*. The changes in an initial estimation are reflected on the possibilities associated with the percentage of raw material orders, delivered by the supplier. Four cases of an unreliable external supplier are modelled by the following fuzzy sets:

Case 1. Uncertain percentage of ordered raw material, delivered by the supplier is modelled by a discrete fuzzy set

$$SP = \{0/60, 0.25/70, 0.5/80, 0.75/90, 1/100\}.$$

This case represents an initial estimation of uncertainty in external supply.

Case 2. The possibilities of the supplier delivering less than 100% of the quantity ordered, are increased by 5% in comparison with Case 1;

$$SP = \{0/60, 0.26/70, 0.52/80, 0.79/90, 1/100\}.$$

Case 3. The possibilities associated with external supply are increased by 10% in comparison with Case 1;

$$SP = \{0/60, 0.27/70, 0.55/80, 0.82/90, 1/100\}.$$

Case 4. The possibilities associated with external supply are increased by 15% in comparison with Case 1;

$$SP = \{0/60, 0.29/70, 0.57/80, 0.86/90, 1/100\}.$$

The four cases differ in the possibilities of the external supplier delivering less than 100% of raw material ordered, i.e., delivering 70%, 80% and 90% of a raw material order, which means that the external supplier reliability is decreasing, from Case 1 to Case 4. The four possibility distributions of uncertain external supply are graphically presented in Figure 8.8.

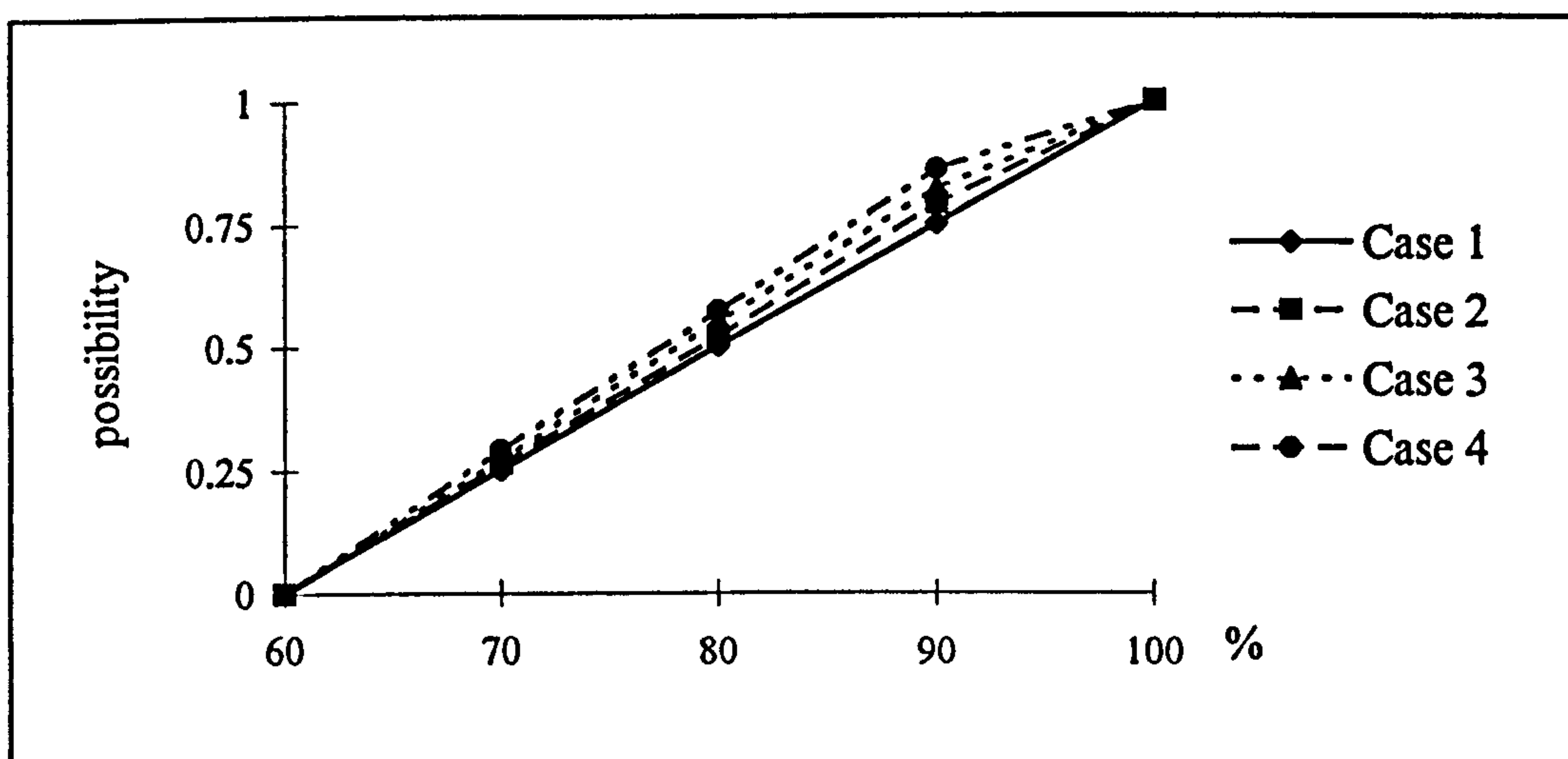


Figure 8.8. Fuzzy sets that model the unreliable external supplier

The *SC 1* performance is measured for each of the four cases of uncertain external supply, by simulating *SC 1* operations over 52 weeks 100 times. The two performance measures, the average fill rate of each inventory and the average holding cost of each inventory per item demanded achieved with the different unreliable supply of raw material are compared in Figure 8.9. and Figure 8.10., respectively. Increasing slightly the possibilities of delivering smaller quantities of raw material than ordered has an insignificant effect on the *SC 1* fill rate. For instance, increasing these possibilities by 5%, 10% and 15%, reduced the *SC 1* fill rate by 1%, 1% and 3%, respectively. Also, the total incurred *SC 1* costs per end-product demanded in the four cases of unreliable external supply are approximately the same.

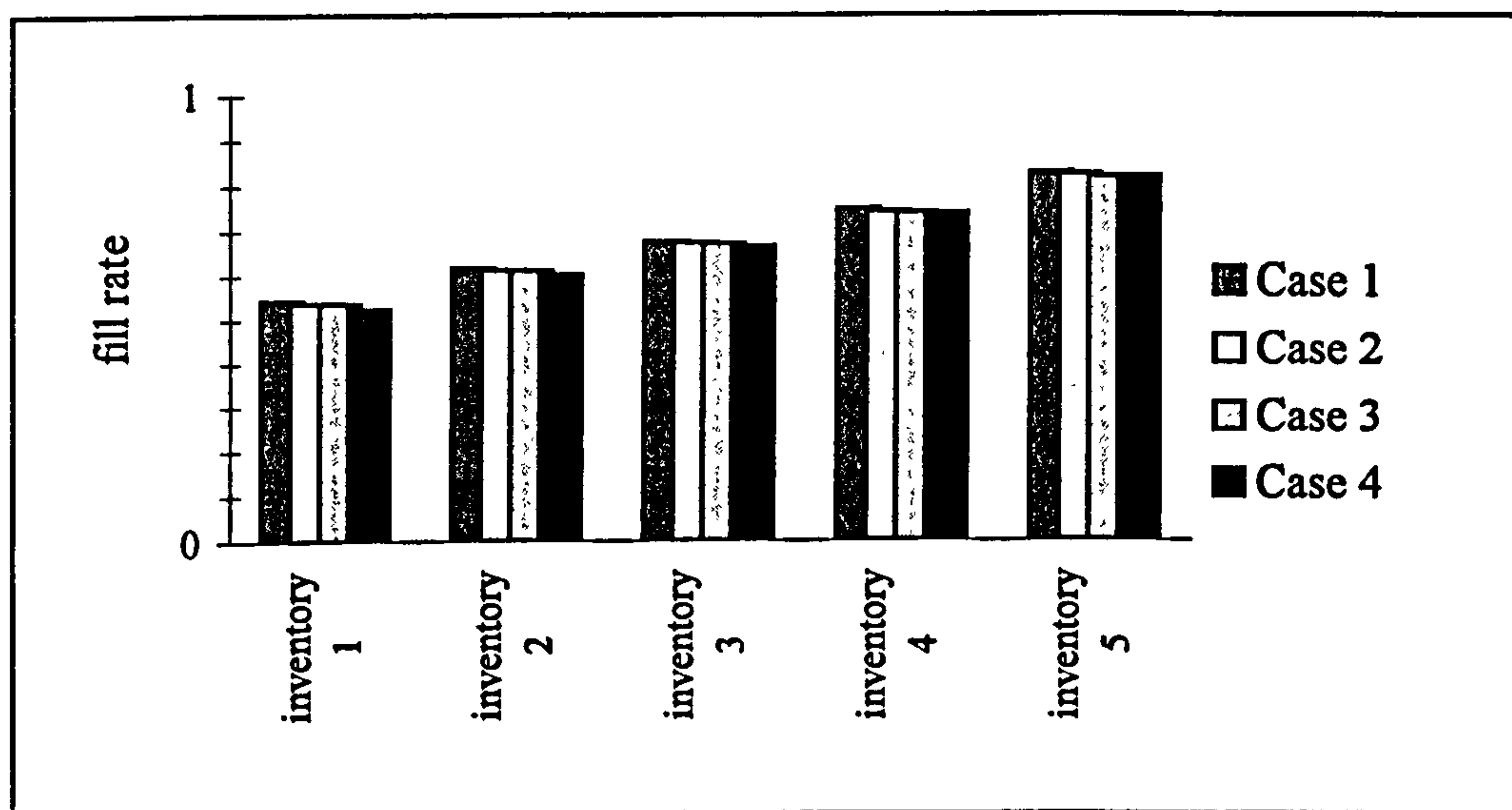


Figure 8.9. The effects of small changes in unreliable external supply on the average fill rates of the inventories



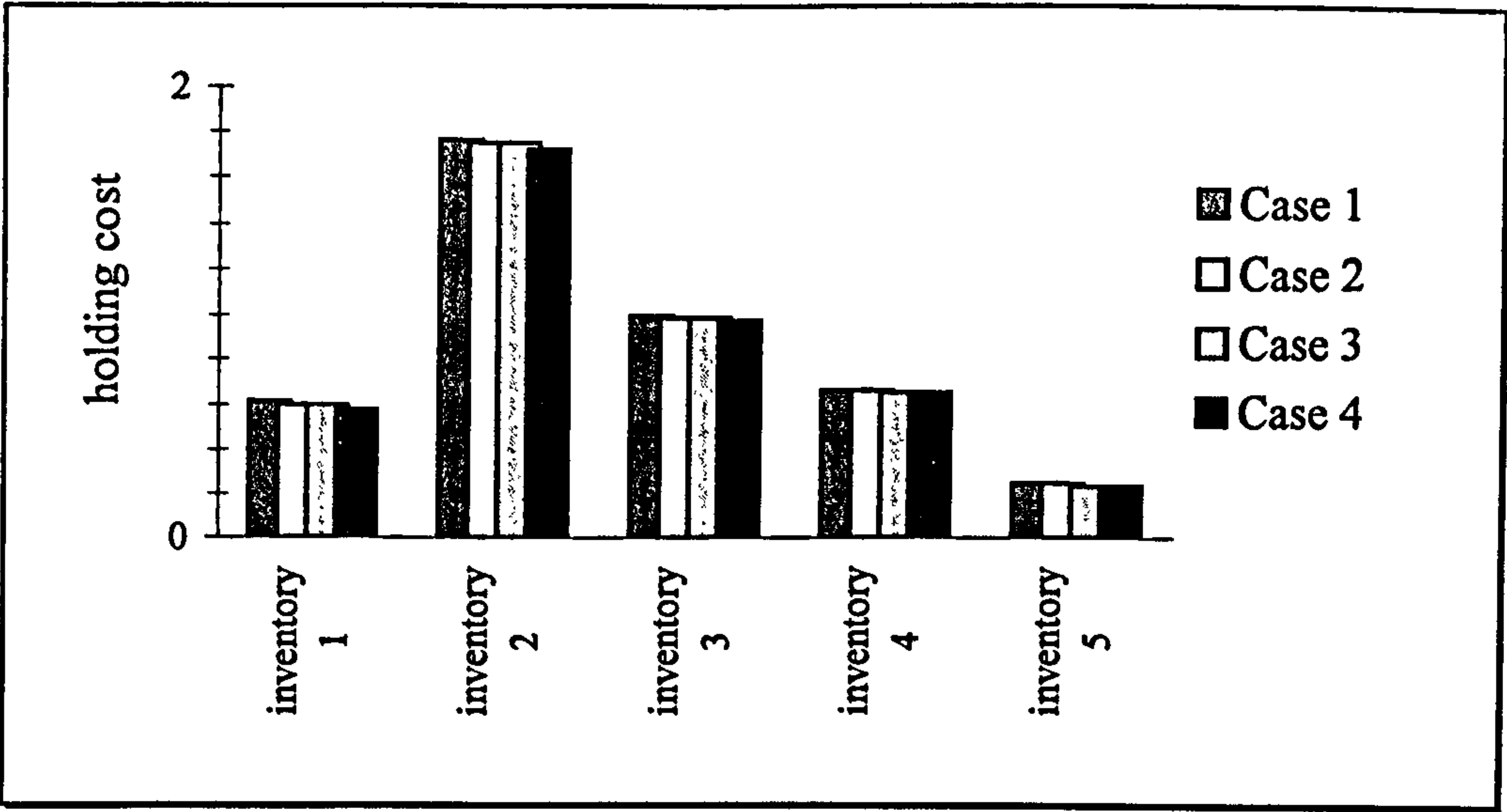


Figure 8.10. The effects of small changes in unreliable external supply on the average holding cost of each inventory per item demanded

## **CHAPTER 9**

### **CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK**

#### **9.1. Conclusions**

Supply chain (SC) modelling is of real importance for effective SC management and control. It contributes to a better understanding of SCs, including their structures, activities, inter-dependencies among SC parts and their relationships with environments in which they operate.

Treating uncertainty is an important issue in SC modelling and analysis of SC dynamic behaviour and performance. The main objectives of this research have been to model and analyse SCs in the presence of uncertainty. Main sources of uncertainty inherent in a serial SC and its environment have been identified, including customer demand, external supply of raw material, supply from one to the succeeding facility in the chain and lead times to the facilities. A new approach to modelling and treating these uncertainties based on fuzzy sets theory has been developed and analysed.

New original SC fuzzy analytical models have been developed to determine order-up-to levels and replenishment quantities which minimise the possible costs over time for all inventories in a serial SC. As the building blocks of the SC fuzzy models, three new fuzzy analytical models for isolated single stocking point inventory control problems have been developed: (1) the first fuzzy model treats uncertainty in customer demand only; (2) the second fuzzy model assumes that uncertainties exist in both customer demand and supply; (3) the third fuzzy model treats uncertainty in customer

demand and uncertainty in a lead time to the inventory, simultaneously. By using these models, order-up-to levels are determined in such a way as to minimise the possible total inventory cost incurred during a finite time horizon.

The fuzzy models for an isolated inventory have been extended to the SC fuzzy models. Two control strategies have been defined and built into the SC fuzzy models: (1) decentralised control and (2) partially coordinated control. Under decentralised control, each inventory is controlled independently taking into account uncertainty in customer and internal demands from one to the preceding facility, or both uncertainty in demands and lead times to the facilities. As an alternative to decentralised control, a new original concept of partially coordinated control has been developed. Under partially coordinated control each inventory is controlled assuming uncertainty in supply along the chain, in addition to uncertainty in demands and lead times. This strategy imposes coordination of each facility in the chain with its immediate supplier.

A family of new SC simulation models has been developed and implemented into an original software package *SCSIM*. The simulation models provide a dynamic view of SC behaviour in the presence of uncertainties described by fuzzy sets. They are linked with the SC fuzzy models in such a way that order-up-to levels determined by the fuzzy models are input data into the *SCSIM*.

Further, different ways of compensating for the negative effects of uncertainty in external supply have been investigated. Two iterative procedures have been developed and tested: (1) one-site compensation, where the order-up-to levels of only one inventory in an SC are increased iteratively, and (2) multi-site compensation, where in each iteration the order-up-to levels of any inventory are increased in such a way as to ensure that the increment in the SC fill rate per cost incurred is the largest.

The following main conclusions have been drawn:

- It has been shown that fuzzy sets theory is an appropriate framework for describing and treating uncertainty in an SC environment, particularly in situations where there is a lack of evidence of the SC parameters, lack of certainty in evidence or



when evidence does not exist. Fuzzy sets have been used to represent different kinds of information imperfection, where natural language expressions are used to describe uncertainty perceived in customer demand, external supply of raw material, supply along a chain and lead times to SC facilities. It has been, further, shown that imprecise SC data can be effectively combined in fuzzy models. In particular, a new approach to modelling fuzzy demand during fuzzy lead time using a level 2 fuzzy set has been proposed, which is less restrictive than the conventional probabilistic approach. The conceptual and computational simplicities of fuzzy sets demonstrated in this thesis are of importance for their applications in considering SC management and control in the presence of uncertainty.

- The new SC fuzzy models developed and applied in this research include and treat simultaneously various sources of uncertainty: (1) in customer and internal demands, (2) in supply along an SC and (3) in lead times to the facilities. A simple optimisation method based on one-dimensional search which operates with fuzzy data has been developed. It has been used for determination of order-up-to levels of all inventories along an SC which minimise the possible total cost.
- The new SC simulation models, developed and tested in this work, provide a dynamic view of SC behaviour and performance over time. They determine SC fill rate and total cost, which cannot be obtained analytically due to the presence of various sources of uncertainty and the complexity of the relations describing SC processes. The simulation models imitate operations of a serial SC in a fuzzy environment. The fuzzy environment is characterised by fuzzy data, such as fuzzy customer demand, fuzzy external supply of raw material and fuzzy lead times to the facilities in the chain. In the simulation models, a theoretical approach to transforming possibility distributions induced by fuzzy sets to the corresponding probability distributions is applied for the first time.

- The two types of models developed, the SC fuzzy analytical and simulation models, have been effectively combined into a powerful tool *SCSIM*. The SC fuzzy models constitute a generative part of the tool, used for determining optimal order-up-to levels and replenishment quantities for inventories in an SC. The SC simulation models constitute an evaluative part, used for assessing SC performance which can be achieved by applying order-up-to levels and replenishment quantities recommended by the fuzzy models. The tool is implemented using object-oriented programming language C++. Modular design of *SCSIM* makes it a flexible tool which enables incorporation of additional models, such as models of various SC configurations including arborescent (distribution) and assembly structures, models of different inventory control strategies, e.g., continuous review, production models, etc.

Development of the SC fuzzy and simulation models is envisaged as a first step towards the development of a decision support system to assist in decision making on operational SC control parameters in an uncertain environment.

- The combined use of SC fuzzy models and SC simulation is a powerful tool which can be used: (1) to gain more insight into SC dynamic behaviour and its performance in an uncertain environment, and (2) to enhance decision making on SC control parameters in order to improve SC performance in an uncertain environment. Various directions in SC analyses have been identified to demonstrate the potential of using the developed tool. The following example applications have been presented:

1. Analysis and comparison of SC control strategies, such as decentralised and partially coordinated control, with respect to SC performance achieved. The advantages of partially coordinated control over decentralised control have been demonstrated by illustrative examples.

2. Analysis and quantification of the effects of changing uncertainty in SC data (e.g., customer demand) on SC behaviour and its performance. This involves selection of an appropriate measure of information imperfection of SC data represented by a fuzzy set. In this research a measure of non-specificity of fuzzy sets has been suggested and used.
3. Analysis of the effects of uncertainty in external supply of raw material and investigation into the ways of making an SC less vulnerable to this uncertainty.
4. Application of the new procedures for one-site and multi-site compensation developed to compensate for the negative effects of uncertainty in external supply. It has been shown that the multi-site approach provides a better compensation with respect to the total SC cost and a good balance between the fill rates of the inventories.

## 9.2. Suggestions for further work

It is recognised that there are still many open questions on both modelling and analyses of SCs in uncertain environments. Several directions for further work can be suggested. They can contribute to both theoretical and practical aspects of SC management and control problems.

The simulation tool developed for SC modelling and analyses enables incorporation of various SC models. In addition to the serial SCs, attention should be paid to other configurations, in particular to arborescent (distribution) and to assembly structures. This requires the development of new models which should reflect specific relationships that exist between facilities in the distribution or assembly structure. These models can then be used as building blocks for an overall SC network model. New strategies for coordination of the facilities in the network need to be developed. Obviously, analysis of SC behaviour in an uncertain environment, effects of spreading



uncertainty along the whole network and ways of reducing negative effects of uncertainty become more complex.

The analyses which have been performed in this research are given for illustrative purposes. Further analyses of SC behaviour and its performance in the presence of different sources of uncertainty are required. This necessitates careful planning and design of simulation experiments which will systematically cover uncertainties in customer demand, external supplier reliability, and lead times. Their simultaneous effects on SC performance need to be analysed.

The fuzzy SC models developed can be modified in several ways. One of the important factors which influence SC operations is limited production capacity. Including limits on production capacity into the fuzzy SC models would bring them nearer to the real world SCs. Problems of coordinated production and inventory control under limited capacity would, however, become more complex and very difficult to solve.

In the current framework for SC modelling and analyses, the simulation models are used either to evaluate the SC performance achieved by setting the order-up-to levels of inventories recommended by the fuzzy SC models, or in the procedures for compensating for the negative effects of uncertainty. An analytical procedure for approximating SC performance would improve and accelerate the analysis of operational SC control parameters to compensate for the negative effects of uncertainty. As one can expect, development of the analytical procedure for approximating SC performance in the presence of uncertainty is a complex problem.

Application of fuzzy models in imitating real SC problems creates a problem of fuzzy sets tuning. Representation of uncertainty in customer demand, external supplier reliability or lead times by means of fuzzy sets should reflect a practitioner's subjective opinion or an aggregation of the opinions of a group of practitioners. Further research is required to investigate the problem of selecting appropriate fuzzy sets.

Finally, to make important steps to include the fuzzy SC models into an existing manufacturing environment, they should be interfaced with some software products in the manufacturing domain, such as Material Requirement Planning (MRP) and Distribution Requirement Planning (DRP) models.

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# APPENDIX 1

## SCSIM

### SCBL simulator

#### general algorithm

read input data

calculate order-up-to levels for the inventories along the SC, depending on the SC control strategy applied

for  $s = 1$  to *number of simulation runs* do

    for time unit  $t = 1$  to  $T$  do

        replenishment phase

        ordering phase

        generate uncertain customer demand

        receiving customer demand

        cost calculation

        print the report on performance of each inventory in the SC

        time increment

    end for

    print the report on the performance achieved during  $T_h$

end for

print the report on the average performance achieved during  $T_h$

replenishment phase

for inventory  $n = N$  to 1 do

if  $repl_t^n = 1$  // beginning of a replenishment period

// quantity received

$$Q_t^n = Q_{t-1}^n + BS_t^{n+1} \quad \text{where } BS_t^{N+1} = 0$$

// inventory level after replenishment

$$I_t^n = I_{t-1}^n + Q_t^n$$

// backlogged quantity ready for delivery

// to the succeeding facility

$$BS_t^n = \begin{cases} BR_{t-1}^n & \text{if } I_t^n > 0 \\ Q_t^n & \text{if } I_t^n \leq 0 \end{cases}$$

// backlogged quantity not available for delivery

// to the succeeding facility

$$BR_t^n = \begin{cases} 0 & \text{if } I_t^n > 0 \\ -I_t^n & \text{if } I_t^n \leq 0 \end{cases}$$

$$repl_{t+1}^n = 0$$

else

$$Q_t^n = Q_{t-1}^n$$

$$I_t^n = I_{t-1}^n$$

$$BS_t^n = BS_{t-1}^n$$

$$BR_t^n = BR_{t-1}^n$$

$$repl_{t+1}^n = repl_t^n$$

end if

end for



ordering phase

for inventory  $n = 1$  to  $N-1$  do

if  $rew_t^n = 0$  //beginning of a review period

// order quantity

$$O_t^n = \max \{ S^n - [I_t^n + BS_t^{n+1} + BR_t^{n+1} - \text{defuzz}(D_{t+1}^n + \dots + D_{t+L^n}^n)], 0 \}$$

// quantity that will be received

$$Q_t^n = \begin{cases} \max(I_t^{n+1}, 0) & \text{if } O_t^n > I_t^{n+1} \\ O_t^n & \text{if } O_t^n \leq I_t^{n+1} \end{cases}$$

$$\text{templ} = I_t^{n+1}$$

// inventory level at the preceding facility

$$I_t^{n+1} = \text{templ} - O_t^n$$

// backlogged quantity not available for delivery

// from the preceding facility

$$BR_t^{n+1} = \begin{cases} BR_t^{n+1} + O_t^n & \text{if } I_t^{n+1} < 0 \text{ and } \text{templ} < 0 \\ BR_t^{n+1} - I_t^{n+1} & \text{if } I_t^{n+1} < 0 \text{ and } \text{templ} \geq 0 \\ BR_t^{n+1} & \text{if } I_t^{n+1} \geq 0 \end{cases}$$

// cumulative number of shortages at the preceding facility

$$NS_t^{n+1} = \begin{cases} NS_{t-1}^{n+1} + O_t^n & \text{if } I_t^{n+1} < 0 \text{ and } \text{templ} < 0 \\ NS_{t-1}^{n+1} - I_t^{n+1} & \text{if } I_t^{n+1} < 0 \text{ and } \text{templ} \geq 0 \\ NS_{t-1}^{n+1} & \text{if } I_t^{n+1} \geq 0 \end{cases}$$

$$\text{repl}_{t+1}^n = 0$$

else

$$NS_t^{n+1} = NS_{t-1}^{n+1}$$

$$O_t^n = 0$$

end if

end for

if  $rew_t^N = 0$  // beginning of a review period of the raw material inventory

$$O_t^N = \max \{ S^N - [I_t^N - \text{defuzz}(D_{t+1}^N + \dots + D_{t+L^N}^N)], 0 \}$$

$$Q_t^N = O_t^N$$

else

$$O_t^N = 0$$

end if

receiving customer demand

$$temple = I_t^1$$

// end - product inventory level

$$I_t^1 = temple - D_t^1$$

// end - product quantity not available for delivery to customers

$$BR_t^1 = \begin{cases} BR_t^1 + D_t^1 & \text{if } I_t^1 < 0 \text{ and } temple < 0 \\ BR_t^1 - I_t^1 & \text{if } I_t^1 < 0 \text{ and } temple \geq 0 \\ BR_t^1 & \text{if } I_t^1 \geq 0 \end{cases}$$

// cumulative number of shortages of the end - products

$$NS_t^1 = \begin{cases} NS_{t-1}^1 + D_t^1 & \text{if } I_t^1 < 0 \text{ and } temple < 0 \\ NS_{t-1}^1 - I_t^1 & \text{if } I_t^1 < 0 \text{ and } temple \geq 0 \\ NS_{t-1}^1 & \text{if } I_t^1 \geq 0 \end{cases}$$

cost calculation

for inventory  $n = 1$  to  $N$  do

// cumulative shortage cost

$$F_{s,t}^n = NS_t^n \cdot c_s^n$$

// cumulative holding cost

$$F_{h,t}^n = \max(I_t^n \cdot c_h^n, 0) + F_{h,t-1}^n$$

// cumulative total cost

$$F_t^n = F_{s,t}^n + F_{h,t}^n$$

end for

time increment

*for inventory  $n = 1$  to  $N$  do*

*// counter of the current time interval within the review period*

$$rew_{t+1}^n = rew_t^n + 1$$

*// beginning of a replenishment period*

$$\text{if } (rew_{t+1}^n = L^n) \quad repl_{t+1}^n = 1$$

*// beginning of a review period*

$$\text{if } (rew_{t+1}^n = R^n) \quad rew_{t+1}^n = 0$$

*end for*

Calculation of performance of each inventory achieved during  $T_h$

*for inventory  $n = 1$  to  $N$  do*

$$F_{T_h}^n = F_T^n \quad // \text{ total cost}$$

$$FD_{T_h}^n = \frac{F_{T_h}^n}{D_{T_h}^n} \quad // \text{ total cost per item demanded}$$

$$FH_{T_h}^n = F_{h,T}^n \quad // \text{ holding cost}$$

$$FHD_{T_h}^n = \frac{FH_{T_h}^n}{D_{T_h}^n} \quad // \text{ holding cost per item demanded}$$

$$FR_{T_h}^n = 1 - \frac{NS_T^n}{D_{T_h}^n} \quad // \text{ fill rate}$$

$$\text{where } D_{T_h}^n = \begin{cases} \sum_{t=1}^T d'_t & n = 1 \\ \sum_{t=1}^T O_t^{n-1} & n = 2, \dots, N \end{cases} \quad // \text{ cumulative demand}$$

*end for*



Calculation of the SC performance achieved during  $T_h$

$$F_{T_h} = \sum_{n=1}^N FH_{T_h}^n + F_{s,T}^1 \quad // \text{ total cost}$$

$$FD_{T_h} = \frac{F_{T_h}}{D_{T_h}^1} \quad // \text{ total cost per end - product demanded}$$

$$FH_{T_h} = \sum_{n=1}^N FH_{T_h}^n \quad // \text{ holding cost}$$

$$FHD_{T_h} = \frac{FH_{T_h}}{D_{T_h}^1} \quad // \text{ holding cost per end - product demanded}$$

$$FR_{T_h} = FR_{T_h}^1 \quad // \text{ fill rate}$$

$$\text{where } D_{T_h}^1 = \sum_{t=1}^T d'_t \quad // \text{ cumulative customer demand}$$

Initial inventory states are specified by

$Q_0^n$  - expected replenishment quantity,

$I_0^n$  - stock level,

$BS_0^n$  - backlogged quantity ready for delivery,

$BR_0^n$  - backlogged quantity not available for delivery,

$rew_1^n$  - counter of time units within the review period,

$repl_1^n$  - indicator of the beginning of a replenishment period .

## SCSUPP simulator

The pseudo-code is the same as for *SCBL* apart from the ordering phase, for  $n=N$ .

### ordering phase

*if*  $rew_t^N = 0$  //beginning of a review period of the raw material inventory

// order quantity

$$O_t^N = \max \{ S^N - [I_t^N - defuzz(D_{t+1}^N + \dots + D_{t+L}^N)], 0 \}$$

generate uncertain percentage of raw material

delivered by the external supplier  $sp'_t$

// quantity that will be received

$$Q_t^N = O_t^N \cdot sp'_t$$

*else*

$$O_t^N = 0$$

*end if*

## ***SCLT* simulator**

### general algorithm

```

read input data
calculate order-up-to levels for the inventories along the SC, depending on the
SC control strategy applied
for  $s = 1$  to number of simulation runs do
    for time unit  $t = 1$  to  $T$  do
        for inventory  $n = 1$  to  $N$  do
            if  $rew_t^n = 0$  then
                generate uncertain lead time  $L^n$  to inventory  $n$ 
            end if
        end for
        replenishment phase
        ordering phase
        generate uncertain customer demand
        receiving customer demand
        cost calculation
        print the report on performance of each inventory in the SC
        time increment
    end for
    print the report on the performance achieved during  $T_h$ 
end for
print the report on the average performance achieved during  $T_h$ 

```



The pseudo-code of all the modules is the same as for *SCBL* apart from the ordering phase. An order quantity  $O_i^n$  is calculated in the ordering phase in the following way.

ordering phase

$$O_i^n = \max \{ S^n - [ I + BS + BR - \text{defuzz} ( s - \text{fuzzif} ( DL ) ) ], 0 \}$$

## APPENDIX 2

### ALGORITHMS FOR COMPENSATION FOR UNCERTAINTY IN EXTERNAL SUPPLY

#### Decentralised control

- Algorithm for one-site compensation

select inventory  $\bar{n}$

for review period  $k^{\bar{n}} = 1$  to  $K^{\bar{n}}$  do

$$S_{do,k^{\bar{n}}}^{\bar{n}} = S_{d,k^{\bar{n}}}^{\bar{n}} \text{ // set the order-up-to levels}$$

end for

do

for review period  $k^{\bar{n}} = 1$  to  $K^{\bar{n}}$  do

// increase the order-up-to levels

$$S_{do,k^{\bar{n}}}^{\bar{n}} = S_{do,k^{\bar{n}}}^{\bar{n}} + 1$$

end for

simulate the SC with the order-up-to levels

$$S_{d,k^n}^n, n = 1, \dots, N, n \neq \bar{n}, k^n = 1, \dots, K^n \text{ and } S_{do,k^{\bar{n}}}^{\bar{n}}, k^{\bar{n}} = 1, \dots, K^{\bar{n}}$$

until the SC fill rate  $FR_{T_h}$  obtained is approximately equal to

the target SC fill rate

the final order-up-to levels are  $S_{d,k^n}^n, n = 1, \dots, N, n \neq \bar{n}, k^n = 1, \dots, K^n$  and

$$S_{do,k^{\bar{n}}}^{\bar{n}}, k^{\bar{n}} = 1, \dots, K^{\bar{n}}$$

- Algorithm for multi-site compensation

for inventory  $n = 1$  to  $N$  do

for review period  $k^n = 1$  to  $K^n$  do

$$S_{dm,k^n}^n = S_{d,k^n}^n \text{ // set the order-up-to levels}$$

end for

end for

do

for inventory  $nn = 1$  to  $N$  do

for  $k^{nn} = 1$  to  $K^{nn}$  do

// increase the order-up-to levels of inventory  $nn$

$$STEMP_{dm,k^{nn}}^{nn} = S_{dm,k^{nn}}^{nn} + 1$$

end for

simulate the SC with the order-up-to levels

$$S_{dm,k^n}^n, n = 1, \dots, N, n \neq nn, k^n = 1, \dots, K^n \text{ and}$$

$$STEMP_{dm,k^{nn}}^{nn}, k^{nn} = 1, \dots, K^{nn} \text{ to obtain:}$$

$$FR_{T_h}, \Delta FR_{T_h}^{nn} \text{ and } FHD_{T_h}$$

$$MF^{nn} = FR_{T_h} \cdot \Delta FR_{T_h}^{nn} / FHD_{T_h}$$

end for

select  $\bar{n}$  for which  $MF^{\bar{n}}$  attains the maximum

for review period  $k^{\bar{n}} = 1$  to  $K^{\bar{n}}$  do

$$S_{dm,k^{\bar{n}}}^{\bar{n}} = STEMP_{dm,k^{\bar{n}}}^{\bar{n}}$$

end for

until the SC fill rate  $FR_{T_h}$  obtained with  $S_{dm,k^n}^n, n = 1, \dots, N, k^n = 1, \dots, K^n$

is approximately equal to the target SC fill rate



the final order-up-to levels are  $S_{dm,k^n}^n, n = 1, \dots, N, k^n = 1, \dots, K^n$

## Partially coordinated control

- Algorithm for one-site compensation

select inventory  $\bar{n}$

for review period  $k^{\bar{n}} = 1$  to  $K^{\bar{n}}$  do

$$S_{po,k^{\bar{n}}}^{\bar{n}} = S_{p,k^{\bar{n}}}^{\bar{n}} \text{ // set the order-up-to levels}$$

end for

do

for review period  $k^{\bar{n}} = 1$  to  $K^{\bar{n}}$  do

// increase the order-up-to levels

$$S_{po,k^{\bar{n}}}^{\bar{n}} = S_{po,k^{\bar{n}}}^{\bar{n}} + 1$$

end for

apply the partial coordination starting from inventory  $\bar{n}$  towards inventory 1 to obtain  $S_{po,k^n}^n, n = \bar{n}, \dots, 1, k^n = 1, \dots, K^n$

simulate the SC with the order-up-to levels

$S_{po,k^n}^n, n = 1, \dots, \bar{n}, k^n = 1, \dots, K^n$  and

$S_{p,k^n}^n, n = \bar{n}, \dots, N, k^n = 1, \dots, K^n$

until  $FR_{T_h}$  is approximately equal to the target SC fill rate

the final order-up-to levels are  $S_{po,k^n}^n, n = 1, \dots, \bar{n}, k^n = 1, \dots, K^n$  and

$S_{p,k^n}^n, n = \bar{n}, \dots, N, k^n = 1, \dots, K^n$